CHAPTER 1
Functions and Their Graphs

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Section 1.1 Functions

- Given a set or an equation, you should be able to determine if it represents a function.
- Given a function, you should be able to do the following.
  (a) Find the domain.
  (b) Evaluate it at specific values.

Solutions to Odd-Numbered Exercises

1. Yes, it does represent a function. Each domain value is matched with only one range value.

3. No, it does not represent a function. The domain values are each matched with three range values.

5. Yes, it does represent a function. Each input value is matched with only one output value.

7. No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.

9. (a) Each element of A is matched with exactly one element of B, so it does represent a function.

(b) The element 1 in A is matched with two elements, -2 and 1 of B, so it does not represent a function.

(c) Each element of A is matched with exactly one element of B, so it does represent a function.

(d) The element 2 of A is not matched to any element of B, so it does not represent a function.

11. Each are functions. For each year there corresponds one and only one circulation.

13. $x^2 + y^2 = 4 \implies y = \pm\sqrt{4 - x^2}$

Thus, y *is not* a function of x. For instance, the values $y = 2$ and -2 both correspond to $x = 0$.

15. $x^2 + y = 4 \implies y = 4 - x^2$

Thus, y *is* a function of x.

17. $2x + 3y = 4 \implies y = \frac{1}{3}(4 - 2x)$

Thus, y *is* a function of x.

19. $y^2 = x^2 - 1 \implies y = \pm\sqrt{x^2 - 1}$

Thus, y *is not* a function of x. For instance, the values $y = \sqrt{3}$ and $-\sqrt{3}$ both correspond to $x = 2$.

21. $y = |4 - x|$ This is a function of x.

23. $x = 4$ does not represent y as a function of x. All values of y correspond to $x = 4$.

25. $f(x) = \dfrac{1}{x + 1}$

(a) $f(4) = \dfrac{1}{(4) + 1} = \dfrac{1}{5}$

(b) $f(0) = \dfrac{1}{(0) + 1} = 1$

(c) $f(4t) = \dfrac{1}{(4t) + 1} = \dfrac{1}{4t + 1}$

(d) $f(x + c) = \dfrac{1}{(x + c) + 1} = \dfrac{1}{x + c + 1}$

27. $f(x) = 2x - 3$

(a) $f(1) = 2(1) - 3 = -1$

(b) $f(-3) = 2(-3) - 3 = -9$

(c) $f(x - 1) = 2(x - 1) - 3 = 2x - 5$

29. $h(t) = t^2 - 2t$

(a) $h(2) = 2^2 - 2(2) = 0$

(b) $h(1.5) = (1.5)^2 - 2(1.5) = -0.75$

(c) $h(x + 2) = (x + 2)^2 - 2(x + 2) = x^2 + 2x$

31. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

33 $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0) = \dfrac{1}{0^2 - 9} = -\dfrac{1}{9}$

(b) $q(3) = \dfrac{1}{3^2 - 9}$ is undefined.

(c) $q(y + 3) = \dfrac{1}{(y + 3)^2 - 9} = \dfrac{1}{y^2 + 6y}$

35. $f(x) = \dfrac{|x|}{x}$

(a) $f(2) = \dfrac{|2|}{2} = 1$

(b) $f(-2) = \dfrac{|-2|}{-2} = -1$

(c) $f(x^2) = \dfrac{|x^2|}{x^2} = 1,\ x \neq 0$

37. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

39. $f(x) = x^2 - 3$

x	-2	-1	0	1	2
$f(x)$	1	-2	-3	-2	1

41. $h(t) = \frac{1}{2}|t + 3|$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

43. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \leq 0 \\ (x - 2)^2, & x > 0 \end{cases}$

x	-2	-1	0	1	2
$f(x)$	5	$\frac{9}{2}$	4	1	0

45. $15 - 3x = 0$

$3x = 15$

$x = 5$

47. $f(x) = \dfrac{3x - 4}{5} = 0$

$3x - 4 = 0$

$3x = 4$

$x = \frac{4}{3}$

49. $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

51. $f(x) = \sqrt{x^2 - 16} = 0$

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$

53. $f(x) = g(x)$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

$x = -1$ or $x = 2$

55. $f(x) = g(x)$

$\sqrt{3x} + 1 = x + 1$

$\sqrt{3x} = x$

$3x = x^2$

$0 = x^2 - 3x$

$0 = x(x - 3)$

$x = 0$ or $x = 3$

57. $f(x) = 5x^2 + 2x - 1$

Since $f(x)$ is a polynomial, the domain is all real numbers x.

59. $h(t) = \frac{4}{t}$

Domain: All real numbers except $t = 0$

61. $g(y) = \sqrt{y - 10}$

Domain: $y - 10 \geq 0$

$y \geq 10$

63. $f(x) = \sqrt[4]{1 - x^2}$

Domain: $1 - x^2 \geq 0$

$x^2 - 1 \leq 0$

$-1 \leq x \leq 1$

65. $g(x) = \frac{1}{x} - \frac{1}{x + 2}$

Domain: All real numbers except

$x = 0,\ x = -2.$

67. $f(s) = \frac{\sqrt{s - 1}}{s - 4}$

Domain: $s - 1 \geq 0$ and $s - 4 \neq 0$. That is, all real numbers $s \geq 1, s \neq 4$.

69. $f(x) = \frac{\sqrt[3]{x - 4}}{x}$. Domain: all $x \neq 0$.

71. $f(x) = x^2$

$\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

73. $f(x) = \sqrt{x + 2}$

$\{(-2, 0), (-1, 1), \left(0, \sqrt{2}\right), \left(1, \sqrt{3}\right), (2, 2)\}$

75. By plotting the points, we have a parabola, so $g(x) = cx^2$. Since $(-4, -32)$ is on the graph, we have $-32 = c(-4)^2 \implies c = -2$. Thus, $g(x) = -2x^2$.

77. Since the function is undefined at 0, we have $r(x) = \frac{c}{x}$. Since $(-8, -4)$ is on the graph, we have $-4 = \frac{c}{-8} \implies c = 32$. Thus, $r(x) = \frac{32}{x}$.

79. $f(x) = 2x$

$f(x + c) = 2(x + c) = 2x + 2c$

$f(x + c) - f(x) = (2x + 2c) - 2x = 2c$

$\frac{f(x + c) - f(x)}{c} = \frac{2c}{c} = 2, c \neq 0$

81. $f(x) = x^2 - x + 1$

$f(2 + h) = (2 + h)^2 - (2 + h) + 1$

$= 4 + 4h + h^2 - 2 - h + 1$

$= h^2 + 3h + 3$

$f(2) = (2)^2 - 2 + 1 = 3$

$f(2 + h) - f(2) = h^2 + 3h$

$\frac{f(2 + h) - f(2)}{h} = h + 3,\ h \neq 0$

83. $f(x) = x^3$

$$f(x + c) = (x + c)^3 = x^3 + 3x^2c + 3xc^2 + c^3$$

$$\frac{f(x + c) - f(x)}{c} = \frac{(x^3 + 3x^2c + 3xc^2 + c^3) - x^3}{c}$$

$$= \frac{c(3x^2 + 3xc + c^2)}{c}$$

$$= 3x^2 + 3xc + c^2, \quad c \neq 0$$

85. $f(t) = \frac{1}{t}$

$f(1) = 1$

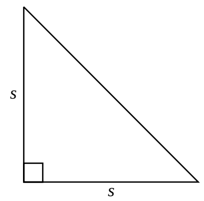
$$\frac{f(t) - f(1)}{t - 1} = \frac{\frac{1}{t} - 1}{t - 1} = \frac{1 - t}{t(t - 1)} = -\frac{1}{t}, \; t \neq 1$$

87. $A = \pi r^2, \quad C = 2\pi r$

$$r = \frac{C}{2\pi}$$

$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$

89. Area $= \text{A} = \frac{1}{2}bh = \frac{1}{2}(s)(s) = \frac{s^2}{2}$

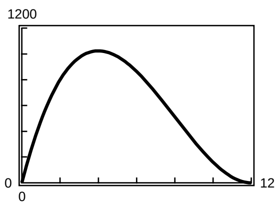


91. (a)

Height, x	Width	Volume, V
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

The volume is maximum when $x = 4$.

(b)



$V = x(24 - 2x)^2$

Domain: $0 < x < 12$

(c) $V(9) = 324$; $V(10) = 160$

(d) $V(9) = 9(24 - 2(9))^2 = 9(36) = 324$

$V(10) = 10(24 - 2(10))^2 = 10(16) = 160$

93. $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}xy.$

Since $(0, y)$, $(2, 1)$ and $(x, 0)$ all lie on the same line, the slopes between any pair of points are equal.

$$\frac{1-y}{2-0} = \frac{1-0}{2-x}$$

$$1 - y = \frac{2}{2-x}$$

$$y = 1 - \frac{2}{2-x} = \frac{x}{x-2}$$

Therefore, $A = \frac{1}{2}xy = \frac{1}{2}x\left(\frac{x}{x-2}\right) = \frac{x^2}{2x-4}$

The domain is $x > 2$, since $A > 0$.

95. **(a)** $V = (\text{length})(\text{width})(\text{height}) = yx^2$

But, $y + 4x = 108$, or $y = 108 - 4x$.

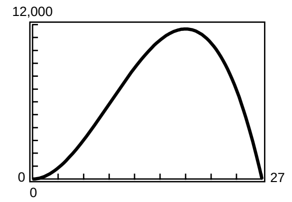
Thus, $V = (108 - 4x)x^2$.

(b) Since $y = 108 - 4x > 0$

$4x < 108$

$x < 27$

Domain: $0 < x < 27$

(c) 

(d) The highest point on the graph occurs at $x = 18$. The dimensions that maximize the volume are $18 \times 18 \times 36$ inches.

97. (a) Cost = variable costs + fixed costs

$C = 12.30x + 98{,}000$

(b) Revenue = price per unit × number of units

$R = 17.98x$

(c) Profit = Revenue − Cost

$P = 17.98x - (12.30x + 98{,}000)$

$P = 5.68x - 98{,}000$

99. (a) $R = (\text{rate})(\text{number of people})$

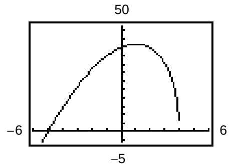
$= [8 - 0.05(n - 80)]n$

$= (12 - 0.05n)n = \dfrac{240n - n^2}{20}$

(b)

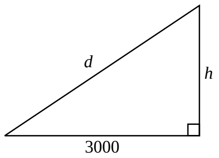
n	90	100	110	120	130	140	150
$R(n)$	675	700	715	720	715	700	675

The revenue increases, and then decreases. The maximum revenue occurs when $n = 120$.

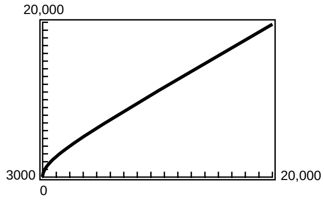
(c) 

The maximum occurs at $n = 120$.

101. (a)



(c)



(b) $3000^2 + h^2 = d^2$

$h^2 = d^2 - 3000^2$

$h = \sqrt{d^2 - 3000^2}, d \geq 3000$

(d) When $d = 10{,}000$, $h \approx 9539.4$ feet.

Algebraically, $h = \sqrt{10{,}000^2 - 3000^2} = \sqrt{91{,}000{,}000}$

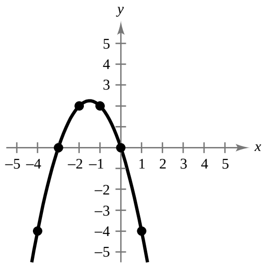
≈ 9539.4 feet

103. False. The range of $f(x)$ is $[-1, \infty)$.

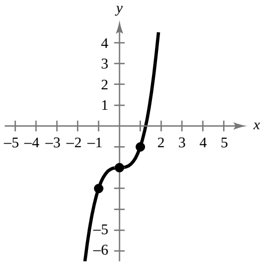
105. No. The element 3 in A has two images in B, u and v.

107. An advantage of function notation is that it gives a name to the relationship so it can easily be referenced. When evaluating a function, you see both the input and output values.

109.



111.



113. Center: $(-8, -5)$

Radius: $\frac{3}{4}$

$$(x - (-8))^2 + (y - (-5))^2 = \left(\frac{3}{4}\right)^2$$

$$(x + 8)^2 + (y + 5)^2 = \frac{9}{16}$$

115. Endpoints of dimeter: $(6, -5), (-2, 7)$

Center: $\left(\frac{6 - 2}{2}, \frac{-5 + 7}{2}\right) = (2, 1)$

Radius: $\sqrt{(6 - 2)^2 + (-5 - 1)^2} = \sqrt{16 + 36}$

$= \sqrt{52}$

$$(x - 2)^2 + (y - 1)^2 = 52$$