111. $2x^2 - x \geq 1$

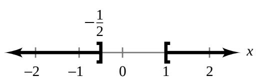
$2x^2 - x - 1 \geq 0$

$(2x + 1)(x - 1) \geq 0$

$[2x + 1 \geq 0 \text{ and } x - 1 \geq 0]$ or $[2x + 1 \leq 0 \text{ and } x - 1 \leq 0]$

$\left[x \geq -\frac{1}{2} \text{ and } x \geq 1\right]$ or $\left[x \leq -\frac{1}{2} \text{ and } x \leq 1\right]$

$x \geq 1$ or $x \leq -\frac{1}{2}$

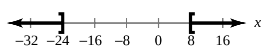


113. $|x + 8| - 1 \geq 15$

$|x + 8| \geq 16$

$x + 8 \geq 16$ or $x + 8 \leq -16$

$x \geq 8$ or $x \leq -24$



115. Vertex: $(0, -8)$

$f(x) = a(x - 0)^2 - 8 = ax^2 - 8$

Point: $(5, 9) \implies 9 = a(5)^2 - 8$

$17 = 25a$

$a = \frac{17}{25}$

$f(x) = \frac{17}{25}x^2 - 8$

117. Vertex: $(-5, -2)$

$f(x) = a(x + 5)^2 - 2$

Point: $(0, 3) \implies 3 = a(0 + 5)^2 - 2$

$5 = 25a$

$a = \frac{1}{5}$

$f(x) = \frac{1}{5}(x + 5)^2 - 2$

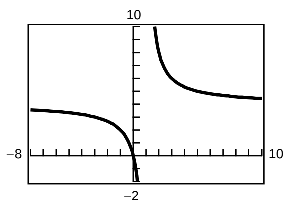
Section 2.3 Real Zeros of Polynomial Functions

You should know the following basic techniques and principles of polynomial division.

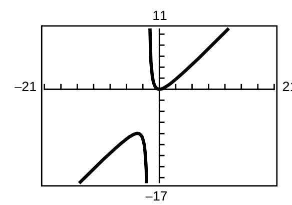
- The Division Algorithm (Long Division of Polynomials)
- Synthetic Division
- $f(k)$ is equal to the remainder of $f(x)$ divided by $(x - k)$.
- $f(k) = 0$ if and only if $(x - k)$ is a factor of $f(x)$.
- The Rational Zero Test
- The Upper and Lower Bound Rule

Solutions to Odd-Numbered Exercises

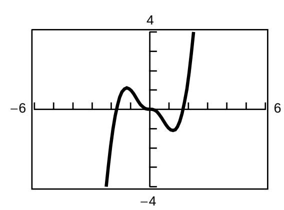
1. $$\begin{aligned} y_2 &= 4 + \frac{4}{x-1} \\ &= \frac{4(x-1)+4}{x-1} \\ &= \frac{4x-4+4}{x-1} \\ &= \frac{4x}{x-1} \\ &= y_1 \end{aligned}$$



3. $$\begin{aligned} y_2 &= x - 2 + \frac{4}{x+2} \\ &= \frac{(x-2)(x+2)+4}{x+2} \\ &= \frac{x^2-4+4}{x+2} \\ &= \frac{x^2}{x+2} \\ &= y_1 \end{aligned}$$



5. $$\begin{aligned} y_2 &= x^3 - 4x + \frac{4x}{x^2+1} \\ &= \frac{(x^3-4x)(x^2+1)+4x}{x^2+1} \\ &= \frac{x^5+x^3-4x^3-4x+4x}{x^2+1} \\ &= \frac{x^5-3x^3}{x^2+1} = y_1 \end{aligned}$$



7. $$\begin{array}{r|l} & 2x + 4 \\ \hline x+3 & 2x^2 + 10x + 12 \\ & -(2x^2 + 6x) \\ \hline & 4x + 12 \\ & -(4x + 12) \\ \hline & 0 \end{array}$$

$$\frac{2x^2+10x+12}{x+3} = 2x + 4$$

9. $$\begin{array}{r|l} & x^2 - 3x + 1 \\ \hline 4x+5 & 4x^3 - 7x^2 - 11x + 5 \\ & -(4x^3 + 5x^2) \\ \hline & -12x^2 - 11x \\ & -(-12x^2 - 15x) \\ \hline & 4x + 5 \\ & -(4x + 5) \\ \hline & 0 \end{array}$$

$$\frac{4x^3-7x^2-11x+5}{4x+5} = x^2 - 3x + 1$$

11.

$$\begin{array}{r} 7 \\ x+2\,\overline{)\;7x+3} \\ -(7x+14) \\ \hline -11 \end{array}$$

$$\frac{7x+3}{x+2} = 7 - \frac{11}{x+2}$$

13.

$$\begin{array}{r} 3x+5 \\ 2x^2+0x+1\,\overline{)\;6x^3+10x^2+x+8} \\ -(6x^3+0x^2+3x) \\ \hline 10x^2-2x+8 \\ -(10x^2+0x+5) \\ \hline -2x+3 \end{array}$$

$$\frac{6x^3+10x^2+x+8}{2x^2+1} = 3x+5-\frac{2x-3}{2x^2+1}$$

15.

$$\begin{array}{r} x^2+2x+4 \\ x^2-2x+3\,\overline{)\;x^4+0x^3+3x^2+0x+1} \\ -(x^4-2x^3+3x^2) \\ \hline 2x^3+0x^2+0x \\ -(2x^3-4x^2+6x) \\ \hline 4x^2-6x+1 \\ -(4x^2-8x+12) \\ \hline 2x-11 \end{array}$$

$$\Longrightarrow \frac{x^4+3x^2+1}{x^2-2x+3} = x^2+2x+4+\frac{2x-11}{x^2-2x+3}$$

17.

$$\begin{array}{r} 2x \\ x^2-2x+1\,\overline{)\;2x^3-4x^2-15x+5} \\ -(2x^3-4x^2+2x) \\ \hline -17x+5 \end{array}$$

$$\frac{2x^3-4x^2-15x+5}{(x-1)^2} = 2x-\frac{17x-5}{(x-1)^2}$$

19.

$$\begin{array}{c|cccc} 4 & 3 & -10 & 12 & -22 \\ & & 12 & 8 & 80 \\ \hline & 3 & 2 & 20 & 58 \end{array}$$

$$\frac{3x^3-10x^2+12x-22}{x-4} = 3x^2+2x+20+\frac{58}{x-4}$$

21.

$$\begin{array}{c|cccc} 3 & 6 & 7 & -1 & 26 \\ & & 18 & 75 & 222 \\ \hline & 6 & 25 & 74 & 248 \end{array}$$

$$\frac{6x^3+7x^2-x+26}{x-3} = 6x^2+25x+74+\frac{248}{x-3}$$

23.

$$\begin{array}{c|cccc} 2 & 9 & -18 & -16 & 32 \\ & & 18 & 0 & -32 \\ \hline & 9 & 0 & -16 & 0 \end{array}$$

$$\frac{9x^3-18x^2-16x+32}{x-2} = 9x^2-16$$

25.

$$\begin{array}{c|cccc} -8 & 1 & 0 & 0 & 512 \\ & & -8 & 64 & -512 \\ \hline & 1 & -8 & 64 & 0 \end{array}$$

$$\frac{x^3+512}{x+8} = x^2-8x+64$$

27.

$$\begin{array}{c|cccc} -\frac{1}{2} & 4 & 16 & -23 & -15 \\ & & -2 & -7 & 15 \\ \hline & 4 & 14 & -30 & 0 \end{array}$$

$$\frac{4x^3+16x^2-23x-15}{x+\frac{1}{2}} = 4x^2+14x-30$$

29. $f(x) = x^3 - x^2 - 14x + 11,\; k = 4$

$$\begin{array}{c|cccc} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$f(x) = (x-4)(x^2+3x-2)+3$

$f(4) = (0)(26) + 3 = 3$

31. $f(x) = x^3 + 3x^2 - 2x - 14,\; k = \sqrt{2}$

$$\begin{array}{c|cccc} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2+3\sqrt{2} & 6 \\ \hline & 1 & 3+\sqrt{2} & 3\sqrt{2} & -8 \end{array}$$

$f(x) = \left(x-\sqrt{2}\right)\left[x^2+\left(3+\sqrt{2}\right)x+3\sqrt{2}\right]-8$

$f\left(\sqrt{2}\right) = (0)\left(4+6\sqrt{2}\right) - 8 = -8$

33. $$\begin{array}{r|rrrr} 1-\sqrt{3} & 4 & -6 & -12 & -4 \\ & & 4-4\sqrt{3} & 10-2\sqrt{3} & 4 \\ \hline & 4 & -2-4\sqrt{3} & -2-2\sqrt{3} & 0 \end{array}$$

$f(x) = \left(x - 1 + \sqrt{3}\right)\left[4x^2 - \left(2 + 4\sqrt{3}\right)x - \left(2 + 2\sqrt{3}\right)\right]$

$f\left(1 - \sqrt{3}\right) = 0$

35. $f(x) = 4x^3 - 13x + 10$

(a) $$\begin{array}{r|rrrr} 1 & 4 & 0 & -13 & 10 \\ & & 4 & 4 & -9 \\ \hline & 4 & 4 & -9 & \underline{1} = f(1) \end{array}$$

(b) $$\begin{array}{r|rrrr} -2 & 4 & 0 & -13 & 10 \\ & & -8 & 16 & -6 \\ \hline & 4 & -8 & 3 & \underline{4} = f(-2) \end{array}$$

(c) $$\begin{array}{r|rrrr} \frac{1}{2} & 4 & 0 & -13 & 10 \\ & & 2 & 1 & -6 \\ \hline & 4 & 2 & -12 & \underline{4} = f\left(\frac{1}{2}\right) \end{array}$$

(d) $$\begin{array}{r|rrrr} 8 & 4 & 0 & -13 & 10 \\ & & 32 & 256 & 1944 \\ \hline & 4 & 32 & 243 & \underline{1954} = f(8) \end{array}$$

37. $h(x) = 3x^3 + 5x^2 - 10x + 1$

(a) $$\begin{array}{r|rrrr} 3 & 3 & 5 & -10 & 1 \\ & & 9 & 42 & 96 \\ \hline & 3 & 14 & 32 & \underline{97} = f(3) \end{array}$$

(b) $$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 5 & -10 & 1 \\ & & 1 & 2 & -\frac{8}{3} \\ \hline & 3 & 6 & -8 & \underline{-\frac{5}{3}} = f\left(\frac{1}{3}\right) \end{array}$$

(c) $$\begin{array}{r|rrrr} -2 & 3 & 5 & -10 & 1 \\ & & -6 & 2 & 16 \\ \hline & 3 & -1 & -8 & \underline{17} = f(-2) \end{array}$$

(d) $$\begin{array}{r|rrrr} -5 & 3 & 5 & -10 & 1 \\ & & -15 & 50 & -200 \\ \hline & 3 & -10 & 40 & \underline{-199} = f(-5) \end{array}$$

39. $$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$x^3 - 7x + 6 = (x - 2)(x^2 + 2x - 3)$

$= (x - 2)(x + 3)(x - 1)$

Zeros: $2, -3, 1$

41. $$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$2x^3 - 15x^2 + 27x - 10$

$= \left(x - \frac{1}{2}\right)(2x^2 - 14x + 20)$

$= (2x - 1)(x - 2)(x - 5)$

Zeros: $\frac{1}{2}, 2, 5$

43. $$\begin{array}{r|rrrr} -2 & 1 & 2 & -2 & -4 \\ & & -2 & 0 & 4 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$x^3 + 2x^2 - 2x - 4 = (x + 2)(x^2 - 2)$

$= (x + 2)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$

Zeros: $-2, \sqrt{2}, -\sqrt{2}$

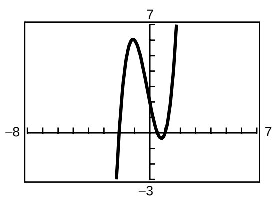
45. (a)
$$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$
$$\begin{array}{r|rrr} 1 & 2 & -3 & 1 \\ & & 2 & -1 \\ \hline & 2 & -1 & 0 \end{array}$$

(b) Remaining factor: $(2x - 1)$

(c) $f(x) = (x + 2)(x - 1)(2x - 1)$

(d) Real zeros: $-2, 1, \frac{1}{2}$

(e)



47. (a)
$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline & 1 & 1 & -10 & 8 & 0 \end{array}$$
$$\begin{array}{r|rrrr} -4 & 1 & 1 & -10 & 8 \\ & & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

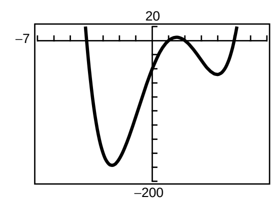
(b) $x^2 - 3x + 2 = (x - 2)(x - 1)$,

Remaining factors: $(x - 2), (x - 1)$

(c) $f(x) = (x - 5)(x + 4)(x - 2)(x - 1)$

(d) Real zeros: $5, -4, 2, 1$

(e)



49. (a)
$$\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 41 & -9 & -14 \\ & & -3 & -19 & 14 \\ \hline & 6 & 38 & -28 & 0 \end{array}$$
$$\begin{array}{r|rrr} \frac{2}{3} & 6 & 38 & -28 \\ & & 4 & 28 \\ \hline & 6 & 42 & 0 \end{array}$$

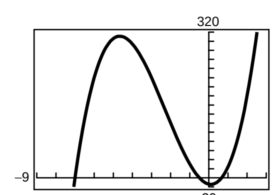
(b) $6x + 42$ Remaining factor $\big($or $6(x + 7)\big)$

(c) $f(x) = (2x + 1)(3x - 2)(x + 7)$

Note: Use $\frac{1}{6}(6x + 42) = x + 7$

(d) Real zeros: $-\frac{1}{2}, \frac{2}{3}, -7$

(e)



51. $f(x) = x^3 + 3x^2 - x - 3$

Possible rational zeros: $\pm 1, \pm 3$

Zeros shown on graph: $-3, -1, 1$

53. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

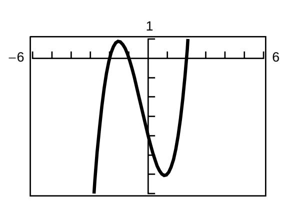
Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

Zeros shown of graph: $-1, \frac{3}{2}, 3, 5$

55. $f(x) = x^3 + x^2 - 4x - 4$

(a) Possible rational zeros: $\pm 1, \pm 2, \pm 4$

(b)

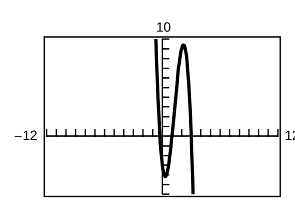


(c) $-2, -1, 2$ on graph

57. $f(x) = -4x^3 + 15x^2 - 8x - 3$

(a) Possible rational zeros: $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm 1, \pm\frac{3}{2}, \pm 3$

(b)



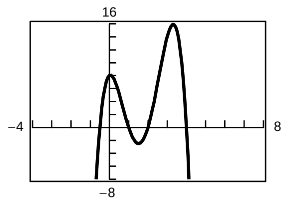
(c) $-\frac{1}{4}, 1, 3$ on graph

59. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) Possible rational zeros:

$\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

(b)



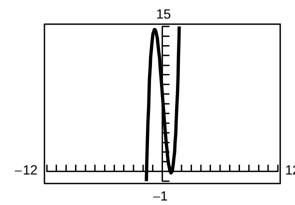
(c) $-\frac{1}{2}$, 1, 2, 4 on graph

61. $f(x) = 6x^3 - x^2 - 13x + 8$

(a) Possible rational zeros:

$\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{2}{3}, \pm 1, \pm\frac{4}{3}, \pm 2, \pm\frac{8}{3}, \pm 4, \pm 8$

(b)



(c) Real zeros: $1, \dfrac{-5 \pm \sqrt{217}}{12}$

$\left[f(x) = (x - 1)(6x^2 + 5x - 8)\right.$; Use Quadratic Formula$\left.\right]$

63. $z^4 - z^3 - 2z - 4 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 2 & -4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$z^4 - z^3 - 2z - 4 = (z + 1)(z - 2)(z^2 + 2) = 0$

The only real zeros are -1 and 2. You can verify this by graphing the function $f(z) = z^4 - z^3 - 2z - 4$.

65. $x^4 - 13x^2 - 12x = 0$

$x(x^3 - 13x - 12) = 0$

$x = 0$ is a solution.

Possible rational zeros of $x^3 - 13x - 12 = 0$ are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6$ and ± 12. Using a graphing utility or synthetic division, you find that the zeros are $0, -1, -3, 4$.

67. $2x^4 - 11x^3 - 6x^2 + 64x + 32 = 0$

Using a graphing utility, you can see that there are three zeros. Using synthetic division, you can verify that these zeros are $-2, -\frac{1}{2}, 4$.

69. $f(x) = x^3 - 2x^2 - 5x + 10$

(a) Zeros: 2, 2.236, -2.236

(b) $$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$ $x = 2$ is a zero

$f(x) = (x - 2)(x^2 - 5)$

$= (x - 2)\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)$

71. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) zeros: -2, 3.732, 0.268

(b) $$\begin{array}{r|rrrr} -2 & 1 & -2 & -7 & 2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$ $t = -2$ is a zero

$h(t) = (t + 2)(t^2 - 4t + 1)$

$= (t + 2)\left[t - \left(\sqrt{3} + 2\right)\right]\left[t + \left(\sqrt{3} - 2\right)\right]$

73. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) $h(x) = x(x^4 - 7x^3 + 10x^2 + 14x - 24)$
From the calculator we have $x = 0, 3, 4$ and $x \approx \pm 1.414$.

(b)

3	1	−7	10	14	−24
		3	−12	−6	24
	1	−4	−2	8	0

4	1	−4	−2	8
		4	0	−8
	1	0	−2	0

$f(x) = x(x - 3)(x - 4)(x^2 - 2)$

$= x(x - 3)(x - 4)(x - \sqrt{2})(x + \sqrt{2})$

The exact roots are $x = 0, 3, 4, \pm\sqrt{2}$.

75. $f(x) = x^4 - 4x^3 + 15$

(a)

4	1	−4	0	0	15
		4	0	0	0
	1	0	0	0	15

4 is an upper bound.

(b)

−1	1	−4	0	0	15
		−1	5	−5	5
	1	−5	5	−5	20

−1 is a lower bound.

77. $f(x) = x^4 - 4x^3 + 16x - 16$

(a)

5	1	−4	0	16	−16
		5	5	25	205
	1	1	5	41	189

5 is an upper bound.

(b)

−3	1	−4	0	16	−16
		−3	21	−63	141
	1	−7	21	−47	125

−3 is a lower bound.

79. $P(x) = x^4 - \frac{25}{4}x^2 + 9$

$= \frac{1}{4}(4x^4 - 25x^2 + 36)$

$= \frac{1}{4}(4x^2 - 9)(x^2 - 4)$

$= \frac{1}{4}(2x + 3)(2x - 3)(x + 2)(x - 2)$

The zeros are $\pm\frac{3}{2}$ and ± 2.

81. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$

$= \frac{1}{4}(4x^3 - x^2 - 4x + 1)$

$= \frac{1}{4}[x^2(4x - 1) - 1(4x - 1)]$

$= \frac{1}{4}(4x - 1)(x^2 - 1)$

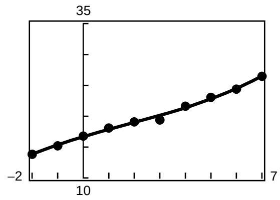
$= \frac{1}{4}(4x - 1)(x + 1)(x - 1)$

The zeros are $\frac{1}{4}$ and ± 1.

83. $f(x) = x^3 - 1 = (x - 1)(x^2 + x + 1)$
Rational zeros: 1 $(x = 1)$
Irrational zeros: 0
Matches (d).

85. $f(x) = x^3 - x = x(x + 1)(x - 1)$
Rational zeros: 3 $(x = 0, \pm 1)$
Irrational zeros: 0
Matches (b).

87. (a)



(b) $R = 0.01326t^3 - 0.06765t^2 + 1.2306t + 16.6770$

(c)

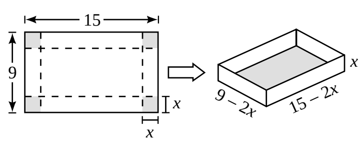
t	-2	-1	0	1	2	3	4	5	6	7
R	13.86	15.21	16.78	18.10	19.08	19.39	21.62	23.07	24.41	26.48
Model	13.84	15.37	16.68	17.85	18.97	20.12	21.37	22.80	24.49	26.52

(d)

12	.01326	-0.06765	1.2306	16.6770
		0.15912	1.09764	27.9389
	.01326	0.09147	2.3282	44.6159

$R(12) \approx 44.62$. No. The model will turn sharply upward.

89. (a)

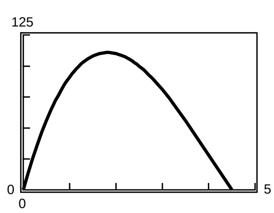


(b) $V = l \cdot w \cdot h = (15 - 2x)(9 - 2x)x$

$= x(9 - 2x)(15 - 2x)$

Since length, width, and height cannot be negative, we have $0 < x < \frac{9}{2}$ for the domain.

(c)



The volume is maximum when $x \approx 1.82$.
The dimensions are: length $= 15 - 2(1.82) = 11.36$
width $= 9 - 2(1.82) = 5.36$
height $= x = 1.82$
1.82 cm × 5.36 cm × 11.36 cm

(d) $56 = x(9 - 2x)(15 - 2x)$

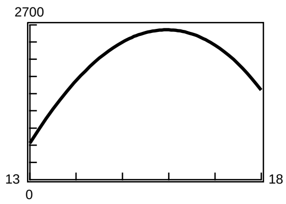
$56 = 135x - 48x^2 + 4x^3$

$0 = 4x^3 - 48x^2 + 135x - 56$

The zeros of this polynomial are $\frac{1}{2}$, $\frac{7}{2}$, and 8.
x cannot equal 8 since it is not in the domain of V.
[The length cannot equal -1 and the width cannot equal -7. The product of $(8)(-1)(-7) = 56$ so it showed up as an extraneous solution.]

91. $y = -5.05x^3 + 3857x - 38{,}411.25,\ 13 \le x \le 18$

(a)



(b) The second air-fuel ration of 16.89 can be obtained by finding the second point where the curves y and $y_1 = 2400$ intersect.

(c) Solve $-5.05x^3 + 3857x - 38{,}411.25 = 2400$ or $-5.05x^3 + 3857x - 40{,}811.25 = 0$. By synthetic division,

15	−5.05	0	3857	−40811.25
		−75.75	−1136.25	40811.25
	−5.05	−75.75	2720.75	0

(d) The positive zero of the quadratic $-5.05x^2 - 75.75x + 2720.75$ can be found by the Quadratic Formula.

$$x \approx \frac{75.75 - \sqrt{(75.75)^2 - 4(-5.05)(2720.75)}}{2(-5.05)} \approx 16.89$$

93. False, $-\frac{4}{7}$ is a root of f.

95.

$\frac{1}{2}$	6	1	−92	45	184	4	−48
		3	2	−45	0	92	48
	6	4	−90	0	184	96	0

True.

97.

$$\begin{array}{r} x^{2n} - x^n + 3 \\ x^n - 2 \overline{\smash{)}\, x^{3n} - 3x^{2n} + 5x^n - 6} \\ \underline{x^{3n} - 2x^{2n}} \qquad\qquad \\ -x^{2n} + 5x^n \qquad \\ \underline{-x^{2n} + 2x^n} \qquad \\ 3x^n - 6 \\ \underline{3x^n - 6} \\ 0 \end{array}$$

Hence, $\dfrac{x^{3n} - 3x^{2n} + 5x^n - 6}{x^n - 2} = x^{2n} - x^n + 3.$

99. You can check polynomial division by multiplying the quotient by the divisor. This should yield the original dividend if the multiplication was performed correctly.

101. (a)
$$\begin{aligned}(f \circ g)(x) &= f(x^2 + 1) \\ &= 2(x^2 + 1) - 5 \\ &= 2x^2 - 3\end{aligned}$$

(b)
$$\begin{aligned}(g \circ f)(x) &= g(2x - 5) \\ &= (2x - 5)^2 + 1 \\ &= 4x^2 - 20x + 26\end{aligned}$$

103. (a)
$$\begin{aligned}(f \circ g)(x) &= f(4x + x^2) \\ &= \frac{1}{4x + x^2}\end{aligned}$$

(b)
$$\begin{aligned}(g \circ f)(x) &= g\left(\frac{1}{x}\right) = 4\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 \\ &= \frac{4x + 1}{x^2}\end{aligned}$$

105.
$$\begin{aligned}f(x) &= (x - 1)(x + 3)(x - 8) \quad \text{[answer not unique]} \\ &= x^3 - 6x^2 - 19x + 24\end{aligned}$$

107.
$$\begin{aligned}f(x) &= \left[x - \left(2 + \sqrt{3}\right)\right]\left[x - \left(2 - \sqrt{3}\right)\right] \\ &= \left[(x - 2) - \sqrt{3}\right]\left[(x - 2) + \sqrt{3}\right] \\ &= (x - 2)^2 - 3 \\ &= x^2 - 4x + 4 - 3 \\ &= x^2 - 4x + 1\end{aligned}$$

[answer not unique]