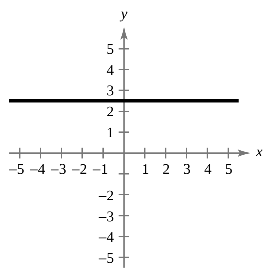
89. $4y - 10 = 0$

$4y = 10$

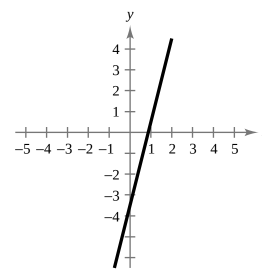
$y = \frac{5}{2}$ horizontal line



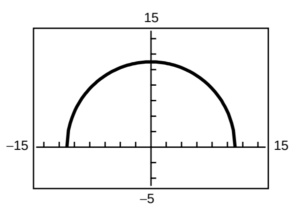
91. $-7 + 8x - 2y = 0$

$-2y = -8x + 7$

$y = 4x - \frac{7}{2}$ line



93.

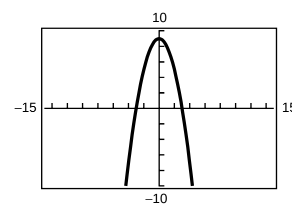


Semicircle

Domain: $-11 \le x \le 11$

Range: $0 \le y \le 11$

95.



Parabola

Domain: all x

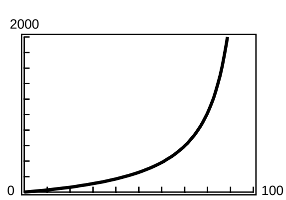
Range: $y \le 9$

Review Exercises for Chapter 2

Solutions to Odd-Numbered Exercises

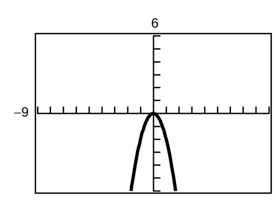
1. (a) $y = 2x^2$

Vertical stretch



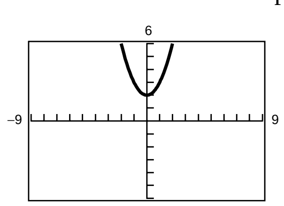
(b) $y = -2x^2$

Vertical stretch and a reflection in the x-axis



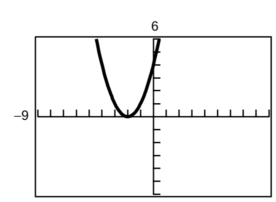
(c) $y = x^2 + 2$

Vertical shift two units upward



(d) $y = (x + 2)^2$

Horizontal shift two units to the left

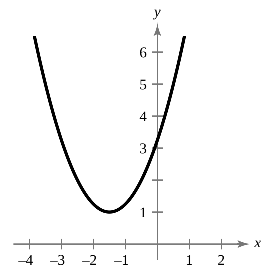


3. $f(x) = \left(x + \frac{3}{2}\right)^2 + 1$

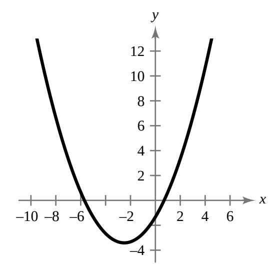
Vertex: $\left(-\frac{3}{2}, 1\right)$

y-intercept: $\left(0, \frac{13}{4}\right)$

No x-intercepts



5. $$\begin{aligned} f(x) &= \frac{1}{3}(x^2 + 5x - 4) \\ &= \frac{1}{3}\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 4\right) \\ &= \frac{1}{3}\left[\left(x - \frac{5}{2}\right)^2 - \frac{41}{4}\right] \\ &= \frac{1}{3}\left(x - \frac{5}{2}\right)^2 - \frac{41}{12} \end{aligned}$$



Vertex: $\left(\frac{5}{2}, -\frac{41}{12}\right)$

y-intercept: $\left(0, -\frac{3}{4}\right)$

x-intercepts: $$\begin{aligned} 0 &= \frac{1}{3}(x^2 + 5x - 4) \\ 0 &= x^2 + 5x - 4 \\ x &= \frac{-5 \pm \sqrt{41}}{2} \quad \text{Use the Quadratic Formula.} \end{aligned}$$

$$\left(\frac{-5 \pm \sqrt{41}}{2}, 0\right)$$

7. Vertex: $(1, -4) \implies f(x) = a(x - 1)^2 - 4$

Point: $(2, -3) \implies -3 = a(2 - 1)^2 - 4$

$1 = a$

Thus, $f(x) = (x - 1)^2 - 4$.

9. $$\begin{aligned} g(x) &= x^2 - 2x \\ &= x^2 - 2x + 1 - 1 \\ &= (x - 1)^2 - 1 \end{aligned}$$

The minimum occurs at the vertex $(1, -1)$.

11. $$\begin{aligned} f(x) &= 6x - x^2 \\ &= -(x^2 - 6x + 9 - 9) \\ &= -(x - 3)^2 + 9 \end{aligned}$$

The maximum occurs at the vertex $(3, 9)$.

13. $$\begin{aligned} f(t) &= -2t^2 + 4t + 1 \\ &= -2(t^2 - 2t + 1 - 1) + 1 \\ &= -2[(t - 1)^2 - 1] + 1 \\ &= -2(t - 1)^2 + 3 \end{aligned}$$

The maximum occurs at the vertex $(1, 3)$.

15. $$\begin{aligned} h(x) &= x^2 + 5x - 4 \\ &= x^2 + 5x + \tfrac{25}{4} - \tfrac{25}{4} - 4 \\ &= \left(x + \tfrac{5}{2}\right)^2 - \tfrac{25}{4} - \tfrac{16}{4} \\ &= \left(x + \tfrac{5}{2}\right)^2 - \tfrac{41}{4} \end{aligned}$$

The minimum occurs at the vertex $\left(-\frac{5}{2}, -\frac{41}{4}\right)$.

17. (a)

x	y	Area
1	$4 - \frac{1}{2}(1)$	$(1)[4 - \frac{1}{2}(1)] = \frac{7}{2}$
2	$4 - \frac{1}{2}(2)$	$(2)[4 - \frac{1}{2}(2)] = 6$
3	$4 - \frac{1}{2}(3)$	$(3)[4 - \frac{1}{2}(3)] = \frac{15}{2}$
4	$4 - \frac{1}{2}(4)$	$(4)[4 - \frac{1}{2}(4)] = 8$
5	$4 - \frac{1}{2}(5)$	$(5)[4 - \frac{1}{2}(5)] = \frac{15}{2}$
6	$4 - \frac{1}{2}(6)$	$(6)[4 - \frac{1}{2}(6)] = 6$

(b) The dimensions that will produce a maximum area are $x = 4$ and $y = 2$.

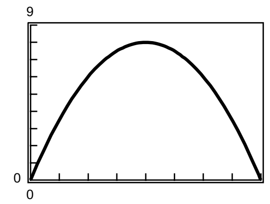
(c) $A = xy = x\left(\dfrac{8 - x}{2}\right)$, since

$x + 2y - 8 = 0 \implies y = \dfrac{8 - x}{2}$.

Since the figure is in the first quadrant and x and y must be positive, the domain of

$A = x\left(\dfrac{8 - x}{2}\right)$ is $0 < x < 8$.

(d)



The maximum area of 8 occurs at the vertex when $x = 4$ and $y = \dfrac{8 - 4}{2} = 2$.

(e) $A = x\left(\dfrac{8 - x}{2}\right)$

$= \dfrac{1}{2}(8x - x^2)$

$= -\dfrac{1}{2}(x^2 - 8x)$

$= -\dfrac{1}{2}(x^2 - 8x + 16 - 16)$

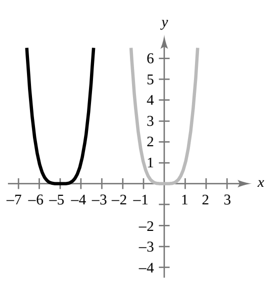
$= -\dfrac{1}{2}[(x - 4)^2 - 16]$

$= -\dfrac{1}{2}(x - 4)^2 + 8$

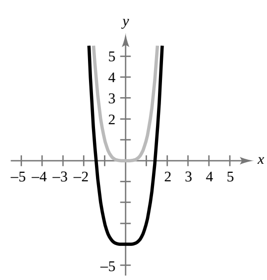
The maximum area of 8 occurs when $x = 4$ and $y = \dfrac{8 - 4}{2} = 2$.

19. $y = x^4$

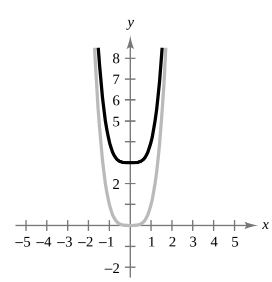
(a)



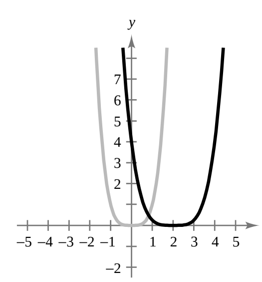
(b)



(c)

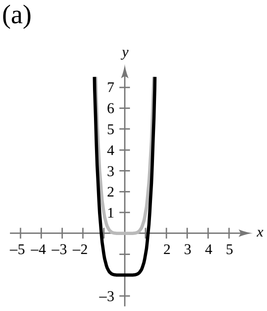


(d)

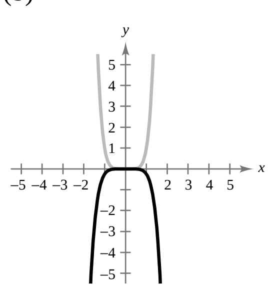


21. $y = x^6$

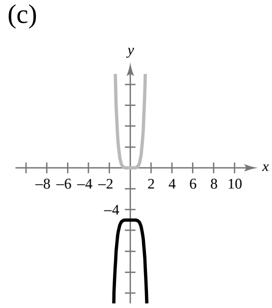
(a)



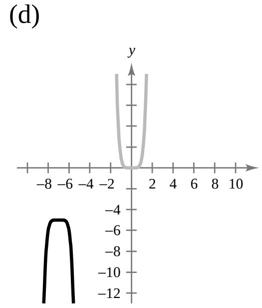
(b)



(c)



(d)



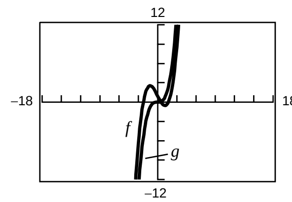
23. $f(x) = -x^2 + 6x + 9$

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

25. $f(x) = \frac{3}{4}(x^4 + 3x^2 + 2)$

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

27. $f(x) = \frac{1}{2}x^3 - 2x + 1;\ g(x) = \frac{1}{2}x^3$



29. $g(x) = x^4 - x^3 - 2x^2$

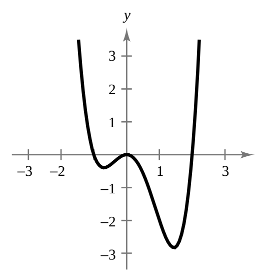
(a) $0 = x^4 - x^3 - 2x^2$

$0 = x^2(x^2 - x - 2)$

$0 = x^2(x - 2)(x + 1)$

Zeros: $0, 0, 2, -1$

(b)



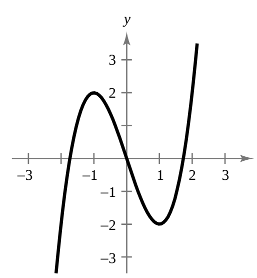
31. $f(t) = t^3 - 3t$

(a) $0 = t^3 - 3t$

$0 = t(t^2 - 3)$

Zeros: $0, \pm\sqrt{3}$

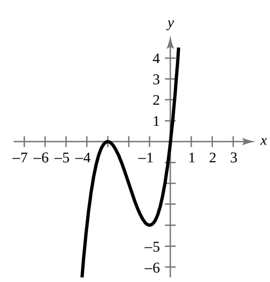
(b)



33. $f(x) = x(x + 3)^2$

(a) Zeros: $0, -3, -3$

(b)



35. (a) The combined length and girth is

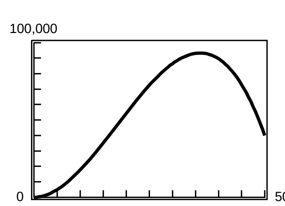
$y + 4x = 216$

$y = 216 - 4x.$

The volume is

$V = x^2y = x^2(216 - 4x).$

(b)

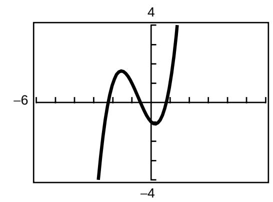


The volume is maximum when $x = 36$ centimeters and $y = 216 - 4(36) = 72$ centimeters.

37. (a) $f(-3) < 0,\ f(-2) > 0 \implies$ zero in $[-3, -2]$

$f(-1) > 0,\ f(0) < 0 \implies$ zero in $[-1, 0]$

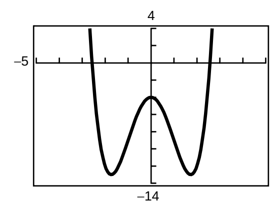
$f(0) < 0,\ f(1) > 0 \implies$ zero in $[0, 1]$



(b) zeros: $-2.247, -0.555, 0.802$

39. (a) $f(-3) > 0,\ f(-2) < 0 \implies$ zero in $[-3, -2]$

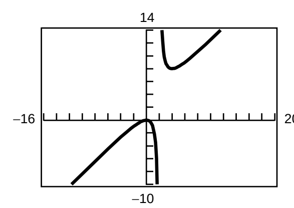
$f(2) < 0,\ f(3) > 0 \implies$ zero in $[2, 3]$



(b) zeros: ± 2.570

41. $y_1 = \dfrac{x^2}{x-2}$

$$\begin{aligned} y_2 &= x + 2 + \frac{4}{x-2} \\ &= \frac{(x+2)(x-2)}{x-2} + \frac{4}{x-2} \\ &= \frac{x^2-4}{x-2} + \frac{4}{x-2} \\ &= \frac{x^2}{x-2} \\ &= y_1 \end{aligned}$$



43.

$$\begin{array}{r} 8x + 5 \\ 3x - 2 \,\overline{)\, 24x^2 - x - 8} \\ \underline{24x^2 - 16x} \\ 15x - 8 \\ \underline{15x - 10} \\ 2 \end{array}$$

Thus, $\dfrac{24x^2 - x - 8}{3x - 2} = 8x + 5 + \dfrac{2}{3x - 2}$.

45.

$$\begin{array}{r} x^2 - 2 \\ x^2 - 1 \,\overline{)\, x^4 - 3x^2 + 2} \\ \underline{x^4 - x^2} \\ -2x^2 + 2 \\ \underline{-2x^2 + 2} \\ 0 \end{array}$$

Thus, $\dfrac{x^4 - 3x^2 + 2}{x^2 - 1} = x^2 - 2 \qquad (x \neq \pm 1)$

47.

$$\begin{array}{r} x^2 - x + 1 \\ x^2 + 2x \,\overline{)\, x^4 + x^3 - x^2 + 2x} \\ \underline{x^4 + 2x^3} \\ - x^3 - x^2 \\ \underline{- x^3 - 2x^2} \\ x^2 + 2x \end{array}$$

Thus, $\dfrac{x^4 + x^3 - x^2 + 2x}{x^2 + 2x} = x^2 - x + 1, \qquad (x \neq 0, -2).$

49.
$$\begin{array}{r|rrrrr} -2 & 0.25 & -4 & 0 & 0 & 0 \\ & & -\frac{1}{2} & 9 & -18 & 36 \\ \hline & \frac{1}{4} & -\frac{9}{2} & 9 & -18 & 36 \end{array}$$

Hence, $\dfrac{0.25x^4 - 4x^3}{x + 2} = \dfrac{1}{4}x^3 - \dfrac{9}{2}x^2 + 9x - 18 + \dfrac{36}{x + 2}$

51.
$$\begin{array}{r|rrrrr} \frac{2}{3} & 6 & -4 & -27 & 18 & 0 \\ & & 4 & 0 & -18 & 0 \\ \hline & 6 & 0 & -27 & 0 & 0 \end{array}$$

Thus, $\dfrac{6x^4 - 4x^3 - 27x^2 + 18x}{x - \left(\frac{2}{3}\right)} = 6x^3 - 27x,\ x \neq \dfrac{2}{3}$

53. (a)
$$\begin{array}{r|rrrr} 4 & 2 & 3 & -20 & -21 \\ & & 8 & 44 & 96 \\ \hline & 2 & 11 & 24 & 75 \end{array} \quad f(4) = 75$$

(b)
$$\begin{array}{r|rrrr} -1 & 2 & 3 & -20 & -21 \\ & & -2 & -1 & 21 \\ \hline & 2 & 1 & -21 & 0 \end{array} \quad f(-1) = 0$$

(c)
$$\begin{array}{r|rrrr} -\frac{7}{2} & 2 & 3 & -20 & -21 \\ & & -7 & 14 & 21 \\ \hline & 2 & -4 & -6 & 0 \end{array} \quad f\left(-\tfrac{7}{2}\right) = 0$$

(d)
$$\begin{array}{r|rrrr} 0 & 2 & 3 & -20 & -21 \\ & & 0 & 0 & 0 \\ \hline & 2 & 3 & -20 & -21 \end{array} \quad f(0) = -21$$

55.
$$\begin{array}{r|rrrrr} -3 & 2 & 5 & -11 & -20 & 12 \\ & & -6 & 3 & 24 & -12 \\ \hline & 2 & -1 & -8 & 4 & 0 \end{array} \quad f(-3) = 0$$

$$\begin{aligned} 2x^4 + 5x^3 - 11x^2 - 20x + 12 &= (x + 3)(2x^3 - x^2 - 8x + 4) \\ &= (x + 3)(2x - 1)(x - 2)(x + 2) \end{aligned}$$

Zeros: $-3, \frac{1}{2}, 2, -2$

57. $f(x) = 4x^3 - 11x^2 + 10x - 3$

Possible Rational Zeros: $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$. Use a graphing utility to see that $x = 1$ is probably a zero.

$$\begin{array}{r|rrrr} 1 & 4 & -11 & 10 & -3 \\ & & 4 & -7 & 3 \\ \hline & 4 & -7 & 3 & 0 \end{array}$$

$4x^3 - 11x^2 + 10x - 3 = (x - 1)(4x^2 - 7x + 3) = (x - 1)^2(4x - 3)$

Thus, the zeros of f are $x = 1$ (repeated) and $x = \frac{3}{4}$.

59. $f(x) = 6x^3 - 5x^2 + 24x - 20$

Graphing $f(x)$ with a graphing utility suggests that $x = \frac{5}{6}$ is a zero.

$$\begin{array}{r|rrrr} \frac{5}{6} & 6 & -5 & 24 & -20 \\ & & 5 & 0 & 20 \\ \hline & 6 & 0 & 24 & 0 \end{array}$$

The quadratic $6x^2 + 24 = 0$ has complex zeros $x = \pm 2i$. Thus, the zeros are $\frac{5}{6}, 2i, -2i$.

61. $f(x) = 6x^4 - 25x^3 + 14x^2 + 27x - 18$

Possible Rational Zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$. Use a graphing utility to see that $x = -1$ and $x = 3$ are probably zeros.

$$\begin{array}{r|rrrrr} -1 & 6 & -25 & 14 & 27 & -18 \\ & & -6 & 31 & -45 & 18 \\ \hline & 6 & -31 & 45 & -18 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 6 & -31 & 45 & -18 \\ & & 18 & -39 & 18 \\ \hline & 6 & -13 & 6 & 0 \end{array}$$

$$\begin{aligned} 6x^4 - 25x^3 + 14x^2 + 27x - 18 &= (x+1)(x-3)(6x^2 - 13x + 6) \\ &= (x+1)(x-3)(3x-2)(2x-3) \end{aligned}$$

Thus, the zeros of f are $x = -1$, $x = 3$, $x = \frac{2}{3}$, and $x = \frac{3}{2}$.

63.

$$\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & -3 \\ & & 4 & 1 & 5 \\ \hline & 4 & 1 & 5 & 2 \end{array}$$

All entries positive. $x = 1$ is upper bound.

$$\begin{array}{r|rrrr} -\frac{1}{4} & 4 & -3 & 4 & -3 \\ & & -1 & 1 & -\frac{5}{4} \\ \hline & 4 & -4 & 5 & -\frac{17}{4} \end{array}$$

Alternating signs. $x = -\frac{1}{4}$ is lower bound.

65. $6 + \sqrt{-25} = 6 + 5i$

67. $-2i^2 + 7i = 2 + 7i$

69. $$\begin{aligned}(7 + 5i) + (-4 + 2i) &= (7 - 4) + (5i + 2i) \\ &= 3 + 7i\end{aligned}$$

71. $5i(13 - 8i) = 65i - 40i^2 = 40 + 65i$

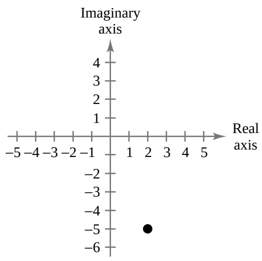
73. $(10 - 8i)(2 - 3i) = 20 - 30i - 16i + 24i^2 = -4 - 46i$

75. $$\begin{aligned}(3 + 7i)^2(3 - 7i)^2 &= (9 + 42i - 49) + (9 - 42i - 49) \\ &= -80\end{aligned}$$

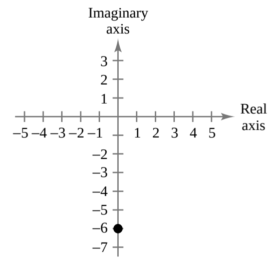
77. $$\begin{aligned}\frac{6+i}{i} &= \frac{6+i}{i} \cdot \frac{-i}{-i} = \frac{-6i - i^2}{-i^2} \\ &= \frac{-6i + 1}{1} = 1 - 6i\end{aligned}$$

79. $\dfrac{4}{-3i} = \dfrac{4}{-3i} \cdot \dfrac{3i}{3i} = \dfrac{12i}{9} = \dfrac{4i}{3} = \dfrac{4}{3}i$

81.

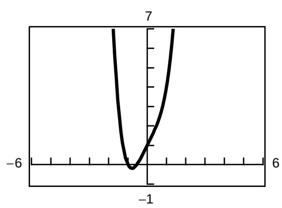


83.



85. $f(x) = x^4 + 2x + 1$

(a)

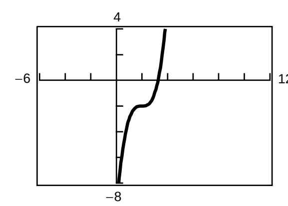


(b) The graph has two x-intercepts, so there are two real zeros.

(c) The zeros are $x = -1$ and $x \approx -0.54$.

87. $h(x) = x^3 - 6x^2 + 12x - 10$

(a)



(b) The graph has one x-intercept, so there is one real zero.

(c) $x \approx 3.26$

89. $f(x) = x^3 - 4x^2 + 6x - 4$

$= (x - 2)(x^2 - 2x + 2)$

Use the Quadratic Formula for $x^2 - 2x + 2$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$$

zeros: $2, 1 + i, 1 - i$

$f(x) = (x - 2)(x - 1 + i)(x - 1 - i)$

91. $f(x) = x^3 + 6x^2 + 11x + 12$

$= (x + 4)(x^2 + 2x + 3)$

Use the Quadratic Formula for $x^2 + 2x + 3$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)}}{2} = \frac{-2 \pm \sqrt{-8}}{2} = -1 \pm \sqrt{2}\, i$$

zeros: $-4, -1 + \sqrt{2}\, i, -1 - \sqrt{2}\, i$

$f(x) = (x + 4)\left(x + 1 + \sqrt{2}\, i\right)\left(x + 1 - \sqrt{2}\, i\right)$

93. $f(x) = x^4 + 34x^2 + 225$

$= (x^2 + 25)(x^2 + 9)$

$= (x + 5i)(x - 5i)(x + 3i)(x - 3i)$

zeros: $x = \pm 5i, \pm 3i$

95. $f(x) = (x + 2)(x + 2)(x + 5i)(x - 5i)$

$= (x^2 + 4x + 4)(x^2 + 25)$

$= x^4 + 4x^3 + 29x^2 + 100x + 100$

97. $f(x) = 3\left(x + \frac{2}{3}\right)(x + 1)\left(x - 3 - \sqrt{2}\, i\right)\left(x - 3 + \sqrt{2}\, i\right)$

$= (3x + 2)(x + 1)((x - 3)^2 + 2)$

$= (3x^2 + 5x + 2)(x^2 - 6x + 11)$

$= 3x^4 - 13x^3 + 5x^2 + 43x + 22$

99. $f(x) = x^4 - x^3 - x^2 + 5x - 20$

(a) $f(x) = (x^2 - 5)(x^2 - x + 4)$

(b) $f(x) = \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)(x^2 - x + 4)$

(c) $f(x) = \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)\left(x - \frac{1}{2} + \frac{\sqrt{15}}{2} i\right)\left(x - \frac{1}{2} - \frac{\sqrt{15}}{2} - 1\right)$

101. Domain: all $x \neq 1$

Horizontal asymptote: $y = -1$

Vertical asymptote: $x = 1$

103. $f(x) = \dfrac{2}{x^2 - 3x - 18} = \dfrac{2}{(x - 6)(x + 3)}$

Domain: all $x \neq 6, \ -3$

Horizontal asymptote: $y = 0$

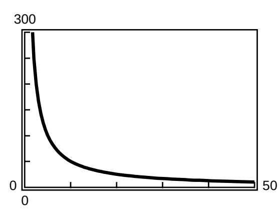
Vertical asymptotes: $x = 6, \ x = -3$

105. $y = -1$ (degree $p(x)$ = degree $q(x)$)

107. $y = \dfrac{4}{2} = 2$ (degree $p(x)$ = degree $q(x)$)

109. $y = 1, \ y = -1$

111. (a)



(b) $\overline{C}(50) = 10.50$ dollars

$\overline{C}(100) = 5.50$ dollars

$\overline{C}(1000) = 1.00$ dollar

$\overline{C}(10{,}000) = 0.55$ dollars

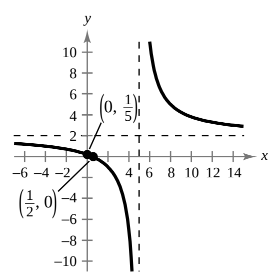
(c) As x increases, the average cost approaches its horizontal asymptote, $\overline{C} = 0.5$.

113. $f(x) = \dfrac{2x - 1}{x - 5}$

Intercepts: $\left(0, \frac{1}{5}\right), \left(\frac{1}{2}, 0\right)$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = 2$



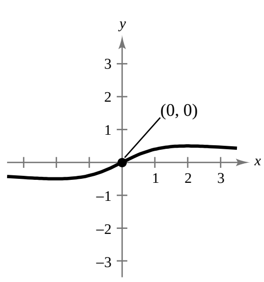
115. $f(x) = \dfrac{2x}{x^2 + 4}$

Intercept: $(0, 0)$

Origin symmetry

Horizontal asymptote: $y = 0$

x	-2	-1	0	1	2
y	$-\frac{1}{2}$	$-\frac{2}{5}$	0	$\frac{2}{5}$	$\frac{1}{2}$

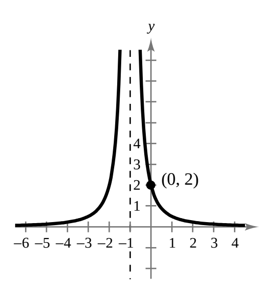


117. $f(x) = \dfrac{2}{(x + 1)^2}$

Intercept: $(0, 2)$

Horizontal asymptote: $y = 0$

Vertical asymptote: $x = -1$

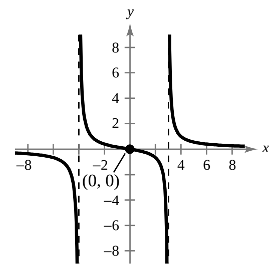


119. $f(x) = \dfrac{2x}{x^2 + x - 12} = \dfrac{2}{(x + 4)(x - 3)}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = -4,\ x = 3$

Horizontal asymptote: $y = 0$



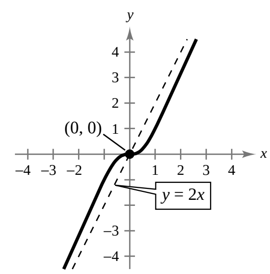
121. $f(x) = \dfrac{2x^3}{x^2 + 1} = 2x - \dfrac{2x}{x^2 + 1}$

Intercept: $(0, 0)$

Origin symmetry

Slant asymptote: $y = 2x$

x	-2	-1	0	1	2
y	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$

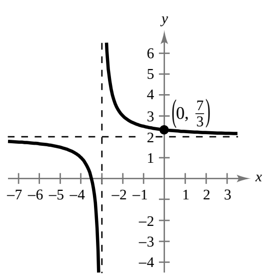


123. $y = \dfrac{1}{x + 3} + 2 = \dfrac{2x + 7}{x + 3}$

Intercepts: $(-3.5, 0), \left(0, 2\frac{1}{3}\right)$

Vertical asymptote: $x = -3$

Horizontal asymptote: $y = 2$



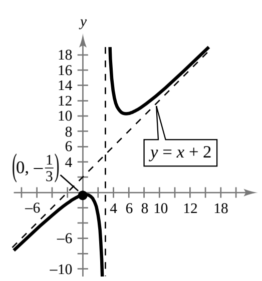
125. $f(x) = \dfrac{x^2 - x + 1}{x - 3}$

$= x + 2 + \dfrac{7}{x - 3}$

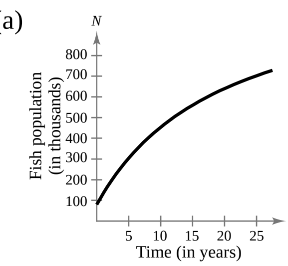
Intercept: $\left(0, -\frac{1}{3}\right)$

Vertical asymptote: $x = 3$

Slant asymptote: $y = x + 2$



127. (a)



(b) $N(5) = 304{,}000$ fish

$N(10) \approx 453{,}333$ fish

$N(25) \approx 702{,}222$ fish

(c) The limit is

$\frac{60}{0.05} = 1{,}200{,}000$ fish,

the horizontal asymptote.

129. True, the graphs are the same. $x^2 = |x^2|$.