Section 3.4 Solving Exponential and Logarithmic Equations

- To solve an exponential equation, isolate the exponential expression, then take the logarithm of both sides. Then solve for the variable.
  1. $\log_a a^x = x$
  2. $\ln e^x = x$
- To solve a logarithmic equation, rewrite it in exponential form. Then solve for the variable.
  1. $a^{\log_a x} = x$
  2. $e^{\ln x} = x$
- If $a > 0$ and $a \neq 1$ we have the following:
  1. $\log_a x = \log_a y \implies x = y$
  2. $a^x = a^y \implies x = y$
- Use your graphing utility to approximate solutions.

Solutions to Odd-Numbered Exercises

1. $4^{2x-7} = 64$

(a) $x = 5$

$4^{2(5)-7} = 4^3 = 64$

Yes, $x = 5$ is a solution.

(b) $x = 2$

$4^{2(2)-7} = 4^{-3} = \frac{1}{64} \neq 64$

No, $x = 2$ is not a solution.

3. $3e^{x+2} = 75$

(a) $x = -2 + e^{25}$

$3e^{(-2+e^{25})+2} = 3e^{e^{25}} \neq 75$

No, $x = -2 + e^{25}$ is not a solution.

(b) $x = -2 + \ln 25$

$3e^{(-2+\ln 25)+2} = 3e^{\ln 25} = 3(25) = 75$

Yes, $x = -2 + \ln 25$ is a solution.

(c) $x \approx 1.2189$

$3e^{1.2189+2} = 3e^{3.2189} \approx 75$

Yes, $x \approx 1.2189$ is a solution.

5. $\log_4(3x) = 3 \implies 3x = 4^3 \implies 3x = 64$

(a) $x \approx 20.3560$

$3(20.3560) = 61.0680 \neq 64$

No, $x \approx 20.3560$ is not a solution.

(b) $x = -4$

$3(-4) = -12 \neq 64$

No, $x = -4$ is not a solution.

(c) $x = \frac{64}{3}$

$3\left(\frac{64}{3}\right) = 64$

Yes, $x = \frac{64}{3}$ is a solution.

7. $\ln(x - 1) = 3.8$

(a) $x = 1 + e^{3.8}$

$\ln(1 + e^{3.8} - 1) = \ln e^{3.8} = 3.8$

Yes, $x = 1 + e^{3.8}$ is a solution.

(b) $x \approx 45.7012$

$\ln(45.7012 - 1) = \ln(44.7012) \approx 3.8$

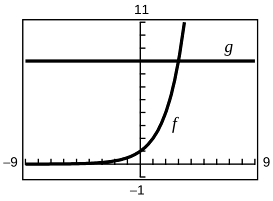
Yes, $x \approx 45.7012$ is a solution.

(c) $x = 1 + \ln 3.8$

$\ln(1 + \ln 3.8 - 1) = \ln(\ln 3.8) \approx 0.289$

No, $x = 1 + \ln 3.8$ is not a solution.

9.



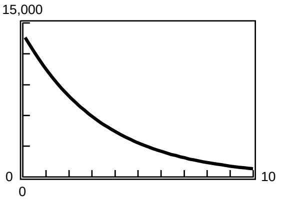
Intersection Point: $(3, 8)$

Algebraically, $2^x = 8$

$2^x = 2^3$

$x = 3 \Longrightarrow y = 8 \Longrightarrow (3, 8)$

11.

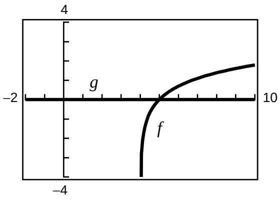


Intersection Point: $(9, 2)$

Algebraically, $\log_3 x = 2$

$x = 3^2 = 9 \Longrightarrow y = 2 \Longrightarrow (9, 2)$

13.



Intersection Point: $(5, 0)$

Algebraically, $\ln(x - 4) = 0$

$x - 4 = e^0 = 1$

$x = 5 \Longrightarrow y = 0 \Longrightarrow (5, 0)$

15. $4^x = 16$

$4^x = 4^2$

$x = 2$

17. $5^x = 625$

$5^x = 5^4$

$x = 4$

19. $8^x = 4$

$8^x = 8^{2/3}$

$x = \frac{2}{3}$

21. $\left(\frac{1}{4}\right)^x = 64$

$\left(\frac{1}{4}\right)^x = 4^3$

$\left(\frac{1}{4}\right)^x = \left(\frac{1}{4}\right)^{-3}$

$x = -3$

23. $3^{x-1} = 27$

$3^{x-1} = 3^3$

$x - 1 = 3$

$x = 4$

25. $\ln x - \ln 5 = 0$

$\ln x = \ln 5$

$x = 5$

27. $e^x = 4$

$x = \ln 4 \approx 1.386$

29. $\ln x = -7$

$x = e^{-7}$

31. $\log_x 625 = 4$

$x^4 = 625$

$x^4 = 5^4$

$x = 5$

33. $\log_{10} x = -1$

$x = 10^{-1}$

$x = \frac{1}{10}$

35. $\ln(2x - 1) = 0$

$e^0 = 2x - 1$

$1 = 2x - 1$

$2 = 2x$

$1 = x$

37. $\ln e^{x^2} = x^2 \ln e = x^2$

39. $e^{\ln(5x+2)} = 5x + 2$

41. $e^{\ln x^2} = x^2$

43.
$$10^x = 570$$
$$\log_{10} 10^x = \log_{10} 570$$
$$x = \log_{10} 570 \approx 2.756$$

45.
$$e^x = 10$$
$$x = \ln 10 \approx 2.303$$

47.
$$5^{-t/2} = 0.20 = \frac{1}{5}$$
$$-\frac{t}{2}\ln 5 = \ln\left(\frac{1}{5}\right)$$
$$-\frac{t}{2}\ln 5 = -\ln 5$$
$$\frac{t}{2} = 1$$
$$t = 2$$

49.
$$2^{3-x} = 565$$
$$(3 - x)\ln 2 = \ln 565$$
$$-x\ln 2 = \ln 565 - 3\ln 2$$
$$x = (3\ln 2 - \ln 565)/\ln 2 \approx -6.142$$

51.
$$500e^{-x} = 300$$
$$e^{-x} = \tfrac{3}{5}$$
$$-x = \ln\tfrac{3}{5}$$
$$x = -\ln\tfrac{3}{5} = \ln\tfrac{5}{3} \approx 0.511$$

53.
$$7 - 2e^x = 5$$
$$-2e^x = -2$$
$$e^x = 1$$
$$x = \ln 1 = 0$$

55.
$$e^{2x} - 4e^x - 5 = 0$$
$$(e^x - 5)(e^x + 1) = 0$$
$$e^x = 5 \text{ or } e^x = -1$$
$$x = \ln 5 \approx 1.609$$

($e^x = -1$ is impossible.)

57.
$$50(120 - e^{x/2}) = 600$$
$$120 - e^{x/2} = 12$$
$$e^{x/2} = 108$$
$$\frac{x}{2} = \ln 108$$
$$x = 2\ln 108 \approx 9.364$$

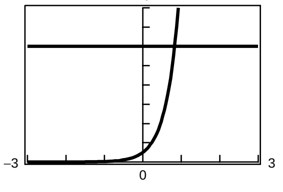
59. Using the root feature of a graphing utility for

$$y = \left(1 + \frac{0.10}{12}\right)^{12t} - 2 = 0,$$

you obtain $t \approx 6.960$.

61.

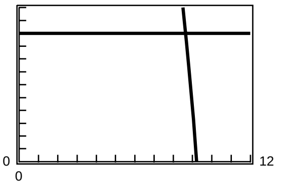
x	0.6	0.7	0.8	0.9	1.0
$f(x)$	6.05	8.17	11.02	14.88	20.09



$x \approx 0.828$

63.

x	5	6	7	8	9
$f(x)$	1756	1598	1338	908	200



$x \approx 8.635$

65. $2^{3x} = 50$

Graphing $y = 2^{3x} - 50$, you obtain $x \approx 1.881$

67. $2^{-3x} = 0.90$

Graphing $y = 2^{-3x} - 0.90$, you obtain $x \approx 0.051$

69. $5(10^{x-6}) = 7$

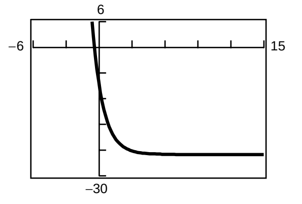
Graphing $y = 5(10^{x-6}) - 7$, you obtain $x \approx 6.146$

71. $\left(1 + \dfrac{0.065}{365}\right)^{365t} = 4 \Longrightarrow t = 21.330$

73. $\dfrac{3000}{2 + e^{2x}} = 2$

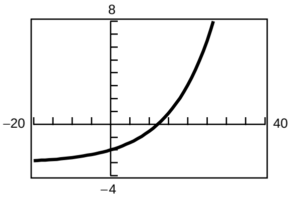
$$1500 = 2 + e^{2x}$$
$$1498 = e^{2x}$$
$$\ln 1498 = \ln e^{2x}$$
$$\ln 1498 = 2x$$
$$\frac{\ln 1498}{2} = x \approx 3.656$$

75. $g(x) = 6e^{1-x} - 25$



zero at $x = -0.427$

77. $g(t) = e^{0.09t} - 3$



zero at $t = 12.207$

79. $\ln x = -3$

$$x = e^{-3} \approx 0.050$$

81. $\ln 4x = 2.1$

$$4x = e^{2.1}$$
$$x = \tfrac{1}{4}e^{2.1}$$
$$\approx 2.042$$

83. $2\ln 3x = 19$

$$\ln 3x = \frac{19}{2} = 9.5$$
$$3x = e^{9.5}$$
$$x = \frac{1}{3}e^{9.5} \approx 4453.242$$

85. $\log_{10}(z - 3) = 2$

$$z - 3 = 10^2$$
$$z = 10^2 + 3 = 103$$

87. $7 \log_4 (0.6x) = 12$

$$\log_4(0.6x) = \frac{12}{7}$$
$$4^{12/7} = 0.6x = \frac{3}{5}x$$
$$x = \frac{5}{3}4^{12/7} \approx 17.945$$

89. $\ln\sqrt{x + 2} = 1$

$$\sqrt{x + 2} = e^1$$
$$x + 2 = e^2$$
$$x = e^2 - 2 \approx 5.389$$

91. $\ln(x + 1)^2 = 2$

$$e^{\ln(x+1)^2} = e^2$$
$$(x + 1)^2 = e^2$$
$$x + 1 = e \text{ or } x + 1 = -e$$
$$x = e - 1 \approx 1.718$$
or
$$x = -e - 1 \approx -3.718$$

93. $\log_4 x - \log_4(x - 1) = \dfrac{1}{2}$

$$\log_4\left(\frac{x}{x - 1}\right) = \frac{1}{2}$$
$$4^{\log_4(x/x-1)} = 4^{1/2}$$
$$\frac{x}{x - 1} = 2$$
$$x = 2(x - 1)$$
$$x = 2x - 2$$
$$2 = x$$

95. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1).$

$$\ln(x+5) = \ln\left(\frac{x-1}{x+1}\right)$$

$$x + 5 = \frac{x-1}{x+1}$$

$$(x+5)(x+1) = x - 1$$

$$x^2 + 6x + 5 = x - 1$$

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0$$

$$x = -2 \text{ or } x = -3$$

Both of these solutions are extraneous, so the equation has no solution.

97. $\log_{10} 8x - \log_{10}(1 + \sqrt{x}) = 2$

$$\log_{10} \frac{8x}{1+\sqrt{x}} = 2$$

$$\frac{8x}{1+\sqrt{x}} = 10^2$$

$$8x = 100 + 100\sqrt{x}$$

$$8x - 100\sqrt{x} - 100 = 0$$

$$2x - 25\sqrt{x} - 25 = 0$$

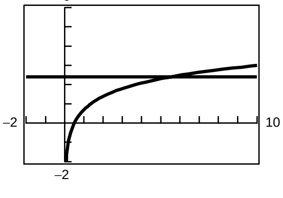
$$\sqrt{x} = \frac{25 \pm \sqrt{25^2 - 4(2)(-25)}}{4}$$

$$= \frac{25 \pm 5\sqrt{33}}{4}$$

Choosing the positive value, we have $\sqrt{x} \approx 13.431$ and $x \approx 180.384$.

99.

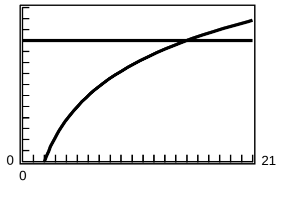
x	2	3	4	5	6
$f(x)$	1.39	1.79	2.08	12.30	2.49



$x \approx 5.512$

101.

x	12	13	14	15	16
$f(x)$	9.79	10.22	10.63	11.00	11.36



$x \approx 14.988$

103. $\log_{10}(z - 4) = 1$

Graphing $y = \log_{10}(z - 4) - 1$, you obtain $z = 14$.

105. $3 \ln x = 5$

Graphing $y = 3 \ln x - 5$, you obtain $x \approx 5.294$

107. $\ln x + \ln(x - 3) = 1$

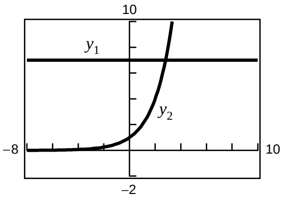
Graphing $y = \ln x + \ln(x - 3) - 1$, you obtain $x \approx 3.729$

109. $\ln(x - 5) = \ln(x - 3) - \ln(x + 3)$

Graphing $y = \ln(x - 5) - \ln(x - 3) + \ln(x + 3)$, you obtain $x \approx 5.275$

111. $y_1 = 7$

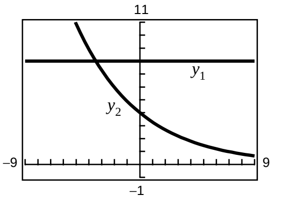
$y_2 = 2^x$



From the graph we have $(x, y) \approx (2.807, 7)$.

113. $y_1 = 8$

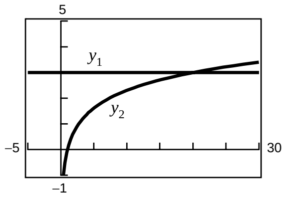
$y_2 = 4e^{-0.2x}$



From the graph, we have $(x, y) \approx (-3.466, 8)$.

115. $y_1 = 3$

$y_2 = \ln x$



From the graph we have $(x, y) \approx (20.086, 3)$.

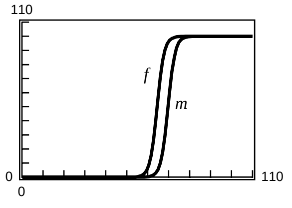
117. (a)

$$\begin{aligned} A &= Pe^{rt} \\ 2000 &= 1000e^{0.085t} \\ 2 &= e^{0.085t} \\ \ln 2 &= 0.085t \\ \frac{\ln 2}{0.085} &= t \\ t &\approx 8.2 \text{ years} \end{aligned}$$

(b)

$$\begin{aligned} 3000 &= 1000e^{0.085t} \\ 3 &= e^{0.085t} \\ \ln 3 &= 0.085t \\ \frac{\ln 3}{0.085} &= t \\ t &\approx 12.9 \text{ years} \end{aligned}$$

119. (a)



(b) From the graph we see horizontal asymptotes at $y = 0$ and $y = 100$. These represent the lower and upper percent bounds.

(c) Males:

$$\begin{aligned} 50 &= \frac{100}{1 + e^{-0.6114(x-69.71)}} \\ 1 + e^{-0.6114(x-69.71)} &= 2 \\ e^{-0.6114(x-69.71)} &= 1 \\ -0.6114(x - 69.71) &= \ln 1 \\ -0.6114(x - 69.71) &= 0 \\ x &= 69.71 \text{ inches} \end{aligned}$$

Females:

$$\begin{aligned} 50 &= \frac{100}{1 + e^{-0.66607(x-64.51)}} \\ 1 + e^{-0.66607(x-64.51)} &= 2 \\ e^{-0.66607(x-64.51)} &= 1 \\ -0.66607(x - 64.51) &= \ln 1 \\ -0.66607(x - 64.51) &= 0 \\ x &= 64.51 \text{ inches} \end{aligned}$$

121. $p = 500 - 0.5(e^{0.004x})$

(a)

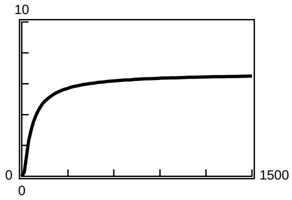
$$\begin{aligned} p &= 350 \\ 350 &= 500 - 0.5(e^{0.004x}) \\ 300 &= e^{0.004x} \\ 0.004x &= \ln 300 \\ x &\approx 1426 \text{ units} \end{aligned}$$

(b)

$$\begin{aligned} p &= 300 \\ 300 &= 500 - 0.5(e^{0.004x}) \\ 400 &= e^{0.004x} \\ 0.004x &= \ln 400 \\ x &\approx 1498 \text{ units} \end{aligned}$$

123. $V = 6.7e^{-48.1/t},\ t > 0$

(a)



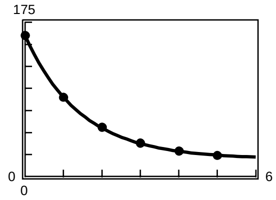
(b) As $t \longrightarrow \infty$, $V \longrightarrow 6.7$.

Horizontal asymptote: $y = 6.7$
The yield will approach
6.7 million cubic feet per acre.

(c)
$$1.3 = 6.7e^{-48.1/t}$$
$$\frac{1.3}{6.7} = e^{-48.1/t}$$
$$\ln\frac{13}{67} = \frac{-48.1}{t}$$
$$t = \frac{-48.1}{\ln\left(\frac{13}{67}\right)} \approx 29.3 \text{ years}$$

125. $T = 20[1 + 7(2^{-h})]$

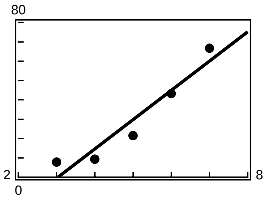
(a)



(b) We see a horizontal asymptote at $y = 20$. This represents the room temperature.

(c)
$$100 = 20[1 + 7(2^{-h})]$$
$$5 = 1 + 7(2^{-h})$$
$$4 = 7(2^{-h})$$
$$\frac{4}{7} = 2^{-h}$$
$$\ln\left(\frac{4}{7}\right) = \ln 2^{-h}$$
$$\ln\left(\frac{4}{7}\right) = -h \ln 2$$
$$\frac{\ln(4/7)}{-\ln 2} = h$$
$$h \approx 0.81 \text{ hour}$$

127. (a)

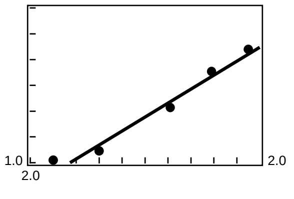


$y = 15.17x - 46.15$

$y = 100$ when $x \approx 9.6$, or during 1999.

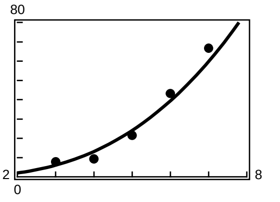
(b)

$\ln x$	1.0986	1.3863	1.6094	1.7918	1.9459
$\ln y$	2.0541	2.2300	3.0681	3.7658	4.2002



(c) $y = e^{2.706 \ln x - 1.175} = e^{\ln x^{2.706} - 1.175} = e^{-1.175}x^{2.706}$

$= 0.309x^{2.706}$



The second model is better.

$$\ln y = 2.706 \ln x - 1.175$$

For $y = 100$, $\ln 100 = 2.706 \ln x - 1.175$
$$5.780 = 2.706 \ln x$$
$$\ln x = 2.136$$
$$x \approx 8.5 \text{ or during 1998}$$