Section 3.3 Properties of Logarithms

- You should know the following properties of logarithms.

  (a) $\log_a x = \dfrac{\log_b x}{\log_b a}$

  (b) $\log_a (uv) = \log_a u + \log_a v$ $\qquad \ln (uv) = \ln u + \ln v$

  (c) $\log_a (u/v) = \log_a u - \log_a v$ $\qquad \ln (u/v) = \ln u - \ln v$

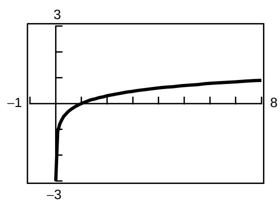
  (d) $\log_a u^n = n \log_a u$ $\qquad \ln u^n = n \ln u$
- You should be able to rewrite logarithmic expressions using these properties.

Solutions to Odd-Numbered Exercises

1. $f(x) = \log_{10} x$

$g(x) = \dfrac{\ln x}{\ln 10}$

$f(x) = g(x)$



3. $\log_3 7 = \dfrac{\ln 7}{\ln 3} \approx 1.771$

5. $\log_{1/2} 4 = \dfrac{\ln 4}{\ln \frac{1}{2}} = -2$

7. $\log_9(0.8) = \dfrac{\ln(0.8)}{\ln 9} \approx -0.102$

9. $\log_{15} 1460 = \dfrac{\ln 1460}{\ln 15} \approx 2.691$

11. (a) $\log_5 x = \dfrac{\log_{10} x}{\log_{10} 5}$

(b) $\log_5 x = \dfrac{\ln x}{\ln 5}$

13. (a) $\log_{1/5} x = \dfrac{\log_{10} x}{\log_{10} \frac{1}{5}} = \dfrac{\log_{10} x}{-\log_{10} 5}$

(b) $\log_{1/5} x = \dfrac{\ln x}{\ln \frac{1}{5}} = \dfrac{\ln x}{-\ln 5}$

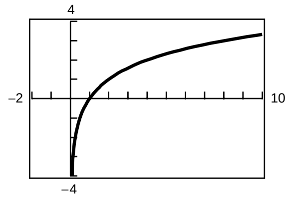
15. (a) $\log_a\left(\dfrac{3}{10}\right) = \dfrac{\log_{10}\left(\frac{3}{10}\right)}{\log_{10} a}$

(b) $\log_a\left(\dfrac{3}{10}\right) = \dfrac{\ln\left(\frac{3}{10}\right)}{\ln a}$

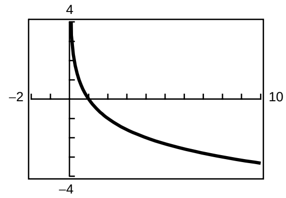
17. (a) $\log_{2.6} x = \dfrac{\log_{10} x}{\log_{10} 2.6}$

(b) $\log_{2.6} x = \dfrac{\ln x}{\ln 2.6}$

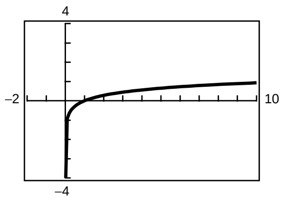
19. $f(x) = \log_2 x = \dfrac{\ln x}{\ln 2}$



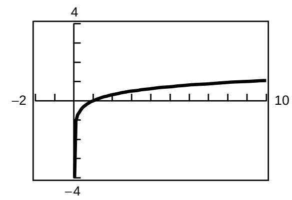
21. $f(x) = \log_{1/2} x = \dfrac{\ln x}{\ln \frac{1}{2}} = -\dfrac{\ln x}{\ln 2}$



23. $f(x) = \log_{11.8} x = \dfrac{\ln x}{\ln 11.8}$



25. $f(x) = \log_3 x^{1/2} = \dfrac{1}{2}\dfrac{\ln x}{\ln 3}$



27. $\log_{10} 5x = \log_{10} 5 + \log_{10} x$

29. $\log_{10} \dfrac{5}{x} = \log_{10} 5 - \log_{10} x$

31. $\log_8 x^4 = 4 \log_8 x$

33. $\ln\sqrt{z} = \ln z^{1/2} = \frac{1}{2} \ln z$

35. $\ln xyz = \ln x + \ln y + \ln z$

37. $\ln\sqrt{a-1} = \frac{1}{2} \ln(a-1)$

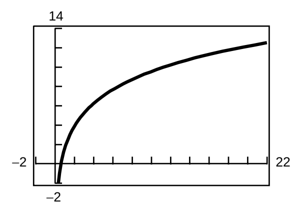
39. $\ln z(z-1)^2 = \ln z + \ln(z-1)^2$
$= \ln z + 2\ln(z-1)$

41. $\ln \sqrt[3]{\dfrac{x}{y}} = \dfrac{1}{3} \ln \dfrac{x}{y}$
$= \dfrac{1}{3}[\ln x - \ln y]$
$= \dfrac{1}{3} \ln x - \dfrac{1}{3} \ln y$

43. $\ln\left(\dfrac{x^4\sqrt{y}}{z^5}\right) = \ln x^4\sqrt{y} - \ln z^5$
$= \ln x^4 + \ln\sqrt{y} - \ln z^5$
$= 4\ln x + \dfrac{1}{2}\ln y - 5\ln z$

45. $\log_b\left(\dfrac{x^2}{y^2z^3}\right) = \log_b x^2 - \log_b y^2z^3$
$= \log_b x^2 - [\log_b y^2 + \log_b z^3]$
$= 2\log_b x - 2\log_b y - 3\log_b z$

47. $y_1 = \ln[x^3(x+4)]$
$y_2 = 3\ln x + \ln(x+4)$
$y_1 = y_2$, for positive values of x.



49. $\ln x + \ln 4 = \ln 4x$

51. $\log_4 z - \log_4 y = \log_4 \dfrac{z}{y}$

53. $2\log_2(x+3) = \log_2(x+3)^2$

55. $\frac{1}{3}\log_3 7x = \log_3(7x)^{1/3} = \log_3 \sqrt[3]{7x}$

57. $\ln x - 3\ln(x+1) = \ln x - \ln(x+1)^3$
$= \ln \dfrac{x}{(x+1)^3}$

59. $\ln(x-2) - \ln(x+2) = \ln\left(\dfrac{x-2}{x+2}\right)$

61. $\ln x - 2[\ln(x+2) + \ln(x-2)] = \ln x - 2\ln[(x+2)(x-2)]$

$= \ln x - 2\ln(x^2 - 4)$

$= \ln x - \ln(x^2-4)^2$

$= \ln \dfrac{x}{(x^2-4)^2}$

63. $\frac{1}{3}[2\ln(x+3) + \ln x - \ln(x^2-1)] = \frac{1}{3}[\ln(x+3)^2 + \ln x - \ln(x^2-1)]$

$= \frac{1}{3}[\ln[x(x+3)^2] - \ln(x^2-1)]$

$= \frac{1}{3}\ln \dfrac{x(x+3)^2}{x^2-1}$

$= \ln \sqrt[3]{\dfrac{x(x+3)^2}{x^2-1}}$

65. $\frac{1}{3}[\ln y + 2\ln(y+4)] - \ln(y-1) = \frac{1}{3}[\ln y + \ln(y+4)^2] - \ln(y-1)$

$= \frac{1}{3}\ln[y(y+4)^2] - \ln(y-1)$

$= \ln\sqrt[3]{y(y+4)^2} - \ln(y-1)$

$= \ln \dfrac{\sqrt[3]{y(y+4)^2}}{y-1}$

67. $2\ln 3 - \frac{1}{2}\ln(x^2+1) = \ln 3^2 - \ln\sqrt{x^2+1}$

$= \ln \dfrac{9}{\sqrt{x^2+1}}$

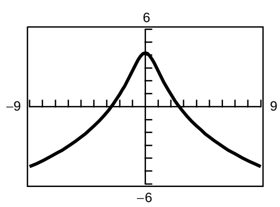
69. $y_1 = 2[\ln 8 - \ln(x^2+1)]$

$y_2 = \ln\left[\dfrac{64}{(x^2+1)^2}\right]$

$y_1 = 2[\ln 8 - \ln(x^2+1)]$

$= 2\ln\left(\dfrac{8}{x^2+1}\right)$

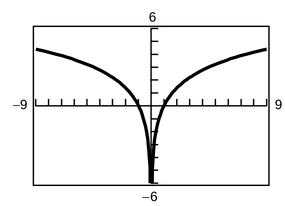
$= \ln\left[\dfrac{64}{(x^2+1)^2}\right] = y_2$



71. $y_1 = \ln x^2$

$y_2 = 2\ln x$

$y_1 = y_2$ for $x > 0$.



They are not equivalent. The domain of $f(x)$ is all real numbers except 0. The domain of $g(x)$ is $x > 0$.

73. $\log_3 9 = 2\log_3 3 = 2$

75. $\log_4 16^{3.4} = 3.4\log_4(4^2) = 6.8\log_4 4 = 6.8$

77. $\log_2(-4)$ is undefined. -4 is not in the domain of $f(x) = \log_2 x$

79. $\log_5 75 - \log_5 3 = \log_5 \frac{75}{3} = \log_5 25 = \log_5 5^2 =$

81. $\ln e^3 - \ln e^7 = 3 - 7 = -4$

83. $\log_{10} 0$ is undefined. 0 is not in the domain of $\log_{10} x$.

85. $\ln e^{8.5} = 8.5$

87. $\log_4 8 = \log_4 2^3 = 3 \log_4 2$

$= 3 \log_4 \sqrt{4} = 3 \log_4 4^{1/2}$

89. $\log_7 \sqrt{70} = \frac{1}{2} \log_7 70 = \frac{1}{2} \log_7(10 \cdot 7)$

$= \frac{1}{2} \log_7 10 + \frac{1}{2} \log_7 7 = \frac{1}{2} \log_7 10 + \frac{1}{2}$

91. $\log_5 \frac{1}{250} = \log_5 1 - \log_5 250 = 0 - \log_5 (125 \cdot 2$

$= -\log_5(5^3 \cdot 2) = -(\log_5 5^3 + \log_5 2)$

$= -(3 \log_5 5 + \log_5 2) = -3 - \log_5 2$

93. $\ln(5e^6) = \ln 5 + \ln e^6 = \ln 5 + 6 = 6 + \ln 5$

95. (a) $\beta = 10 \cdot \log_{10}\left(\frac{I}{10^{-12}}\right) = 10(\log_{10} I - \log_{10} 10^{-12})$

$= 10[\log_{10} I - (-12) \log_{10} 10]$

$= 10 (\log_{10} I + 12) = 120 + 10 \cdot \log_{10} I$

(b)

I	10^{-4}	10^{-6}	10^{-8}	10^{-10}	10^{-12}	10^{-14}
β	80	60	40	20	0	-20

(c) $\beta(10^{-4}) = 120 + 10 \cdot \log_{10} 10^{-4} = 120 - 40 = 80$

$\beta(10^{-6}) = 120 + 10 \cdot \log_{10} 10^{-6} = 120 - 60 = 60$

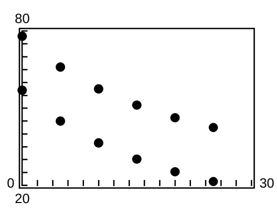
$\beta(10^{-8}) = 120 + 10 \cdot \log_{10} 10^{-8} = 120 - 80 = 40$

$\beta(10^{-10}) = 120 + 10 \cdot \log_{10} 10^{-10} = 120 - 100 = 20$

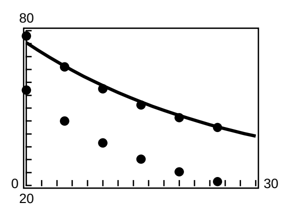
$\beta(10^{-12}) = 120 + 10 \cdot \log_{10} 10^{-12} = 120 - 120 = 0$

$\beta(10^{-14}) = 120 + 10 \cdot \log_{10} 10^{-14} = 120 - 140 = -20$

97. (a)



(b) The data $(t, T - 21)$ fits the exponential model $T - 21 = 54.4380(0.9635)^t$. For the original data the model is $T = 54.4380(0.9635)^t$.

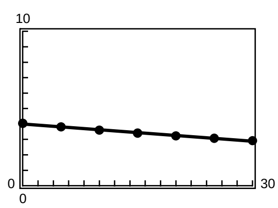


(c) $\ln(T - 21) = -0.03721t + 3.9971$

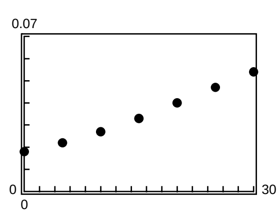
$T - 21 = e^{-0.0377t + 3.9971}$

$T = 21 + 54.44(e^{-0.0377t})$

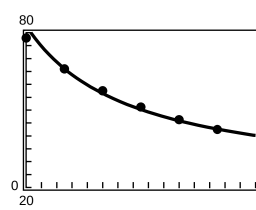
$= 21 + 54.44(0.9635)^t$



(d) $T = \dfrac{4960}{6t + 80} + 21$



(e)



99. $f(x) = \ln x$

False, $f(0) \neq 0$ since 0 is not in the domain of $f(x)$. $f(1) = \ln 1 = 0$

101. False, $f(x) - f(2) = \ln x - \ln 2 = \ln \dfrac{x}{2} \neq \ln(x - 2)$.

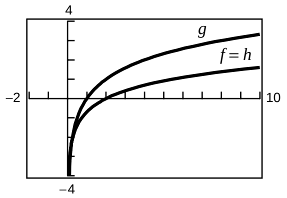
103. False, $f(u) = 2f(v) \implies \ln u = 2 \ln v \implies \ln u = \ln v^2 \implies u = v^2$.

105. $f(x) = \ln \dfrac{x}{2}$

$g(x) = \dfrac{\ln x}{\ln 2}$

$h(x) = \ln x - \ln 2$

$f(x) = h(x)$ by Property 2.



107. Let $x = \log_b u$ and $y = \log_b v$, then $b^x = u$ and $b^y = v$.

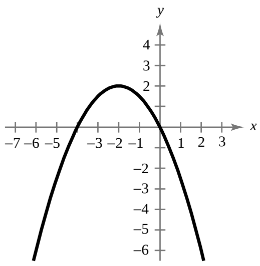
$$\frac{u}{v} = \frac{b^x}{b^y} = b^{x-y}$$

$$\log_b\left(\frac{u}{v}\right) = \log_b(b^{x-y}) = x - y = \log_b u - \log_b v$$

109. $f(x) = -\frac{1}{2}(x^2 + 4x)$

Intercepts: $(0, 0), (-4, 0)$

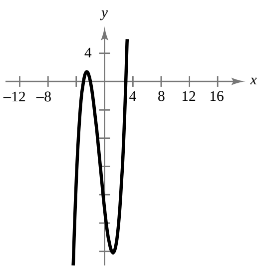
Parabola opening downward.



111. $f(x) = x^3 + 2x^2 - 9x - 18 = x^2(x + 2) - 9(x +$

$= (x + 2)(x - 3)(x + 3)$

Zeros: $x = -2, -3, 3$



113. $x^2 - 6x + 2 = 0$

$$x = \frac{6 \pm \sqrt{36 - 4(2)}}{2} = 3 \pm \sqrt{7}$$

115. $x^4 - 19x^2 + 48 = 0$

$(x^2 - 16)(x^2 - 3) = 0$

$(x - 4)(x + 4)\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) = 0$

$x = \pm 4, \pm 3$

117. $x^3 - 6x^2 - 4x + 24 = 0$

$x^2(x - 6) - 4(x - 6) = 0$

$(x^2 - 4)(x - 6) = 0$

$(x - 2)(x + 2)(x - 6) = 0$

$x = 2, -2, 6$

119. $1.6^{-2\pi} \approx 0.052$

121. $260^{\sqrt{3}} \approx 15{,}235.494$

123. $\log_{10}(220) \approx 2.342$

125. $\ln 2.008 \approx 0.697$