Section 1.2 Graphs of Functions

- You should be able to determine the domain and range of a function from its graph.
- You should be able to use the vertical line test for functions.
- You should be able to determine when a function is constant, increasing, or decreasing.
- You should be able to find relative maximum and minimum values of a function.
- You should know that f is
  (a) Odd if $f(-x) = -f(x)$.
  (b) Even if $f(-x) = f(x)$.

Solutions to Odd-Numbered Exercises

1. $g(x) = 1 - x^2$

Domain: All real numbers

Range: $(-\infty, 1]$

3. $f(x) = \sqrt{x^2 - 1}$

Domain: $(-\infty, -1] \cup [1, \infty)$

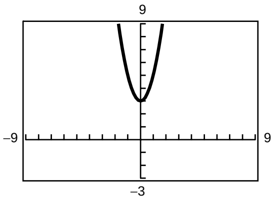
Range: $[0, \infty)$

5. $f(x) = \frac{1}{2}|x - 2|$

Domain: All real numbers

Range: $[0, \infty]$

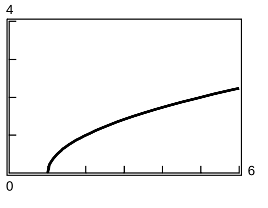
7. $f(x) = 2x^2 + 3$



Domain: All real numbers

Range: $[3, \infty]$

9. $f(x) = \sqrt{x - 1}$



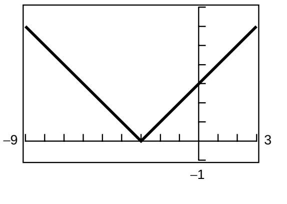
Domain: $x - 1 \geq 0 \Rightarrow x \geq 1$ or $[1, \infty)$

Range: $[0, \infty)$

11. $f(x) = |x + 3|$

Domain: All real numbers

Range: $[0, \infty)$



13. $y = \frac{1}{2}x^2$

A vertical line intersects the graph just once, so y is a function of x.

15. $x - y^2 = 1 \quad \Rightarrow \quad y = \pm\sqrt{x - 1}$

y is not a function of x. Graph $y_1 = \sqrt{x - 1}$ and $y_2 = -\sqrt{x - 1}$.

17. $x^2 = 2xy - 1$

A vertical line intersects the graph just once, so y is a function of x. Solve for y and graph

$$y = \frac{x^2 + 1}{2x}$$

19. $f(x) = \frac{3}{2}x$

(a) f is increasing on $(-\infty, \infty)$.

(b) Since $f(-x) = -f(x)$, f is odd.

21. $f(x) = x^3 - 3x^2 + 2$

(a) f is increasing on $(-\infty, 0)$ and $(2, \infty)$.

f is decreasing on $(0, 2)$.

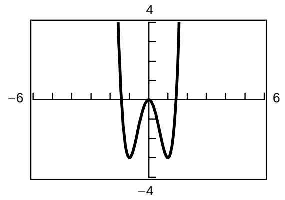
(b) $f(-x) \neq -f(x)$

$f(-x) \neq f(x)$

f is neither odd nor even.

23 . $f(x) = 3x^4 - 6x^2$

(a)



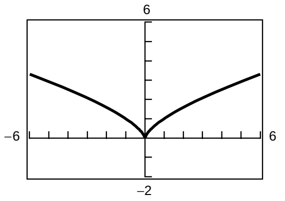
(b) Increasing on $(-1, 0)$ and $(1, \infty)$

Decreasing on $(-\infty, -1)$ and $(0, 1)$

(c) Since $f(-x) = f(x)$, f is even.

25. $f(x) = x^{2/3}$

(a)



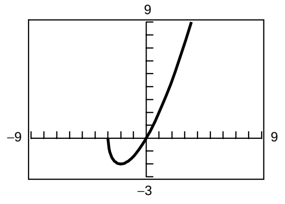
(b) Increasing on $(0, \infty)$

Decreasing on $(-\infty, 0)$

(c) $f(-x) = (-x)^{2/3} = x^{2/3} = f(x) \Longrightarrow$
The function is even.

27. $f(x) = x\sqrt{x + 3}$

(a)



(b) Increasing on $(-2, \infty)$

Decreasing on $(-3, -2)$

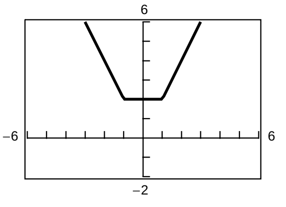
(c) $f(-x) \neq -f(x)$

$f(-x) \neq f(x)$

f is neither odd nor even.

29. $f(x) = |x + 1| + |x - 1|$

(a)



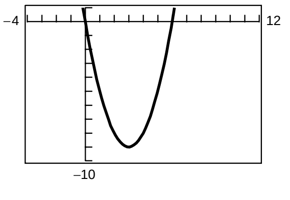
(b) Increasing on $(1, \infty)$, constant on $(-1, 1)$, decreasing on $(-\infty, -1)$

(c)
$$\begin{aligned} f(-x) &= |-x + 1| + |-x - 1| \\ &= |(-1)(x - 1)| + |(-1)(x + 1)| \\ &= |x - 1| + |x + 1| = f(x) \end{aligned}$$

$\Longrightarrow$ The function is even.

31. $f(x) = x^2 - 6x$

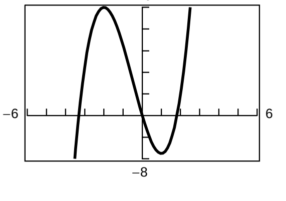
Relative minimum: $(3, -9)$



33. $y = 2x^3 + 3x^2 - 12x$

Relative minimum: $(1, -7)$

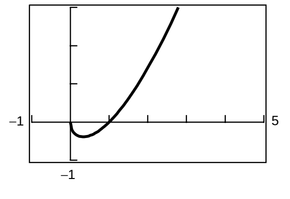
Relative maximum: $(-2, 20)$



35. $h(x) = (x - 1)\sqrt{x}$

Relative minimum: $(0.33, -0.38)$

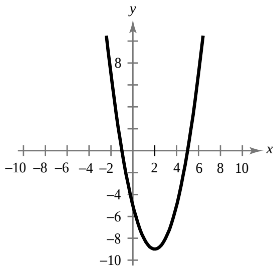
Relative maximum: $(-2, 20)$



((0, 0) is not a relative maximum because it occurs at the endpoint of the domain $[0, \infty)$.)

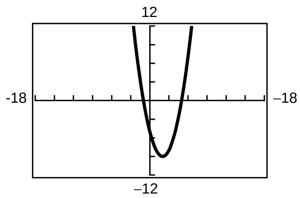
37. $f(x) = x^2 - 4x - 5$

(a)



Minimum: $(2, -9)$

(b)

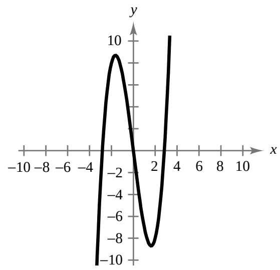


Minimum: $(2, -9)$

(c) Answers are the same

39. $f(x) = x^3 - 8x$

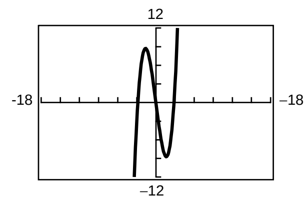
(a)



Maximum: Approximately $(-2, 9)$

Minimum: Approximately $(2, -9)$

(b)



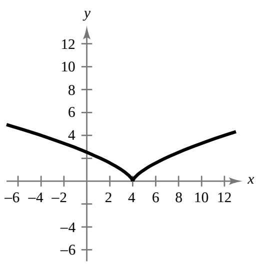
Maximum: $(-1.63, 8.71)$

Minimum: $(1.63, -8.71)$

(c) The answers are similar.

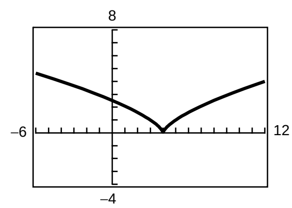
41. $f(x) = (x - 4)^{2/3}$

(a)



Minimum: $(4, 0)$

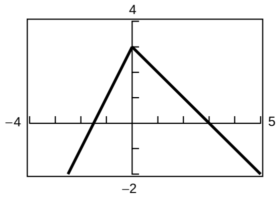
(b)



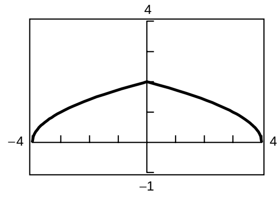
Minimum: $(4, 0)$

(c) The answers are the same.

43. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \geq 0 \end{cases}$



45. $f(x) = \begin{cases} \sqrt{x + 4} & x < 0 \\ \sqrt{4 - x} & x \geq 0 \end{cases}$



47. $f(-t) = (-t)^2 + 2(-t) - 3$

$= t^2 - 2t - 3$

$\neq f(t) \neq -f(t)$

f is neither even nor odd.

49. $g(-x) = (-x)^3 - 5(-x)$

$= -x^3 + 5x$

$= -g(x)$

g is odd.

51. $f(-x) = (-x)\sqrt{1-(-x)^2} = -x\sqrt{1-x^2} = -f(x)$

The function is odd.

53. $g(-s) = 4(-s)^{2/3} = 4s^{2/3} = g(s)$

The function is even.

55. $\left(-\frac{3}{2}, 4\right)$

(a) If f is even, another point is $\left(\frac{3}{2}, 4\right)$.

(b) If f is odd, another point is $\left(\frac{3}{2}, -4\right)$.

57. $(4, 9)$

(a) If f is even, another point is $(-4, 9)$.

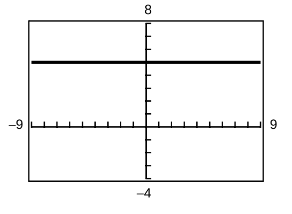
(b) If f is odd, another point is $(-4, -9)$.

59. $(x, -y)$

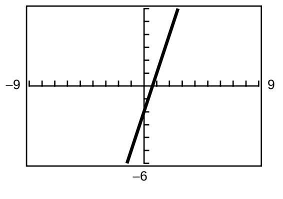
(a) If f is even, another point is $(-x, -y)$.

(b) If f is odd, another point is $(-x, y)$.

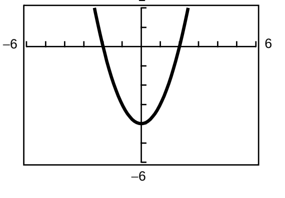
61. $f(x) = 5$, even



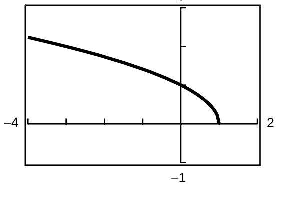
63. $f(x) = 3x - 2$, neither even nor odd



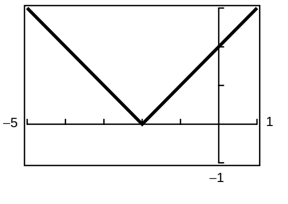
65. $h(x) = x^2 - 4$, even



67. $f(x) = \sqrt{1-x}$, neither even nor odd

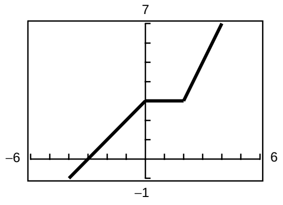


69. $f(x) = |x + 2|$, neither even nor odd



71. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2, \\ 2x - 1, & x > 2 \end{cases}$

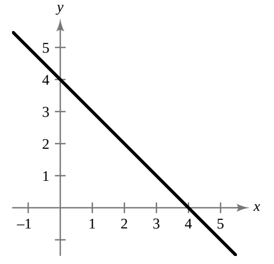
Neither even nor odd



73. $f(x) = 4 - x \ge 0$

$4 \ge x$

$(-\infty, 4]$

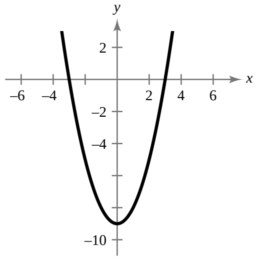


75. $f(x) = x^2 - 9 \ge 0$

$x^2 \ge 9$

$x \ge 3$ or $x \le -3$

$[3, \infty)$ or $(-\infty, -3]$

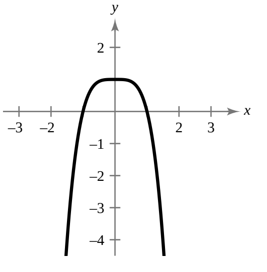


77. $f(x) = 1 - x^4 \ge 0$

$1 \ge x^4$

$-1 \le x \le 1$

$[-1, 1]$

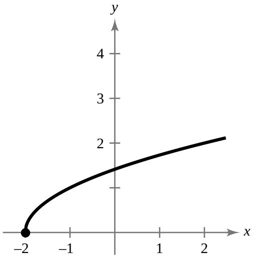


79. $f(x) = \sqrt{x + 2} \ge 0$

$x + 2 \ge 0$

$x \ge -2$

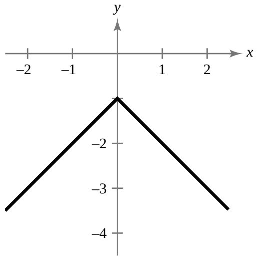
$[-2, \infty)$



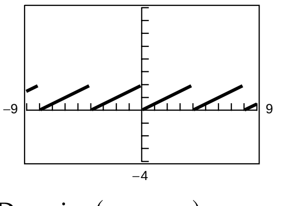
81. $f(x) = -(1 + |x|)$

$f(x)$ is never greater than 0.

$f(x) < 0$ for all x.



83. $s(x) = 2\left(\frac{1}{4}x - \left[\!\left[\frac{1}{4}x\right]\!\right]\right)$



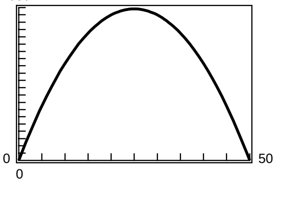
Domain: $(-\infty, \infty)$

Range: $[0, 2)$

Sawtooth pattern

85. (a) Let x and y be the length and width of the rectangle. Then $100 = 2x + 2y$ or $y = 50 - x$. Thus, the area is $A = xy = x(50 - x)$.

(b)

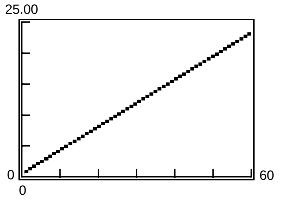


(c) The maximum area is 625 m^2 when $x = y = 25$ m. That is, the rectangle is a square.

87. (a) The second model is correct. For instance,

$$C_2\left(\tfrac{1}{2}\right) = 1.05 - 0.38[[-\left(\tfrac{1}{2} - 1\right)]]$$

$$= 1.05 - 0.38\left[\left[\tfrac{1}{2}\right]\right] = 1.05.$$

(b) 

The cost of an 18-minute 45-second call is

$C_2\left(18\frac{45}{60}\right)$

$= C_2(18.75) = 1.05 - 0.38[[-(18.75 - 1)]]$

$= 1.05 - 0.38[[-17.75]] = 1.05 - 0.38(-18)$

$= 1.05 + 0.38(18) = \$7.89$

89. $h = \text{top} - \text{bottom}$

$= (-x^2 + 4x - 1) - 2$

$= -x^2 + 4x - 3, 1 \le x \le 3$

91. $h = \text{top} - \text{bottom}$

$= (4x - x^2) - 2x$

$= 2x - x^2, 0 \le x \le 2$

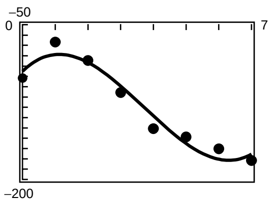
93. $L = \text{right} - \text{left}$

$= \frac{1}{2}y^2 - 0$

$= \frac{1}{2}y^2, 0 \le y \le 4$

95. (a) $y = 1.473x^3 + 16.411x^2 + 31.242x - 95.195$

(b) Domain: $[0, 7]$

(c) 

(d) Most accurate: 1992 $[\text{error} = -84.5 - (-86.57) = 2.071]$

Least accurate: 1991 $[\text{error} = -66.7 - (-78.89) = 12.191]$

(e) Yes, the deficit would decrease as x increases because of the positive x^3 coefficient.

97. False. The domain of $f(x) = \sqrt{x^2}$ is the set of all real numbers.

99. $f(x) = a_{2n+1}x^{2n+1} + a_{2n-1}x^{2n-1} + \cdots + a_3x^3 + a_1x$

$f(-x) = a_{2n+1}(-x)^{2n+1} + a_{2n-1}(-x)^{2n-1} + \cdots + a_3(-x)^3 + a_1(-x)$

$= -a_{2n+1}x^{2n+1} - a_{2n-1}x^{2n-1} - \cdots - a_3x^3 - a_1x = -f(x)$

Therefore, $f(x)$ is odd.