91. (a) Yes, f^{-1} exists because for each value of $f(t)$, there corresponds a unique value of t.

(b) f^{-1} indicates the year t corresponds to the total value of new car sales.

(c) $f^{-1}(456.2) = 5$ (or 1995)

(d) No, in this case the function f would not be one-to-one. $f(4) = f(8) = 430.6$.

93. True. If $(0, b)$ is the y-intercept of f, then $(b, 0)$ is the x-intercept of f^{-1}.

95. If f is one-to-one, then f^{-1} exists. If f is odd, then $f(-x) = -f(x)$. Consider $f(x) = y \leftrightarrow f^{-1}(y) = x$. Then $f^{-1}(-y) = f^{-1}(-f(x)) = f^{-1}(f(-x)) = -x = -f^{-1}(y)$. Thus, f^{-1} is odd.

97. $(f + g)(-(-2)) = (f + g)(2) = f(2) + g(2) = 3 + (-1) = 2$

99. $(fg)(-(-3)) = (fg)(3) = f(3)g(3) = (13)(0) = 0$

101. $y = 12x$

$x = 12y$

$y = \frac{1}{12}x$

$f^{-1}(x) = \frac{1}{12}x$

103. $y = x^3 + 7$

$x = y^3 + 7$

$y^3 = x - 7$

$y = \sqrt[3]{x - 7}$

$f^{-1}(x) = \sqrt[3]{x - 7}$

Review Exercises for Chapter 1

Solutions to Odd-Numbered Exercises

1. (a) Not a function. 20 is assigned two different values.

(b) Function

(c) Function

(d) Not a function. No value is assigned to 30.

3. $16x - y^4 = 0$

$y^4 = 16x$

$y = \pm 2\sqrt[4]{x}$

y is **not** a function of x. Some x-values correspond to two y-values.

For example, $x = 1$ corresponds to $y = 2$ and $y = -2$.

5. $y = \sqrt{1 - x}$

Each x value, $x \le 1$, corresponds to only one y value so y **is** a function of x.

7. $f(x) = x^2 + 1$

(a) $f(2) = 2^2 + 1 = 5$

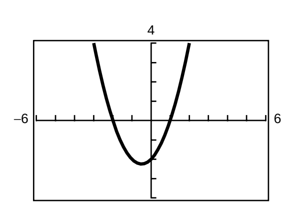
(b) $f(-4) = (-4)^2 + 1 = 17$

(c) $f(t^2) = (t^2)^2 + 1 = t^4 + 1$

(d) $-f(x) = -(x^2 + 1) = -x^2 - 1$

9. $f(x) = (x - 1)(x + 2)$ is defined for all real numbers.

Domain: $(-\infty, \infty)$



11. $f(x) = \sqrt{25 - x^2}$

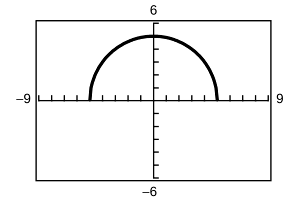
Domain: $25 - x^2 \geq 0$

$(5 + x)(5 - x) \geq 0$

Critical numbers: $x = \pm 5$

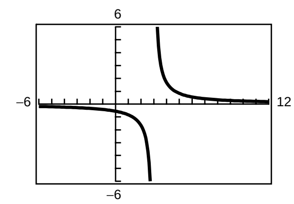
Test intervals: $(-\infty, -5)$, $(-5, 5)$, $(5, \infty)$

Solution set: $[-5, 5]$



13. $g(s) = \dfrac{5}{3s - 9} = \dfrac{5}{3(s - 3)}$

Domain: All real numbers except $s = 3$



15. (a) $C(x) = 16{,}000 + 5.35x$

(b) $P(x) = R(x) - C(x) = 8.20x - [16{,}000 - 5.35x] = 2.85x - 16{,}000$

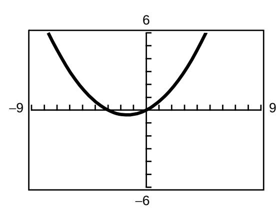
17. Domain: all real numbers

Range: all $y \leq 3$

19. Domain: $36 - x^2 \geq 0 \Longrightarrow x^2 \leq 36 \Longrightarrow -6 \leq x \leq 6$

Range: $0 \leq y \leq 6$

21. (a)

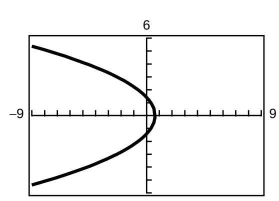


(b) y is a function of x

23. (a) $3x + y^2 = 2$

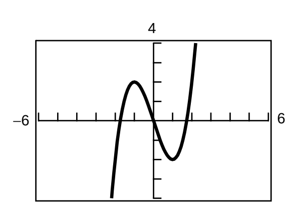
$y^2 = 2 - 3x$

$y = \pm\sqrt{2 - 3x}$



(b) y is not a function of x

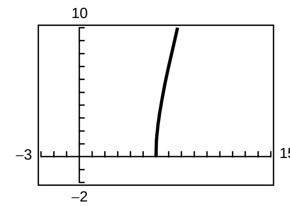
25. $f(x) = x^3 - 3x$



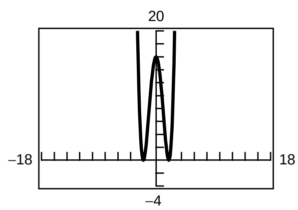
Increasing on $(-\infty, -1)$ and $(1, \infty)$. Decreasing on $(-1, 1)$.

27. $f(x) = x\sqrt{x - 6}$

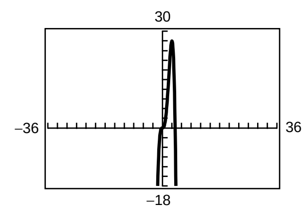
Increasing on $(6, \infty)$



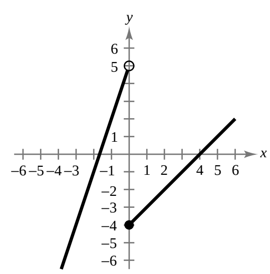
29. $f(x) = (x^2 - 4)^2$. Relative minimums at $(-2, 0)$ and $(2, 0)$. Relative maximum at $(0, 16)$.



31. $h(x) = 4x^3 - x^4$. Relative maximum $(3, 27)$



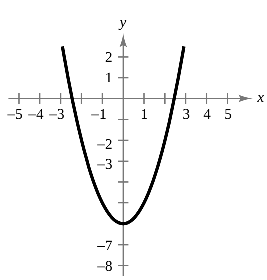
33. $f(x) = \begin{cases} 3x + 5, & x < 0 \\ x - 4, & x \geq 0 \end{cases}$



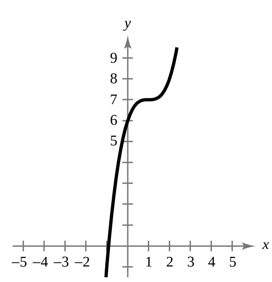
35. $f(-x) = ((-x)^2 - 8)^2 = (x^2 - 8)^2 = f(x)$. f is even.

37. $g(x) = |x| + 3$ is obtained from $f(x) = |x|$ by a vertical shift 3 units upwards. $g(x) = f(x) + 3$

39.



41.



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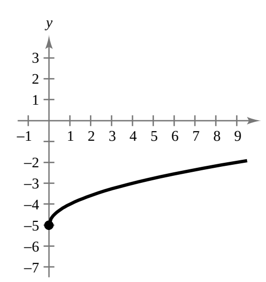

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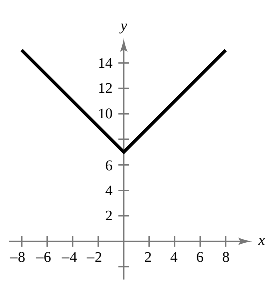

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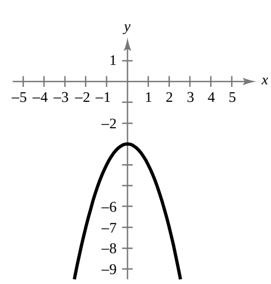

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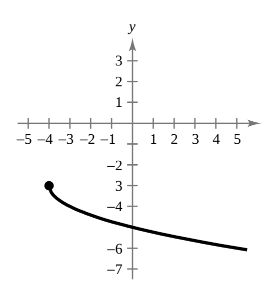
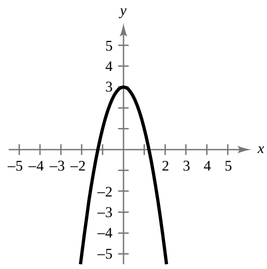
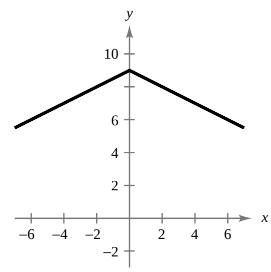


51.



53.



55. $(f - g)(4) = f(4) - g(4)$

$= [3 - 2(4)] - \sqrt{4}$

$= -5 - 2$

$= -7$

57. $(fh)(1) = f(1)h(1) = (3 - 2(1))(3(1)^2 + 2)$

$= (1)(5) = 5$

59. $(h \circ g)(7) = h(g(7))$

$= h(\sqrt{7})$

$= 3(\sqrt{7})^2 + 2$

$= 23$

61. $y_1 = 0.380t^2 + 3.754t + 16.896$

$y_2 = 0.146t^2 + 0.302t + 23.231$

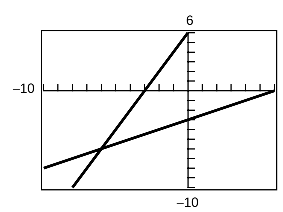
63. $f(x) = 6x \Longrightarrow f^{-1}(x) = \frac{1}{6}x$

65. $f(x) = x - 7 \Longrightarrow f^{-1}(x) = x + 7$

67. (a)

$$f(x) = \tfrac{1}{2}x - 3$$
$$y = \tfrac{1}{2}x - 3$$
$$x = \tfrac{1}{2}y - 3$$
$$x + 3 = \tfrac{1}{2}y$$
$$2(x + 3) = y$$
$$f^{-1}(x) = 2x + 6$$

(b)



(c)

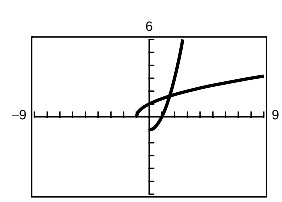
$$f^{-1}(f(x)) = f^{-1}\left(\tfrac{1}{2}x - 3\right)$$
$$= 2\left(\tfrac{1}{2}x - 3\right) + 6$$
$$= x - 6 + 6$$
$$= x$$
$$f(f^{-1}(x)) = f(2x + 6)$$
$$= \tfrac{1}{2}(2x + 6) - 3$$
$$= x + 3 - 3$$
$$= x$$

69. (a)

$$f(x) = \sqrt{x + 1}$$
$$y = \sqrt{x + 1}$$
$$x = \sqrt{y + 1}$$
$$x^2 = y + 1,\ x \ge 0$$
$$x^2 - 1 = y$$
$$f^{-1}(x) = x^2 - 1,\ x \ge 0$$

Note: The inverse must have a restricted domain.

(b)



(c)

$$f^{-1}(f(x)) = f^{-1}\left(\sqrt{x + 1}\right)$$
$$= \left(\sqrt{x + 1}\right)^2 - 1$$
$$= x + 1 - 2$$
$$= x$$
$$f^{-1} = f(x^2 - 1)$$
$$= \sqrt{(x^2 - 1) + 1}$$
$$= \sqrt{x^2} = x \text{ for } x \ge 0$$

71.
$$f(x) = \frac{x}{12}$$
$$y = \frac{x}{12}$$
$$x = \frac{y}{12}$$
$$12x = y$$
$$f^{-1}(x) = 12x$$

73.
$$f(x) = 4x^3 - 3$$
$$y = 4x^3 - 3$$
$$x = 4y^3 - 3$$
$$x + 3 = 4y^3$$
$$\frac{x+3}{4} = y^3$$
$$f^{-1}(x) = \sqrt[3]{\frac{x+3}{4}}$$

75.
$$f(x) = \sqrt{x + 10}$$
$$y = \sqrt{x + 10}, x \geq -10, y \geq 0$$
$$x = \sqrt{y + 10}, y \geq -10, x \geq 0$$
$$x^2 = y + 10$$
$$x^2 - 10 = y$$
$$f^{-1}(x) = x^2 - 10, x \geq 0$$

77. True. $f^{-1}(x) = x^{1/n}$, n odd

79. The vertical line $x = c$ is not a function because it does not pass the Vertical Line Test. All other lines are functions.