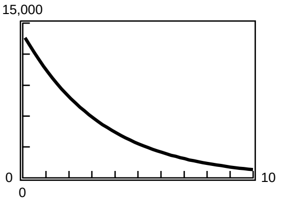
87.



y_3 is the best approximation of $y = e^x$.

89. f is one-to-one, so it has an inverse.

$$f(x) = -\tfrac{2}{3}x + \tfrac{5}{2}$$
$$y = -\tfrac{2}{3}x + \tfrac{5}{2}$$
$$x = -\tfrac{2}{3}y + \tfrac{5}{2}$$
$$x - \tfrac{5}{2} = -\tfrac{2}{3}y$$
$$-\tfrac{3}{2}\left(x - \tfrac{5}{2}\right) = y$$
$$f^{-1}(x) = -\tfrac{3}{2}x + \tfrac{15}{4}$$

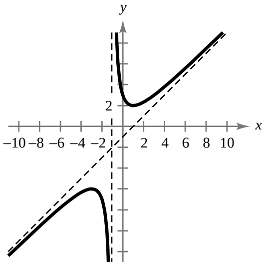
91. f is not one-to-one, so it does not have an inverse.

93. $f(x) = \dfrac{x^2 + 3}{x + 1} = x - 1 + \dfrac{4}{x + 1}$

Slant asymptote: $y = x - 1$

Vertical asymptote: $x = -1$

Intercept: $(0, 3)$

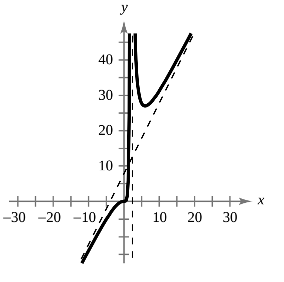


95. $f(x) = \dfrac{2x^3}{(x - 2)^2} = 2x + 8 + \dfrac{24x - 32}{x^2 - 4x + 4}$

Slant asymptote: $y = 2x + 8$

Vertical asymptote: $x = 2$

Intercept: $(0, 0)$



Section 3.2 Logarithmic Functions and Their Graphs

- You should know that a function of the form $y = \log_a x$, where $a > 0$, $a \neq 1$, and $x > 0$, is called a logarithm of x to base a.
- You should be able to convert from logarithmic form to exponential form and vice versa.

  $y = \log_a x \iff a^y = x$
- You should know the following properties of logarithms.
  - (a) $\log_a 1 = 0$ since $a^0 = 1$.
  - (b) $\log_a a = 1$ since $a^1 = a$.
  - (c) $\log_a a^x = x$ since $a^x = a^x$.
  - (d) If $\log_a x = \log_a y$, then $x = y$.

—CONTINUED—

—CONTINUED—

- You should know the definition of the natural logarithmic function.
  $\log_e x = \ln x,\ x > 0$
- You should know the properties of the natural logarithmic function.
  (a) $\ln 1 = 0$ since $e^0 = 1$.
  (b) $\ln e = 1$ since $e^1 = e$.
  (c) $\ln e^x = x$ since $e^x = e^x$.
  (d) If $\ln x = \ln y$, then $x = y$.
- You should be able to graph logarithmic functions.

Solutions to Odd-Numbered Exercises

1. $\log_4 64 = 3 \implies 4^3 = 64$

3. $\log_7 \frac{1}{49} = -2 \implies 7^{-2} = \frac{1}{49}$

5. $\log_{32} 4 = \frac{2}{5} \implies 32^{2/5} = 4$

7. $\ln 1 = 0 \implies e^0 = 1$

9. $5^3 = 125 \implies \log_5 125 = 3$

11. $81^{1/4} = 3 \implies \log_{81} 3 = \frac{1}{4}$

13. $6^{-2} = \frac{1}{36} \implies \log_6 \frac{1}{36} = -2$

15. $e^3 = 20.0855\ldots \implies \ln 20.0855\ldots = 3$

17. $e^{2.6} = 13.463\ldots \implies \ln 13.463\ldots = 2.6$

19. $\log_2 16 = \log_2 2^4 = 4$

21. $\log_{16}\left(\frac{1}{4}\right) = \log_{16}1 - \log_{16}4 = 0 - \log_{16}16^{1/2} = -\frac{1}{2}$

23. $\log_{10} 0.01 = \log_{10} 10^{-2} = -2$

25. $\log_7 x = \log_7 9$
$x = 9$

27. $\ln e^8 = x$
$8 \cdot \ln e = x$
$8 = x$

29. $\log_6 6^2 = x$
$2\log_6 6 = x$
$2 = x$

31. $\log_{10} 345 \approx 2.538$

33. $\log_{10}\left(\frac{4}{5}\right) \approx -0.097$

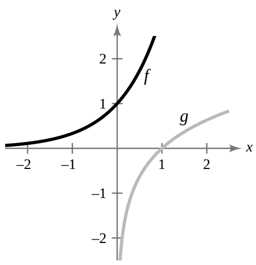
35. $\ln\left(4 + \sqrt{3}\right) \approx 1.746$

37. $\ln\sqrt{42} \approx 1.869$

39. $6 \log_{10} 14.8 \approx 7.022$

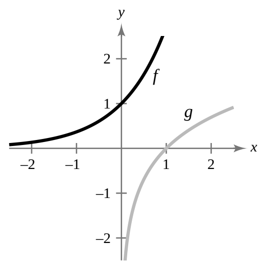
41. $12 \ln 6.4 \approx 22.276$

43. $f(x) = 3^x$, $g(x) = \log_3 x$



f and g are inverses. Their graphs are reflected about the line $y = x$.

45. $f(x) = e^x$, $g(x) = \ln x$



f and g are inverses. Their graphs are reflected about the line $y = x$.

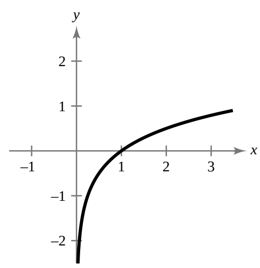
47. $f(x) = \log_3 x + 2$
Asymptote: $x = 0$
Point on graph: $(1, 2)$
Matches graph (c).

49. $f(x) = -\log_3(x + 2)$
Asymptote: $x = -2$
Point on graph: $(-1, 0)$
Matches graph (d).

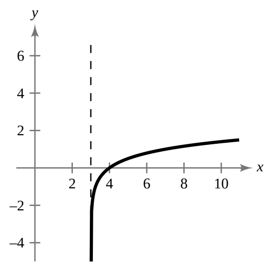
51. $f(x) = \log_3(1 - x)$
Asymptote: $x = 1$
Point on graph: $(0, 0)$
Matches graph (b).

53. $f(x) = \log_4 x$
Domain: $x > 0 \implies$ The domain is $(0, \infty)$.
Vertical asymptote: $x = 0$
x-intercept: $(1, 0)$
$y = \log_4 x \implies 4^y = x$

x	$\frac{1}{4}$	1	4	2
y	-1	0	1	$\frac{1}{2}$



55. $h(x) = \log_4(x - 3)$
Domain: $x - 3 > 0$ or $(3, \infty)$
Vertical asymptote: $x = 3$
Intercept: $(4, 0)$

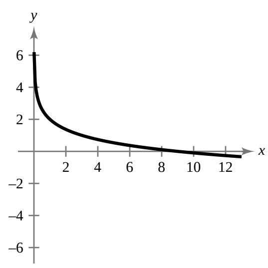


57. $y = -\log_3 x + 2$
Domain: $(0, \infty)$
Vertical asymptote: $x = 0$
x-intercept: $-\log_3 x + 2 = 0$

$$2 = \log_3 x$$
$$3^2 = x$$
$$9 = x$$

The x-intercept is $(9, 0)$.
$y = -\log_3 x + 2$
$\log_3 x = 2 - y \implies 3^{2-y} = x$

x	27	9	3	1	$\frac{1}{3}$
y	-1	0	1	2	3



59. $f(x) = 6 + \log_6(x - 3)$

Domain: $(3, \infty)$

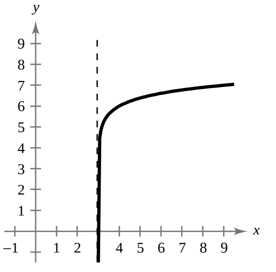
Vertical asymptote: $x = 3$

x-intercept: $\log_6(x - 3) = -6$

$$6^{-6} = x - 3$$

$$x = 3 + 6^{-6} \approx 3$$

x	4	9	$3\frac{1}{6}$
y	6	7	5



61. $y = \log_{10}\left(\frac{x}{5}\right)$

Domain: $\frac{x}{5} > 0 \implies x > 0$

The domain is $(0, \infty)$.

Vertical asymptote: $\frac{x}{5} = 0 \implies x = 0$

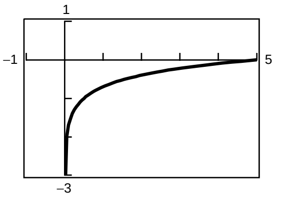
The vertical asymptote is the y-axis.

x-intercept: $\log_{10}\left(\frac{x}{5}\right) = 0$

$$\frac{x}{5} = 10^0$$

$$\frac{x}{5} = 1 \implies x = 5$$

The x-intercept is $(5, 0)$.



63. $f(x) = \ln(x - 2)$

Domain: $x - 2 > 0 \implies x > 2$

The domain is $(2, \infty)$.

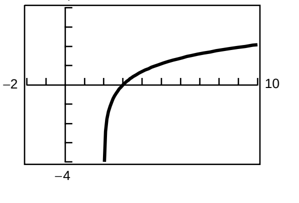
Vertical asymptote: $x - 2 = 0 \implies x = 2$

x-intercept: $0 = \ln(x - 2)$

$$e^0 = x - 2$$

$$3 = x$$

The x-intercept is $(3, 0)$.



65. $g(x) = \ln(-x)$

Domain: $-x > 0 \implies x < 0$

The domain is $(-\infty, 0)$.

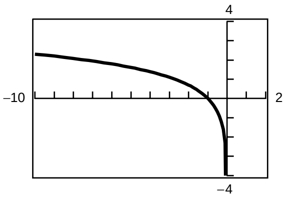
Vertical asymptote: $-x = 0 \implies x = 0$

x-intercept: $0 = \ln(-x)$

$e^0 = -x$

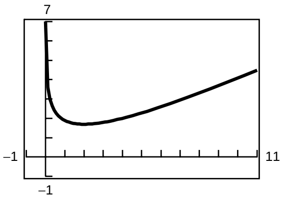
$-1 = x$

The x-intercept is $(-1, 0)$.



67. $f(x) = \dfrac{x}{2} - \ln\dfrac{x}{4}$

(a)



(b) Domain: $(0, \infty)$

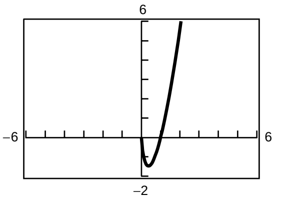
(c) Increasing on $(2, \infty)$

Decreasing on $(0, 2)$

(d) Relative minimum: $(2, 1.693)$

69. $h(x) = 4x \ln x$

(a)



(b) Domain: $(0, \infty)$

(c) Increasing on $(0.368, \infty)$

Decreasing on $(0, 0.368)$

(d) Relative minimum: $(0.368, -1.472)$

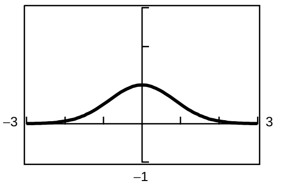
71. $t = \dfrac{10 \ln 2}{\ln 67 - \ln 50} \approx 23.68$ years

73. (a) $f(0) = 80 - 17 \log_{10}(0 + 1) = 80$

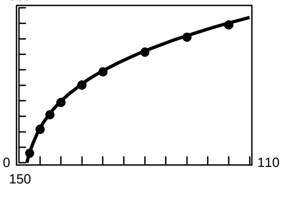
(b) $f(4) = 80 - 17 \log_{10}(4 + 1) \approx 68.1$

(c) $f(10) = 80 - 17 \log_{10}(10 + 1) \approx 62.3$

(d)



75. (a)



(b) $T > 300°$F when $p > 67.3$ pounds per square inch

[The graph of T and $y = 300$ intersect at $p = 67.3$.]

(c) $T(74) = 306.48°$F

77.

r	0.005	0.010	0.015
t	138.6 yr	69.3 yr	46.2 yr

r	0.020	0.025	0.030
t	34.7 yr	27.7 yr	23.1 yr

The doubling time decreases as r increases.

79. $\beta = 10 \log_{10}\left(\frac{I}{10^{-12}}\right) =$

(a) $I = 1$: $\beta = 10 \log_{10}\left(\frac{1}{10^{-12}}\right) = 10 \cdot \log_{10}(10^{12}) = 10(12) = 120$ decibels

(b) $I = 10^{-2}$: $\beta = 10 \log_{10}\left(\frac{10^{-2}}{10^{-12}}\right) = 10 \log_{10}(10^{10}) = 10(10) = 100$ decibels

(c) No, this is a logarithmic scale.

81. $y = 80.4 - 11 \ln x,\ \ 100 \le x \le 1500$

(a) $\frac{450 \text{ cubic ft per minute}}{30 \text{ children}} = 15$ cubic feet per minute per child

(b) From the graph, for $y = 15$ you get $x \approx 382$ cubic feet.

(c) If ceiling height is 30, then 382 square feet of floor space is needed.

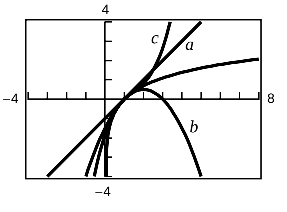
83. $t = 16.625 \ln\left(\frac{1659.24}{1659.24 - 750}\right) \approx 10$ years

85. Total amount $= (1659.24)(10)(12) \approx \$199,108.80$

Interest $= 199,108.80 - 150,000 = \$49,108.80$

87. True. $\log_3(27) = \log_3 3^3 = 3$

89.

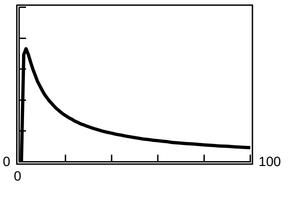


91. (a)

x	1	5	10	10^2	10^4	10^6
$f(x)$	0	0.322	0.230	0.046	0.00092	0.0000138

(b) As x increases without bound, $f(x)$ approaches 0.

(c)



93. Vertical asymptote: $x = 0$

No horizontal asymptote

95. Vertical asymptote: $x = 7$

No horizontal asymptote

97. $e^{7/2} \approx 33.115$

99. $6e^{-8} \approx 0.002$