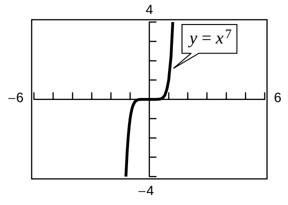
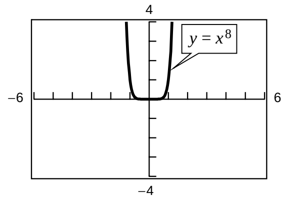
81. False. $f(x) = x^2$ is transformed to $g(x) = -[(x - 6)^2 + 3]$. But, $g(-1) = -52 \neq 28$.

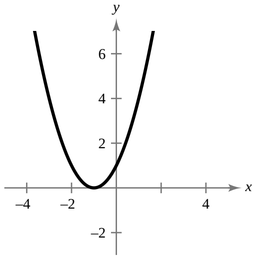
83. $y = x^7$ will look like $y = x^5$, but flatter in $-1 < x < 1$, and steeper for $x < -1$ and $x > 1$.

$y = x^8$ will look like $y = x^6$, but flatter in $-1 < x < 1$, and steeper for $x < -1$ and $x > 1$.

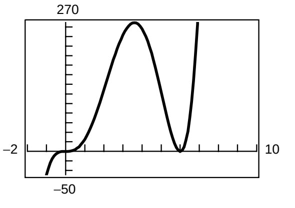




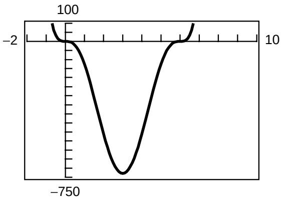
85. $y = (x + 1)^2$



87. $f(x) = x^3(x - 6)^2$



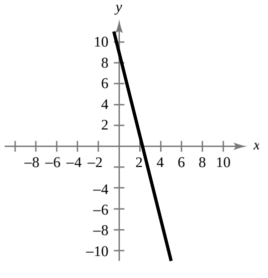
89. $f(x) = x^3(x - 6)^3$



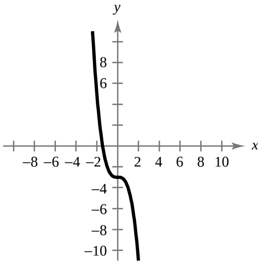
91. $x - 5 \geq 0$ and $x \neq 7 \Longrightarrow$ Domain: $x \geq 5, x \neq 7$

93. Domain: all x

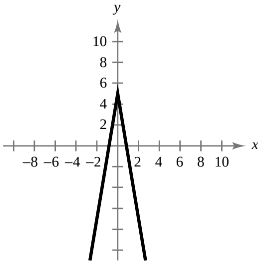
95. $y = 9 - 4x$



97. $y = -x^3 - 3$



99. $y = 5 - 2|3x| = 5 - 6|x|$



Section 1.4 Combinations of Functions

- Given two functions, f and g, you should be able to form the following functions (if defined):
  1. Sum: $(f + g)(x) = f(x) + g(x)$
  2. Difference: $(f - g)(x) = f(x) - g(x)$
  3. Product: $(fg)(x) = f(x)g(x)$
  4. Quotient: $(f/g)(x) = f(x)/g(x), g(x) \neq 0$
  5. Composition of f with g: $(f \circ g)(x) = f(g(x))$
  6. Composition of g with f: $(g \circ f)(x) = g(f(x))$

Solutions to Odd-Numbered Exercises

1. $f(x) = x + 1, g(x) = x - 1$

(a) $(f + g)(x) = f(x) + g(x) = (x + 1) + (x - 1) = 2x$

(b) $(f - g)(x) = f(x) - g(x) = (x + 1) - (x - 1) = 2$

(c) $(fg)(x) = f(x) \cdot g(x) = (x + 1)(x - 1) = x^2 - 1$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x + 1}{x - 1}, \; x \neq 1$

(e) Domain: all $x \neq 1$.

3. $f(x) = x^2, g(x) = 1 - x$

(a) $(f + g)(x) = f(x) + g(x) = x^2 + (1 - x) = x^2 - x + 1$

(b) $(f - g)(x) = f(x) - g(x) = x^2 - (1 - x) = x^2 + x - 1$

(c) $(fg)(x) = f(x) \cdot g(x) = x^2(1 - x) = x^2 - x^3$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{1 - x}, x \neq 1$

(e) Domain: all $x \neq 1$.

5. $f(x) = x^2 + 5, g(x) = \sqrt{1 - x}$

(a) $(f + g)(x) = f(x) + g(x) = (x^2 + 5) + \sqrt{1 - x}$

(b) $(f - g)(x) = f(x) - g(x) = (x^2 + 5) - \sqrt{1 - x}$

(c) $(fg)(x) = f(x) \cdot g(x) = (x^2 + 5)\sqrt{1 - x}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 + 5}{\sqrt{1 - x}}, \; x < 1$

(e) Domain: $x < 1$.

7. $f(x) = \frac{1}{x}, g(x) = \frac{1}{x^2}$

(a) $(f + g)(x) = f(x) + g(x) = \frac{1}{x} + \frac{1}{x^2} = \frac{x + 1}{x^2}$

(b) $(f - g)(x) = f(x) - g(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x - 1}{x^2}$

(c) $(fg)(x) = f(x) \cdot g(x) = \frac{1}{x}\left(\frac{1}{x^2}\right) = \frac{1}{x^3}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{1/x}{1/x^2} = \frac{x^2}{x} = x, \; x \neq 0$

(e) Domain: $x \neq 0$.

9. $(f + g)(3) = f(3) + g(3) = (3^2 + 1) + (3 - 4) = 9$

11. $(f - g)(0) = f(0) - g(0) = [0^2 + 1] - (0 - 4) = 5$

13. $(fg)(4) = f(4)g(4) = (4^2 + 1)(4 - 4) = 0$

15. $\left(\frac{f}{g}\right)(5) = \frac{f(5)}{g(5)} = \frac{5^2 + 1}{5 - 4} = 26$

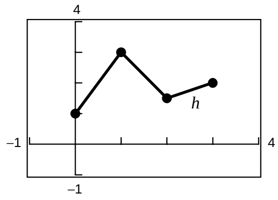
17. $(f - g)(2t) = f(2t) - g(2t) = [(2t)^2 + 1] - (2t - 4) = 4t^2 - 2t + 5$

19. $(fg)(-5t) = f(-5t)g(-5t) = [(-5t)^2 + 1][(-5t) - 4]$

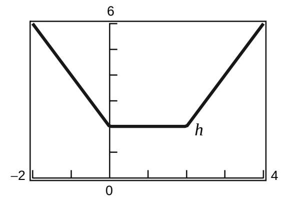
$= (25t^2 + 1)(-5t - 4) = -125t^3 - 100t^2 - 5t - 4$

21. $\left(\frac{f}{g}\right)(-t) = \frac{f(-t)}{g(-t)} = \frac{(-t)^2 + 1}{-t - 4} = \frac{t^2 + 1}{-t - 4}, t \neq -4$

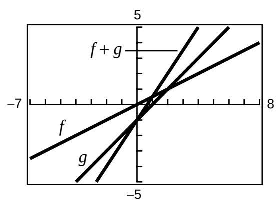
23.



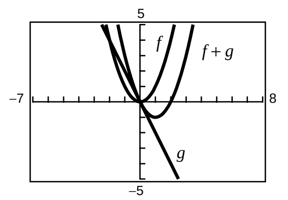
25.



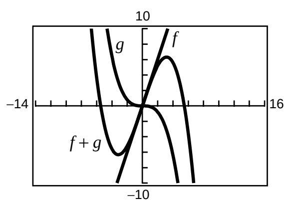
27. $f(x) = \frac{1}{2}x, g(x) = x - 1, (f + g)(x) = \frac{3}{2}x - 1$



29. $f(x) = x^2, g(x) = -2x, (f + g)(x) = x^2 - 2x$

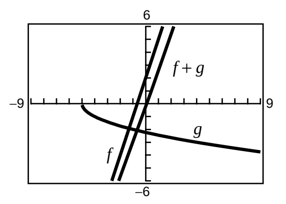


31. $f(x) = 3x, g(x) = -\frac{x^3}{10}, (f + g)(x) = 3x - \frac{x^3}{10}$



For $0 \leq x \leq 2$, $f(x)$ contributes more to the magnitude.
For $x > 6$, $g(x)$ contributes more to the magnitude.

33. $f(x) = 3x + 2, g(x) = -\sqrt{x + 5}, (f + g)(x) = 3x + 2 - \sqrt{x + 5}$



$f(x) = 3x + 2$ contributes more to the magnitude in both intervals.

35. $f(x) = x^2$, $g(x) = x - 1$

(a) $(f \circ g)(x) = f(g(x)) = f(x - 1) = (x - 1)^2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 1$

37. $f(x) = 3x + 5$, $g(x) = 5 - x$

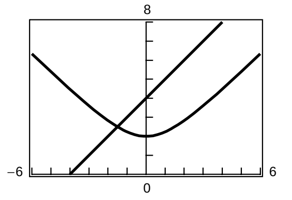
(a) $(f \circ g)(x) = f(g(x)) = f(5 - x) = 3(5 - x) + 5 = 20 - 3x$

(b) $(g \circ f)(x) = g(f(x)) = g(3x + 5) = 5 - (3x + 5) = -3x$

39. (a) $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 + 4}$

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x + 4}\right) = \left(\sqrt{x + 4}\right)^2$

$= x + 4, \ x \geq -4$

(b) 

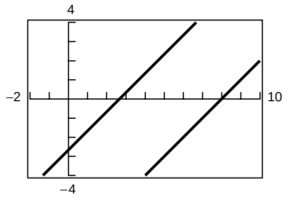
They are not equal.

41. (a) $(f \circ g)(x) = f(g(x)) = f(3x + 1)$

$= \frac{1}{3}(3x + 1) - 3 = x - \frac{8}{3}$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{3}x - 3\right)$

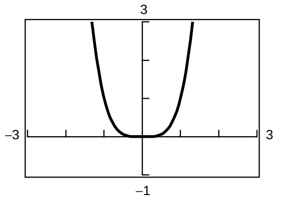
$= 3\left(\frac{1}{3}x - 3\right) + 1 = x - 8$

(b) 

They are not equal.

43. (a) $(f \circ g)(x) = f(g(x)) = f(x^6) = (x^6)^{2/3} = x^4$

$(g \circ f)(x) = g(f(x)) = g(x^{2/3}) = (x^{2/3})^6 = x^4$

(b) 

They are equal.

45. (a) $(f \circ g)(x) = f(g(x)) = f(4 - x) = 5(4 - x) + 4 = 24 - 5x$

$(g \circ f)(x) = g(f(x)) = g(5x + 4) = 4 - (5x + 4) = -5x$

(b) No, $(f \circ g)(x) \neq (g \circ f)(x)$ because $24 - 5x \neq -5x$.

(c)

x	$f(g(x))$	$g(f(x))$
0	24	0
1	19	-5
2	14	-10
3	9	-15

47. (a) $(f \circ g)(x) = f(g(x)) = f(x^2 - 5) = \sqrt{(x^2 - 5) + 6} = \sqrt{x^2 + 1}$

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x + 6}\right) = \left(\sqrt{x + 6}\right)^2 - 5 = (x + 6) - 5 = x + 1,\ x \geq -6$

(b) No, $(f \circ g)(x) \neq (g \circ f)(x)$ because $\sqrt{x^2 + 1} \neq x + 1$

(c)

x	$f(g(x))$	$g(f(x))$
0	1	1
-2	$\sqrt{5}$	-1
3	$\sqrt{10}$	4

49. (a) $(f \circ g)(x) = f(g(x)) = f(2x - 1) = |(2x - 1) + 3| = |2x + 2| = 2|x + 1|$

$(g \circ f)(x) = g(f(x)) = g(|x + 3|) = 2|x + 3| - 1$

(b) No, $(f \circ g)(x) \neq (g \circ f)(x)$ because $2|x + 1| \neq 2|x + 3| - 1$

(c)

x	$f(g(x))$	$g(f(x))$
-1	0	3
0	2	5
1	4	7

51. (a) $(f + g)(3) = f(3) + g(3) = 2 + 1 = 3$

(b) $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{0}{2} = 0$

53. (a) $(f \circ g)(2) = f(g(2)) = f(2) = 0$

(b) $(g \circ f)(2) = g(f(2)) = g(0) = 4$

55. (a) $(f \circ f)(3) = f(f(3)) = f(2) = 0$

(b) $(f \circ f)(4) = f(f(4)) = f(4) = 4$

57. Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$. This is not a unique solution. For example, if $f(x) = (x + 1)^2$ and $g(x) = 2x$, then $(f \circ g)(x) = h(x)$ as well.

59. Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$. This answer is not unique. Other possibilities may be:

$f(x) = \sqrt[3]{x - 4}$ and $g(x) = x^2$ or
$f(x) = \sqrt[3]{-x}$ and $g(x) = 4 - x^2$ or
$f(x) = \sqrt[9]{x}$ and $g(x) = (4 - x^2)^3$

61. Let $f(x) = 1/x$ and $g(x) = x + 2$, then $(f \circ g)(x) = h(x)$. Again, this is not a unique solution. Other possibilities may be:

$f(x) = \dfrac{1}{x + 2}$ and $g(x) = x$

or $f(x) = \dfrac{1}{x + 1}$ and $g(x) = x + 1$

63. Let $f(x) = x^2 + 2x$ and $g(x) = x + 4$. Then $(f \circ g)(x) = h(x)$. (Answer is not unique.)

65. (a) The domain of $f(x) = \sqrt{x}$ is $x \geq 0$.

(b) The domain of $g(x) = x^2 + 1$ is all real numbers.

(c) $(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}$

The domain of $f \circ g$ is all real numbers.

67. (a) The domain of $f(x) = \dfrac{1}{x}$ is all $x \neq 0$.

(b) The domain of $g(x) = x + 3$ is all real numbers.

(c) The domain of $(f \circ g)(x) = f(x + 3) = \dfrac{1}{x + 3}$ is all $x \neq -3$.

69. (a) The domain of $f(x) = \dfrac{2}{|x|}$ is all $x \neq 0$.

(b) The domain of $g(x) = x - 1$ is all real numbers.

(c) The domain of $(f \circ g)(x) = f(x - 1) = \dfrac{2}{|x - 1|}$ is all $x \neq 1$.

71. $f(x) = 3x - 4$

$$\frac{f(x + h) - f(x)}{h} = \frac{[3(x + h) - 4] - (3x - 4)}{h}$$
$$= \frac{3x + 3h - 4 - 3x + 4}{h}$$
$$= \frac{3h}{h}$$
$$= 3, \ h \neq 0$$

73. $f(x) = 1 - x^2$

$$\frac{f(x + h) - f(x)}{h} = \frac{[1 - (x + h)^2] - [1 - x^2]}{h}$$
$$= \frac{1 - x^2 - 2xh - h^2 - 1 + x^2}{h}$$
$$= \frac{-2xh - h^2}{h}$$
$$= \frac{(-2x - h)h}{h} = -2x - h, h \neq 0$$

75. $f(x) = \dfrac{4}{x}$

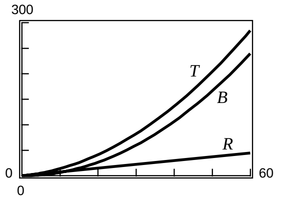
$$\frac{f(x + h) - f(x)}{h} = \frac{\dfrac{4}{x + h} - \dfrac{4}{x}}{h} = \frac{\dfrac{4x - 4(x + h)}{x(x + h)}}{\dfrac{h}{1}}$$
$$= \frac{4x - 4x - 4h}{x(x + h)} \cdot \frac{1}{h}$$
$$= \frac{-4\cancel{h}}{x(x + h)} \cdot \frac{1}{\cancel{h}}$$
$$= \frac{-4}{x(x + h)}, \ h \neq 0$$

77. $f(x) = \sqrt{2x + 1}$

$$\frac{f(x + h) - f(x)}{h} = \frac{\sqrt{2(x + h) + 1} - \sqrt{2x + 1}}{h} \cdot \frac{\sqrt{2(x + h) + 1} + \sqrt{2x + 1}}{\sqrt{2(x + h) + 1} + \sqrt{2x + 1}}$$
$$= \frac{[2(x + h) + 1] - [2x + 1]}{h\left[\sqrt{2x + 2h + 1} + \sqrt{2x + 1}\right]}$$
$$= \frac{2h}{h\left[\sqrt{2x + 2h + 1} + \sqrt{2x + 1}\right]}$$
$$= \frac{2}{\sqrt{2x + 2h + 1} + \sqrt{2x + 1}}, h \neq 0$$

79. (a) $T(x) = R(x) + B(x) = \frac{3}{4}x + \frac{1}{15}x^2$

(b)



(c) $B(x)$ contributes more to $T(x)$ at higher speeds.

81. $y_1 = -0.587t^2 + 7.661t + 144.905$

$y_2 = 16.579t + 245.064$

$y_3 = 1.836t + 21.921$

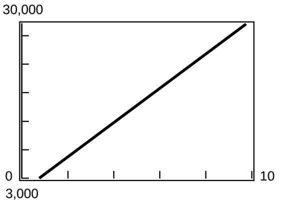
83. $(A \circ r)(t)$ gives the area of the circle as a function of time.

$$\begin{aligned}(A \circ r)(t) &= A(r(t))\\ &= A(0.6t)\\ &= \pi(0.6t)^2 = 0.36\pi t^2\end{aligned}$$

85. (a) $$\begin{aligned}(C \circ x)(t) &= C(x(t))\\ &= 60(50t) + 750\\ &= 3000t + 750\end{aligned}$$

$C \circ x$ represents the cost after t production hours.

(b)

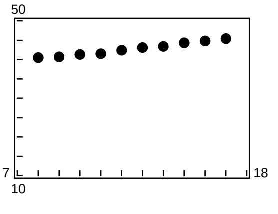


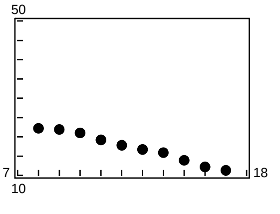
The cost increases to $15,000 when $t = 4.75$ hours.

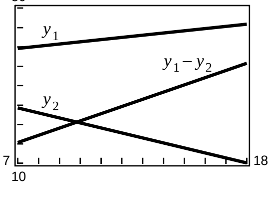
87. $g(f(x)) = g(x - 500{,}000) = 0.03(x - 500{,}000)$

represents 3 percent of the amount over $500,000.

89.







$y_1 = 0.57x + 35.6$

$y_2 = -1.29x + 33.3$

Both data sets appear to be linear.

The difference between circulations is increasing.

91. True. $(f \circ g)(x) = f(g(x))$ is only defined if $g(x)$ is the domain of f.

93. The product of an odd function and an even function is odd. Let $f(x)$ be even, $g(x)$ odd and $h(x) = f(x)g(x)$ their product. Then $h(-x) = f(-x)g(-x) = f(x)(-g(x)) = -f(x)g(x) = -h(x)$. Thus, h is odd.

95. $f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$

$= g(x) \qquad + \quad h(x)$

where g is even and h is odd by Exercise 94.

97. $(0, -5), (1, -5), (2, -7)$ (other answers possible)

99. $\left(\sqrt{24}, 0\right), \left(-\sqrt{24}, 0\right), \left(0, \sqrt{24}\right)$
(other answers possible)

101. $y - (-2) = \dfrac{8 - (-2)}{-3 - (-4)}(x - (-4))$

$y + 2 = 10(x + 4)$

$y - 10x - 38 = 0$

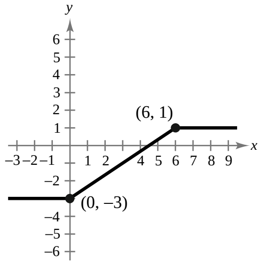
103. $y - (-1) = \dfrac{4 - (-1)}{-\frac{1}{3} - \frac{3}{2}}\left(x - \frac{3}{2}\right)$

$y + 1 = \dfrac{5}{-\frac{11}{6}}\left(x - \frac{3}{2}\right) = -\frac{30}{11}\left(x - \frac{3}{2}\right)$

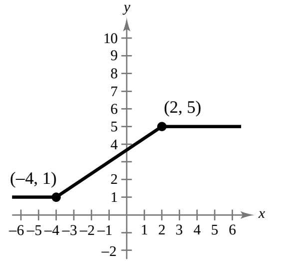
$11y + 11 = -30x + 45$

$30x + 11y - 34 = 0$

105. $f(2) = 1$ and $f(-4) = -3$. Thus, for $g(x) = f(x - 4)$, $g(6) = f(2) = 1$ and $g(0) = f(-4) = -3$.



107. $f(2) = 1$ and $f(-4) = -3$. Thus, $g(x) = f(x) + 4$ satisfies $g(2) = 5$ and $g(-4) = 1$.



109. $f(2) = 1$ and $f(-4) = -3$. Thus, $g(x) = 2f(x)$ satisfies $g(2) = 2$ and $g(-4) = -6$.