101. f is an even function.

(a) $g(x) = -f(x)$ is even because $g(-x) = -f(-x) = -f(x) = g(x)$.

(b) $g(x) = f(-x)$ is even because $g(-x) = f(-(-x)) = f(x) = f(-x) = g(x)$.

(c) $g(x) = f(x) - 2$ is even because $g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.

(d) $g(x) = -f(x - 2)$ is neither even nor odd because $g(-x) = -f(-x-2) = -f(x+2) \neq g(x)$ nor $-g(x)$.

103. No, $x^2 + y^2 = 25$ does not represent x as a function of y. For instance, $(-3, 4)$ and $(3, 4)$ both lie on the graph.

105. (a) $d = \sqrt{(-5-3)^2 + (0-6)^2} = \sqrt{64+36} = \sqrt{100} = 10$

(b) midpoint $= \left(\dfrac{-5+3}{2}, \dfrac{0+6}{2}\right) = (-1, 3)$

107. (a) $d = \sqrt{\left(-6 - \dfrac{3}{4}\right)^2 + \left(\dfrac{2}{3} - \dfrac{1}{6}\right)^2} = \sqrt{\left(\dfrac{-27}{4}\right)^2 + \left(\dfrac{1}{2}\right)^2} = \dfrac{\sqrt{733}}{4}$

(b) midpoint $= \left(\dfrac{-6 + \frac{3}{4}}{2}, \dfrac{\frac{2}{3} + \frac{1}{6}}{2}\right) = \left(\dfrac{-21}{8}, \dfrac{5}{12}\right)$

109. $f(x) = -x^2 - x + 3$

(a) $f(4) = -(4)^2 - 4 + 3 = -17$

(b) $f(-2) = -(-2)^2 - (-2) + 3 = 1$

(c) $f(x-2) = -(x-2)^2 - (x-2) + 3$
$= -(x^2 - 4x + 4) - x + 2 + 3$
$= -x^2 + 3x + 1$

111. $f(x) = -\frac{1}{2}x|x+1|$

(a) $f(-4) = -\frac{1}{2}(-4)|-4+1| = 2(3) = 6$

(b) $f(10) = -\frac{1}{2}(10)|10+1| = -5(11) = -55$

(c) $f\left(-\frac{2}{3}\right) = -\frac{1}{2}\left(-\frac{2}{3}\right)\left|-\frac{2}{3}+1\right| = \frac{1}{3}\left(\frac{1}{3}\right) = \frac{1}{9}$

113. $f(x) = 5 + 6x - x^2$

$f(6+h) = 5 + 6(6+h) - (6+h)^2 = 5 + 36 + 6h - (36 + 12h + h^2) = -h^2 - 6h + 5$

$f(6) = 5 + 6(6) - 6^2 = 5$

$\dfrac{f(x+6) - f(6)}{h} = \dfrac{(-h^2 - 6h + 5) - 5}{h} = \dfrac{h(-h-6)}{h} = -h - 6, h \neq 0$

Section 1.3 Shifting, Reflecting, and Stretching Graphs

- You should know the graphs of the most commonly used functions in algebra, and be able to reproduce them on your graphing utility.
  (a) Constant function: $f(x) = c$
  (b) Identity function: $f(x) = x$
  (c) Absolute value function: $f(x) = |x|$
  (d) Square root function: $f(x) = \sqrt{x}$
  (e) Squaring function: $f(x) = x^2$
  (f) Cubing function: $f(x) = x^3$

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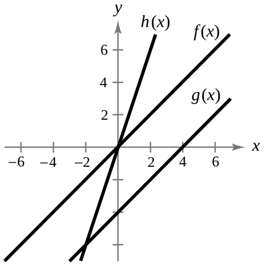
- You should know how the graph of a function is changed by vertical and horizontal shifts.
- You should know how the graph of a function is changed by reflection.
- You should know how the graph of a function is changed by nonrigid transformations, like stretches and shrinks.
- You should know how the graph of a function is changed by a sequence of transformations.

Solutions to Odd-Numbered Exercises

1.



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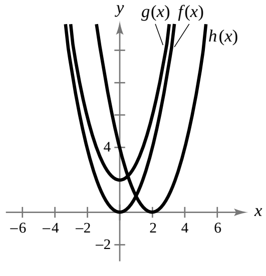

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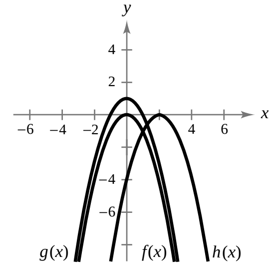

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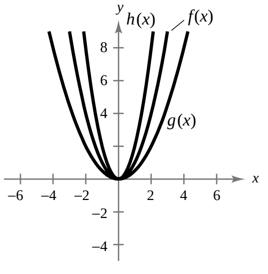

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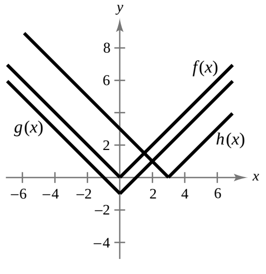
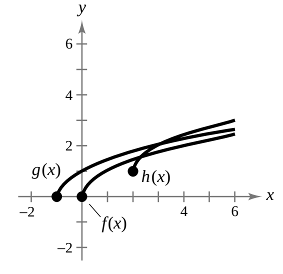
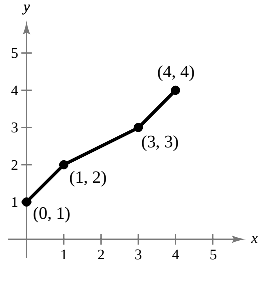


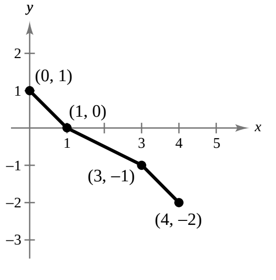
11.



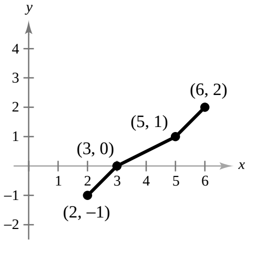
13. (a) $y = f(x) + 2$



(b) $y = -f(x)$



(c) $y = f(x - 2)$



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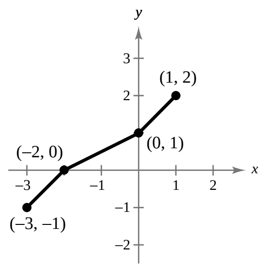
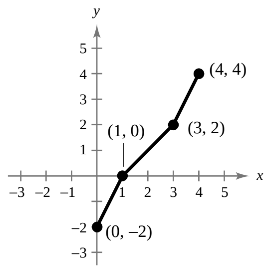
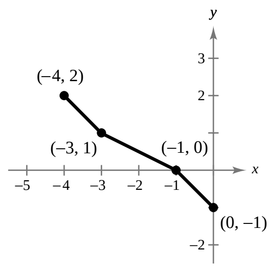
(d) $y = f(x + 3)$



(e) $y = 2f(x)$



(f) $y = f(-x)$



15. Vertical shrink of $y = x$: $y = \frac{1}{2}x$

17. Constant function: $y = 7$

19. Reflection in the x-axis and a vertical shift one unit upward of $y = \sqrt{x}$: $g = 1 - \sqrt{x}$

21. Horizontal shift of $y = |x|$: $y = |x + 2|$

23. Vertical shift one unit downward of $y = x^2$
$y = x^2 - 1$

25. Reflection in the x-axis and a vertical shift one unit upward

$y = 1 - x^3$

27. $y = \sqrt{x} + 2$ is $f(x)$ shifted up two units.

29. $y = \sqrt{x - 2}$ is $f(x)$ shifted right two units.

31. $y = 2\sqrt{x}$ is a vertical stretch of $f(x)$ by 2.

33. $y = |x + 2|$ is $f(x)$ shifted left two units.

35. $y = -|x|$ is $f(x)$ reflected in the x-axis.

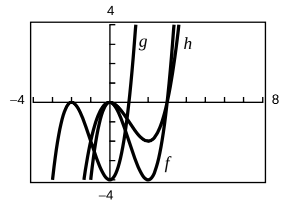
37. $y = \frac{1}{3}|x|$ is a vertical shrink of $f(x)$.

39. $g(x) = 4 - x^3$ is obtained from $f(x)$ by a reflection in the x-axis followed by a vertical shift upward of four units.

41. $h(x) = \frac{1}{4}(x + 2)^3$ is obtained from $f(x)$ by a left shift of two units and a vertical shrink by a factor of $\frac{1}{4}$.

43. $p(x) = \frac{1}{3}x^3 + 2$ is obtained from $f(x)$ by a vertical shrink, followed by a vertical shift of two units upward.

45. $f(x) = x^3 - 3x^2$



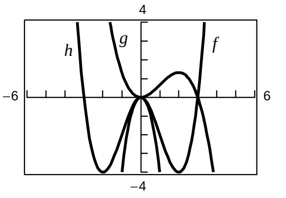
$g(x) = f(x + 2) = (x + 2)^3 - 3(x + 2)^2$ is a horizontal shift 2 units to left

$h(x) = \frac{1}{2}f(x) = \frac{1}{2}(x^3 - 3x^2)$ is a vertical shrink.

47. $f(x) = x^3 - 3x^2$

$g(x) = -\frac{1}{3}f(x) = -\frac{1}{3}(x^3 - 3x^2)$ reflection in the x-axis and vertical shrink

$h(x) = f(-x) = (-x)^3 - 3(-x)^2$ reflection in the y-axis

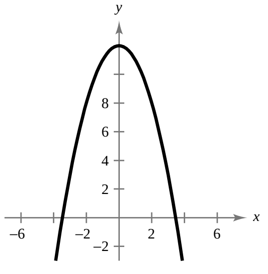


49. The graph of g is obtained from that of f by first negating f, and then shifting vertically one unit upward: $g(x) = -x^3 + 3x^2 + 1$.

51. (a) $f(x) = x^2$

(b) $g(x) = 12 - x^2$ is obtained from f by a reflection in the x-axis followed by a vertical shift upward 12 units.

(c)

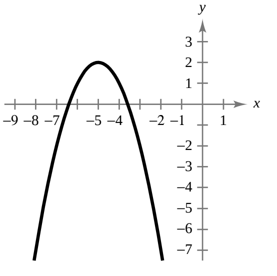


(d) $g(x) = 12 - f(x)$

53. (a) $f(x) = x^2$

(b) $g(x) = 2 - (x + 5)^2$ is obtained from f by a horizontal shift to the left 5 units, a reflection in the x-axis, and a vertical shift upward 2 units.

(c)

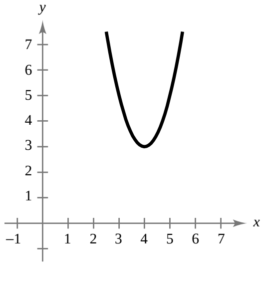


(d) $g(x) = 2 - f(x + 5)$

55. (a) $f(x) = x^2$

(b) $g(x) = 3 + 2(x - 4)^2$ is obtained from f by a horizontal shift 4 units to the right, a vertical stretch of 2, and a vertical shift upward 3 units.

(c)

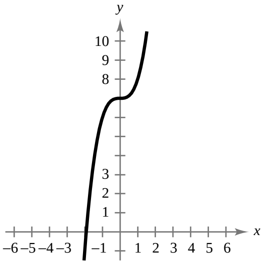


(d) $g(x) = 3 + 2f(x - 4)$

57. (a) $f(x) = x^3$

(b) $g(x) = x^3 + 7$ is obtained from f by a vertical shift upward 7 units.

(c)

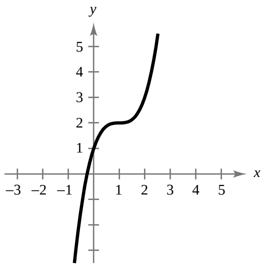


(d) $g(x) = f(x) + 7$

59. (a) $f(x) = x^3$

(b) $g(x) = (x - 1)^3 + 2$ is obtained from f by a horizontal shift 1 unit to the right, and a vertical shift upward 2 units.

(c)

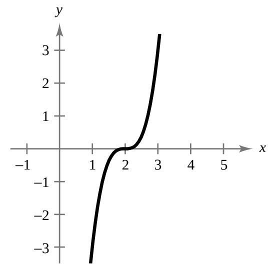


(d) $g(x) = f(x - 1) + 2$

61. (a) $f(x) = x^3$

(b) $g(x) = 3(x - 2)^3$ is obtained from f by a horizontal shift 2 units to the right followed by a vertical stretch of 3.

(c)

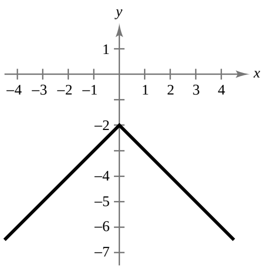


(d) $g(x) = 3f(x - 2)$

63. (a) $f(x) = |x|$

(b) $g(x) = -|x| - 2$ is obtained from f by a reflection in the x-axis, followed by a vertical shift 2 units downward.

(c)

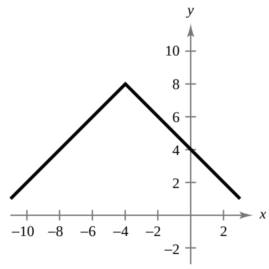


(d) $g(x) = -f(x) - 2$

65. (a) $f(x) = |x|$

(b) $g(x) = -|x + 4| + 8$ is obtained from f by a horizontal shift 4 units to the left, a reflection in the x-axis, followed by a vertical shift 8 units upward.

(c)

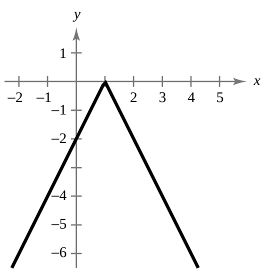


(d) $g(x) = -f(x + 4) + 8$

67. (a) $f(x) = |x|$

(b) $g(x) = -2|x - 1|$ is obtained from f by a horizontal shift 1 unit to the right, a vertical stretch of 2, followed by a reflection in the x-axis.

(c)

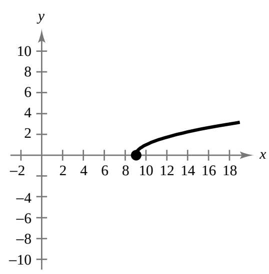


(d) $g(x) = -2f(x - 1)$

69. (a) $f(x) = \sqrt{x}$

(b) $g(x) = \sqrt{x - 9}$ is obtained from f by a horizontal shift 9 units to the right.

(c)

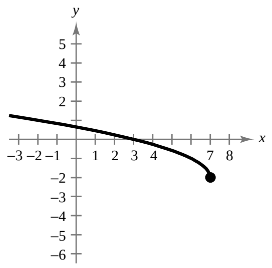


(d) $g(x) = f(x - 9)$

71. (a) $f(x) = \sqrt{x}$

(b) $g(x) = \sqrt{7-x} - 2$ is obtained from f by a reflection in the y-axis, a horizontal shift 7 to the right, followed by a vertical shift 2 units downward. Equivalently, use a horizontal shift left 7 units, a reflection in the y-axis, and a vertical shift down 2 units.

(c)

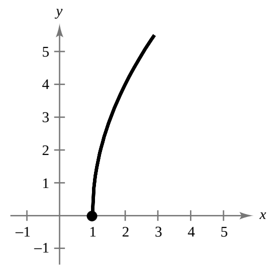


(d) $g(x) = f(7-x) - 2$

73. (a) $f(x) = \sqrt{x}$

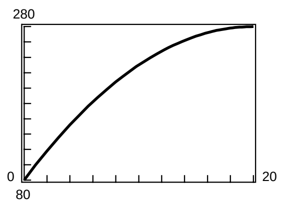
(b) $g(x) = 4\sqrt{x-1}$ is obtained from f by a horizontal shift 1 unit to the right, followed by a vertical stretch of 4 units.

(c)



(d) $g(x) = 4f(x-1)$

75. (a) $P(x) = 80 + 20x - 0.5x^2,\ 0 \le x \le 20$



(b) $P(x)$ is shifted downward by a vertical shift of -2500.

$P(x) = -2420 + 20x - 0.5x^2,\ 0 \le x \le 20$

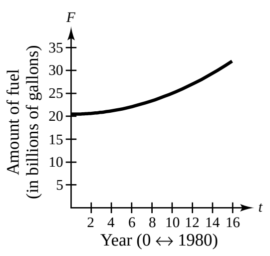
(c) $P(x)$ is changed by a *horizontal stretch.*

$$P(x) = 80 + 20\left(\frac{x}{100}\right) - 0.5\left(\frac{x}{100}\right)^2$$

$$= 80 + 0.2x - 0.00005x^2$$

77. $F(t) = 20.46 + 0.04t^2,\ 0 \le t \le 16,$
$t = 0$ corresponds to 1980

(a) F is obtained from $f(t) = t^2$ by a vertical shrink of 0.04 followed by a vertical shift 20.46 units upward.



(b) $G(t) = F(t+10) = 20.46 + 0.04(t+10)^2,$
$-10 \le t \le 6.$

$G(0) = F(10)$ corresponds to 1990.

79. (a) For each time t there corresponds one and only one temperature T.

(b) $T(4) \approx 60°,\ \ T(15) \approx 72°$

(c) All the temperature changes would be one hour later.

(d) The temperature would be decreased by one degree.