85. True. $\left(-i\sqrt{6}\right)^4 - \left(-i\sqrt{6}\right)^2 + 14 = 36 + 6 + 14 = 56$

87. (a) $i^{40} = (i^4)^{10} = 1^{10} = 1$

(b) $i^{25} = (i^{24})(i) = (i^4)^6(i) = 1(i) = i$

(c) $i^{50} = (i^{48})(i^2) = (i^4)^{12}(-1) = 1(-1) = -1$

(d) $i^{67} = (i^{64})(i^3) = (i^4)^{16}(i^2)(i) = 1(-1)(i) = -i$

89. $(a + bi)(a - bi) = a^2 + abi - abi - b^2i^2 = a^2 + b^2$ which is a real number

91. $2x + 3y = 5$

$3y = -2x + 5$

$y = -\frac{2}{3}x + \frac{5}{3}$ Slope: $-\frac{2}{3}$

(a) Parallel line: $y - 3 = -\frac{2}{3}[x - (-8)]$

$3y - 9 = -2x - 16$

$3y + 2x = -7$

(b) Perpendicular line: $y - 3 = \frac{3}{2}[x - (-8)]$

$2y - 6 = 3x + 24$

$2y - 3x = 30$

93. $y = x^2 + 2x - 8$

Let $y = 0$: $x^2 + 2x - 8 = (x + 4)(x - 2) = 0 \Longrightarrow x = -4, 2$. x-intercepts: $(-4, 0), (2, 0)$

Let $x = 0$: $y = -8$. y-intercept: $(0, -8)$

94. $y = |x| - 1$

Let $y = 0$: $|x| = 1 \Longrightarrow x = \pm 1$. x-intercepts: $(1, 0), (-1, 0)$

Let $x = 0$: $y = -1$. y-intercept: $(0, -1)$

Section 2.5 The Fundamental Theorem of Algebra

- You should know that if f is a polynomial of degree $n > 0$, then f has at least one zero in the complex number system. (Fundamental Theorem of Algebra)
- You should know that if $a + bi$ is a complex zero of a polynomial f, with real coefficients, then $a - bi$ is also a complex zero of f.
- You should know the difference between a factor that is irreducible over the rationals (such as $x^2 - 7$) and a factor that is irreducible over the reals (such as $x^2 + 9$).

Solutions to Odd-Numbered Exercises

1. $f(x) = x(x - 6)^2 = x(x - 6)(x - 6)$
The three zeros are $x = 0$, $x = 6$, and $x = 6$.

3. $g(x) = (x - 2)(x + 4)^3$

The four zeros are $x = 2$, $x = -4$, $x = -4$, and $x = -4$.

5. $f(x) = (x + 6)(x + i)(x - i)$

The three zeros are $x = -6$, $x = -i$, and $x = i$.

7. $f(x) = (x - 2)(x + 3 - 5i)(x + 3 + 5i)$

The three zeros are $x = 2$, $x = -3 + 5i$, and $x = -3 - 5i$.

9. $f(x) = x^3 - 4x^2 + x - 4 = x^2(x - 4) + 1(x - 4)$
$= (x - 4)(x^2 + 1)$ zeros: $4, \pm i$

The only real zero of $f(x)$ is $x = 4$. This corresponds to the x-intercept of $(4, 0)$ on the graph.

11. $f(x) = x^4 + 4x^2 + 4 = (x^2 + 2)^2$
zeros: $\pm\sqrt{2}i, \pm\sqrt{2}i$

$f(x)$ has no real zeros and the graph of $f(x)$ has no x-intercepts.

13. $h(x) = x^2 - 4x + 1$

h has no rational zeros. By the Quadratic Formula, the zeros are $x = \dfrac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$.

$h(x) = \left[x - \left(2 + \sqrt{3}\right)\right]\left[x - \left(2 - \sqrt{3}\right)\right] = \left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right)$

15. $f(x) = x^2 - 12x + 26$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(26)}}{2} = 6 \pm \sqrt{10}.$$

$f(x) = \left[x - \left(6 + \sqrt{10}\right)\right]\left[x - \left(6 - \sqrt{10}\right)\right]$
$= \left(x - 6 - \sqrt{10}\right)\left(x - 6 + \sqrt{10}\right)$

17. $f(x) = x^2 + 25$
$= (x + 5i)(x - 5i)$

The zeros of $f(x)$ are $x = \pm 5i$.

19. $f(x) = x^4 - 81$
$= (x^2 - 9)(x^2 + 9)$
$= (x + 3)(x - 3)(x + 3i)(x - 3i)$

The zeros of $f(x)$ are $x = \pm 3$ and $x = \pm 3i$.

21. $f(z) = z^2 - 2z + 2$

f has no rational zeros. By the Quadratic Formula, the zeros are $z = \dfrac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i$.

$f(z) = [z - (1 + i)][z - (1 - i)]$
$= (z - 1 - i)(z - 1 + i)$

23. $f(t) = t^3 - 3t^2 - 15t + 125$

Possible rational zeros: $\pm 1, \pm 5, \pm 25, \pm 125$

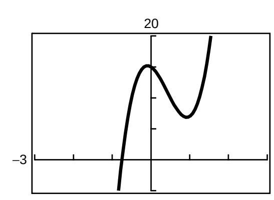
-5	1	-3	-15	125
		-5	40	-125
	1	-8	25	0

By the Quadratic Formula, the zeros of $t^2 - 8t + 25$ are $t = \dfrac{8 \pm \sqrt{64 - 100}}{2} = 4 \pm 3i$.

The zeros of $f(t)$ are $t = -5$ and $t = 4 \pm 3i$.

$f(t) = [t - (-5)][t - (4 + 3i)][t - (4 - 3i)]$
$= (t + 5)(t - 4 - 3i)(t - 4 + 3i)$

25. $f(x) = 16x^3 - 20x^2 - 4x + 15$



The graph reveals one zero at $x = -\frac{3}{4}$.

$-\frac{3}{4}$	16	-20	-4	15
		-12	24	-15
	16	-32	20	0

By the Quadratic Formula, the zeros of $16x^2 - 32x + 20 = 4(4x^2 - 8x + 5)$ are

$$x = \frac{8 \pm \sqrt{64 - 80}}{8} = 1 \pm \frac{1}{2}i.$$

The zeros of $f(x)$ are $x = -\frac{3}{4}$ and $x = 1 \pm \frac{1}{2}i$.

$$16\left(x + \frac{3}{4}\right)\left(x - 1 + \frac{1}{2}i\right)\left(x - 1 - \frac{1}{2}i\right)$$

27. $f(x) = x^4 + 10x^2 + 9$

$= (x^2 + 1)(x^2 + 9)$

$= (x + i)(x - i)(x + 3i)(x - 3i)$

The zeros of $f(x)$ are $x = \pm i$ and $x = \pm 3i$.

29. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

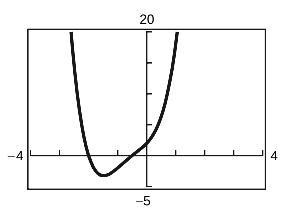
2	1	−4	8	−16	16
		2	−4	8	−16
2	1	−2	4	−8	0
		2	0	8	
	1	0	4	0	

$g(x) = (x - 2)(x - 2)(x^2 + 4)$

$= (x - 2)^2(x + 2i)(x - 2i)$

The zeros of g are 2, 2, and $\pm 2i$.

31. $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$



The graph reveals one zero at $x = -2$ and $x = -\frac{1}{2}$.

−2	2	5	4	5	2
		−4	−2	−4	−2
	2	1	2	1	0

$-\frac{1}{2}$	2	1	2	1
		−1	0	−1
	2	0	2	0

The zeros of $2x^2 + 2 = 2(x^2 + 1)$ are $x = \pm i$.

The zeros of $f(x)$ are $-2, -\frac{1}{2}, \pm i$.

$f(x) = (x + 2)(2x + 1)(x - i)(x + i)$

33. (a) $f(x) = x^2 - 14x + 46$. By the Quadratic Formula

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(46)}}{2} = 7 \pm \sqrt{3}.$$

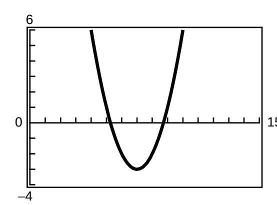
The zeros are $7 + \sqrt{3}$ and $7 - \sqrt{3}$.

(b) $f(x) = \left[x - \left(7 + \sqrt{3}\right)\right]\left[x - \left(7 - \sqrt{3}\right)\right]$

$= \left(x - 7 - \sqrt{3}\right)\left(x - 7 + \sqrt{3}\right)$

(c) x-intercepts: $\left(7 + \sqrt{3}, 0\right)$ and $\left(7 - \sqrt{3}, 0\right)$

(d)



35. (a) $f(x) = x^2 + 14x + 44$. By the Quadratic Formula,

$$x = \frac{-14 \pm \sqrt{14^2 - 4(44)}}{2} = -7 \pm \sqrt{5}$$

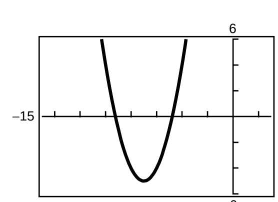
The zeros are $-7 + \sqrt{5}$ and $-7 - \sqrt{5}$.

(c) x-intercepts: $\left(-7 + \sqrt{5}, 0\right), \left(-7 - \sqrt{5}, 0\right)$

(b) $f(x) = \left[x - \left(-7 + \sqrt{5}\right)\right]\left[x - \left(-7 - \sqrt{5}\right)\right]$

$= \left(x + 7 - \sqrt{5}\right)\left(x + 7 + \sqrt{5}\right)$

(d)



37. (a) $f(x) = x^3 - 11x + 150$

$= (x + 6)(x^2 - 6x + 25).$

Use the Quadratic Formula to find the zeros of $x^2 - 6x + 25$:

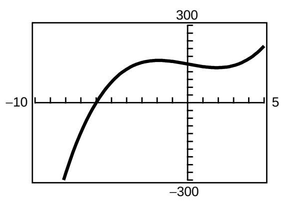
$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(25)}}{2} = 3 \pm 4i.$$

The zeros are -6, $3 + 4i$, and $3 - 4i$.

(b) $f(x) = (x + 6)(x - 3 + 4i)(x - 3 - 4i)$

(c) x-intercept: $(-6, 0)$

(d)



39. (a) $f(x) = x^4 + 25x^2 + 144$

$= (x^2 + 9)(x^2 + 16)$

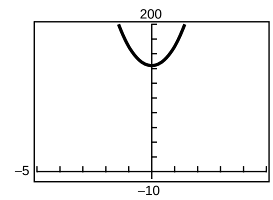
The zeros are $\pm 3i, \pm 4i$.

(b) $f(x) = (x^2 + 9)(x^2 + 16)$

$= (x + 3i)(x - 3i)(x + 4i)(x - 4i)$

(c) No x-intercepts

(d)



41. $f(x) = (x - 1)(x - 5i)(x + 5i)$

$= (x - 1)(x^2 + 25)$

$= x^3 - x^2 + 25x - 25$

Note: $f(x) = a(x^3 - x^2 + 25x - 25)$, where a is any nonzero real number, has the zero, 1 and $\pm 5i$.

43. $f(x) = (x - 2)(x - 4 - i)(x - 4 + i)$

$= (x - 2)(x^2 - 8x + 17)$

$= x^3 - 10x^2 + 33x - 34$

Note: $f(x) = a(x^3 - 10x^2 + 33x - 34)$, where a is any nonzero real number, has these zeros.

45. $f(x) = (x - i)(x + i)(x - 6i)(x + 6i)$

$= (x^2 + 1)(x^2 + 36)$

$= x^4 + 37x^2 + 36$

Note: $f(x) = a(x^4 + 37x^2 + 36)$, where a is any nonzero real number, has the zeros $\pm i$ and $\pm 6i$.

47. If $1 + \sqrt{3}i$ is a zero, so is its conjugate $1 - \sqrt{3}i$.

$f(x) = (x + 5)^2(x - 1 + \sqrt{3}i)(x - 1 - \sqrt{3}i)$

$= (x^2 + 10x + 25)(x^2 - 2x + 4)$

$= x^4 + 8x^3 + 9x^2 - 10x + 100$

Note: $f(x) = a(x^4 + 8x^3 + 9x^2 - 10x + 100)$, where a is any nonzero real number, has these

49. $f(x) = x^4 - 6x^2 - 7$

(a) $f(x) = (x^2 - 7)(x^2 + 1)$

(b) $f(x) = (x - \sqrt{7})(x + \sqrt{7})(x^2 + 1)$

(c) $f(x) = (x - \sqrt{7})(x + \sqrt{7})(x + i)(x - i)$

51. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$

(a) $f(x) = (x^2 - 6)(x^2 - 2x + 3)$

(b) $f(x) = (x + \sqrt{6})(x - \sqrt{6})(x^2 - 2x + 3)$

(c) $f(x) =$

$(x + \sqrt{6})(x - \sqrt{6})(x - 1 - \sqrt{2}i)(x - 1 + \sqrt{2}i)$

53. $f(x) = 2x^3 + 3x^2 + 50x + 75$

Since $5i$ is a zero, so is $-5i$.

$$\begin{array}{r|rrrr} 5i & 2 & 3 & 50 & 75\\ & & 10i & -50+15i & -75\\ \hline & 2 & 3+10i & 15i & 0 \end{array}$$

$$\begin{array}{r|rrr} -5i & 2 & 3+10i & 15i\\ & & -10i & -15i\\ \hline & 2 & 3 & 0 \end{array}$$

The zero of $2x + 3$ is $x = -\frac{3}{2}$. The zeros of f are $x = -\frac{3}{2}$ and $x = \pm 5i$.

Alternate Solution

Since $x = \pm 5i$ are zeros of $f(x)$, $(x + 5i)(x - 5i) = x^2 + 25$ is a factor of $f(x)$. By long division we have:

$$\begin{array}{r} 2x + 3 \\ x^2 + 0x + 25 \overline{\smash{\big)}\, 2x^3 + 3x^2 + 50x + 75} \\ \underline{2x^3 + 0x^2 + 50x } \\ 3x^2 + 0x + 75 \\ \underline{3x^2 + 0x + 75} \\ 0 \end{array}$$

Thus, $f(x) = (x^2 + 25)(2x + 3)$ and the zeros of f are $x = \pm 5i$ and $x = -\frac{3}{2}$.

55. $g(x) = x^3 - 7x^2 - x + 87$. Since $5 + 2i$ is a zero, so is $5 - 2i$.

$$\begin{array}{r|rrrr} 5+2i & 1 & -7 & -1 & 87\\ & & 5+2i & -14+6i & -87\\ \hline & 1 & -2+2i & -15+6i & 0 \end{array}$$

$$\begin{array}{r|rrr} 5-2i & 1 & -2+2i & -15+6i\\ & & 5-2i & 15-6i\\ \hline & 1 & 3 & 0 \end{array}$$

The zero of $x + 3$ is $x = -3$.

The zeros of f are $-3, 5 \pm 2i$.

57. $h(x) = 3x^3 - 4x^2 + 8x + 8$. Since $1 - \sqrt{3}i$ is a zero, so is $1 + \sqrt{3}i$.

$$\begin{array}{r|rrrr} 1-\sqrt{3}i & 3 & -4 & 8 & 8\\ & & 3-3\sqrt{3}i & -10-2\sqrt{3}i & -8\\ \hline & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1+\sqrt{3}i & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i\\ & & 3+3\sqrt{3}i & 2+2\sqrt{3}i\\ \hline & 3 & 2 & 0 \end{array}$$

The zero of $3x + 2$ is $x = -\frac{2}{3}$.

The zeros of h are $x = -\frac{2}{3}, 1 \pm \sqrt{3}i$.

59. $h(x) = 8x^3 - 14x^2 + 18x - 9$. Since $\frac{1}{2}\left(1 - \sqrt{5}i\right)$ is a zero, so is $\frac{1}{2}\left(1 + \sqrt{5}i\right)$.

$$\begin{array}{r|rrrr} \frac{1}{2}\left(1-\sqrt{5}i\right) & 8 & -14 & 18 & -9\\ & & 4-4\sqrt{5}i & -15+3\sqrt{5}i & 9\\ \hline & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{1}{2}\left(1+\sqrt{5}i\right) & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i\\ & & 4+4\sqrt{5}i & -3-3\sqrt{5}i\\ \hline & 8 & -6 & 0 \end{array}$$

The zero of $8x - 6$ is $x = \frac{3}{4}$.

The zeros of h are $x = \frac{3}{4}, \frac{1}{2}\left(1 \pm \sqrt{5}i\right)$.

61. (a) The root feature yields the real roots 1 and 2, and the complex roots $-3 \pm 1.414i$.

(b) By synthetic division,

$$\begin{array}{r|rrrrr} 1 & 1 & 3 & -5 & -21 & 22\\ & & 1 & 4 & -1 & -22\\ \hline & 1 & 4 & -1 & -22 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -1 & -22\\ & & 2 & 12 & 22\\ \hline & 1 & 6 & 11 & 0 \end{array}$$

The complex roots of $x^2 + 6x + 11$ are

$$x = \frac{-6 \pm \sqrt{6^2 - 4(11)}}{2} = -3 \pm \sqrt{2}i.$$

63. (a) The root feature yields the real root 0.75, and the complex roots $0.5 \pm 1.118i$.

(b) By synthetic division,

$$\begin{array}{c|cccc} \frac{3}{4} & 8 & -14 & 18 & -9 \\ & & 6 & -6 & 9 \\ \hline & 8 & -8 & 12 & 0 \end{array}$$

The complex roots of $8x^2 - 8x + 12$ are

$$x = \frac{8 \pm \sqrt{64 - 4(8)(12)}}{2(8)} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}i.$$

65. $-16t^2 + 48t = 64, \quad 0 \le t \le 3$

$$-16t^2 + 48t - 64 = 0$$

$$t = \frac{-48 \pm \sqrt{1792}i}{-32}$$

Since the roots are imaginary, the ball never will reach a height of 64 feet. You can verify this graphically by observing that $y_1 = -16t^2 + 48t$ and $y_2 = 64$ do not intersect.

67. False, a third degree polynomial must have at least one real zero.

69. $f(x) = x^4 - 4x^2 + k$

(a) f has four real zeros for $0 < k < 4$.

(b) f has two real zeros each of multiplicity 2 for $k = 4$: $f(x) = x^4 - 4x^2 + 4 = (x^2 - 2)^2$.

(c) f has two real zeros and two complex zeros if $k < 0$.

(d) f has four complex zeros if $k > 4$.

71. $f(x) = (x - \sqrt{b}i)(x + \sqrt{b}i) = x^2 + b$

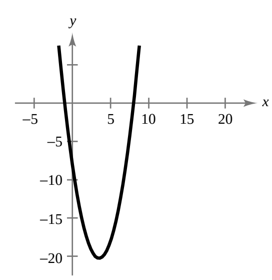
73. $f(x) = x^2 - 7x - 8 = \left(x^2 - 7x + \frac{49}{4}\right) - 8 - \frac{49}{4}$

$= \left(x - \frac{7}{2}\right)^2 - \frac{81}{4}$

Vertex: $\left(\frac{7}{2}, -\frac{81}{4}\right)$

$f(x) = (x - 8)(x + 1)$

Intercepts: $(8, 0), (-1, 0), (0, -8)$



75. $f(x) = 6x^2 + 5x - 6 = (3x - 2)(2x + 3)$

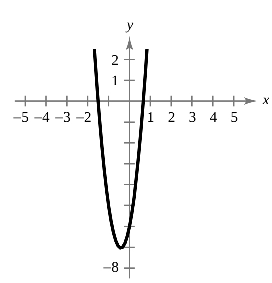
Intercepts: $\left(\frac{2}{3}, 0\right), \left(-\frac{3}{2}, 0\right), (0, -6)$

$f(x) = 6x^2 + 5x - 6$

$= 6\left(x^2 + \frac{5}{6}x + \frac{25}{144}\right) - 6 - \frac{25}{24}$

$= 6\left(x + \frac{5}{12}\right)^2 + \frac{169}{24}$

Vertex: $\left(-\frac{5}{12}, -\frac{169}{24}\right)$



77. $12 - (-7 + 5i) + (-2 + 3i) = 17 - 2i$

79. $(-3 - 8i)^2 = 9 + 48i + (-8i)^2$

$= 9 + 48i - 64$

$= -55 + 48i$