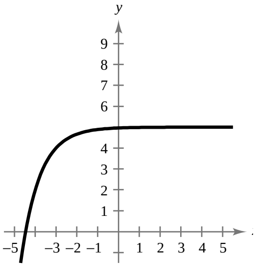
129. False. A logarithmic equation can have any number of extraneous solutions

131. To find the length of time it takes for an investment P to double to $2P$, solve

$$2P = Pe^{rt}$$
$$2 = e^{rt}$$
$$\ln 2 = rt$$
$$\frac{\ln 2}{r} = t.$$

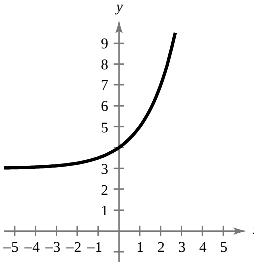
Thus, you can see that the time is not dependent on the size of the investment, but rather the interest rate.

133. $f(x) = -3^{-x-3} + 5$



135. $f(x) = \left(\frac{1}{2}\right)^{-x} + 3$

$= 2^x + 3$



137. $\log_3 22 = \frac{\ln 22}{\ln 3} \approx 2.814$

139. $\log_{21} 140 = \frac{\ln 140}{\ln 21} \approx 1.623$

Section 3.5 Exponential and Logarithmic Models

- You should be able to solve compound interest problems.
  1. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
  2. $A = Pe^{rt}$
- You should be able to solve growth and decay problems.
  (a) Exponential growth if $b > 0$ and $y = ae^{bx}$.
  (b) Exponential decay if $b > 0$ and $y = ae^{-bx}$.
- You should be able to use the Gaussian model
  $y = ae^{-(x-b)^2/c}$.
- You should be able to use the logistics growth model
  $y = \frac{a}{1 + be^{-(x-c)/d}}$.
- You should be able to use the logarithmic models
  $y = \ln(ax + b)$ and $y = \log_{10}(ax + b)$.

Solutions to Odd-Numbered Exercises

1. $y = 2e^{x/4}$

This is an exponential growth model. Matches graph (c).

3. $y = 6 + \log_{10}(x + 2)$

This is a logarithmic model, and contains $(-1, 6)$. Matches graph (b).

5. $y = \ln(x + 1)$

This is a logarithmic model. Matches graph (d).

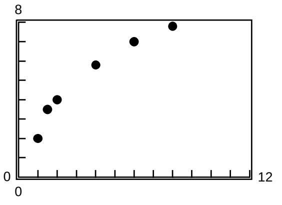
7. A logarithmic model seems best.

9. A Gaussian model seems best.

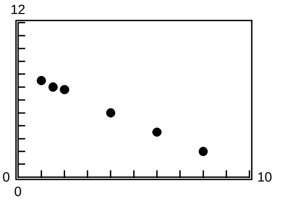
11. An exponential model seems best.

13. A Gaussian model seems best.

15. Logarithmic model



17. Linear model



19. Since $A = 1000e^{0.12t}$, the time to double is given by $2000 = 1000e^{0.12t}$ and we have

$$t = \frac{\ln 2}{0.12} \approx 5.78 \text{ years.}$$

Amount after 10 years: $A = 1000e^{1.2} \approx \$3320.12$

21. Since $A = 750e^{rt}$ and $A = 1500$ when $t = 7.75$, we have the following.

$$15000 = 750e^{7.75r}$$

$$r = \frac{\ln 2}{7.75} \approx 0.0894 = 8.94\%$$

Amount after 10 years: $A = 750e^{0.0894(10)} \approx \$1833.67$

23. Since $A = 500e^{rt}$ and $A = 1292.85$ when $t = 10$, we have the following.

$$1292.85 = 500e^{10r}$$

$$r = \frac{\ln(1292.85/500)}{10} \approx 0.0950 = 9.5\%$$

The time to double is given by

$$1000 = 500e^{0.095t}$$

$$t = \frac{\ln 2}{0.095} \approx 7.30 \text{ years.}$$

25. Since $A = Pe^{0.045t}$ and $A = 10{,}000.00$ when $t = 10$, we have the following.

$$10{,}000.00 = Pe^{0.045(10)}$$

$$\frac{10{,}000.00}{e^{0.045(10)}} = P \approx 6376.28$$

The time to double is given by

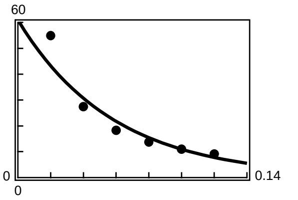
$$t = \frac{\ln 2}{0.045} \approx 15.40 \text{ years.}$$

27. (a)

$$3P = Pe^{rt}$$
$$3 = e^{rt}$$
$$\ln 3 = rt$$
$$\frac{\ln 3}{r} = t$$

r	2%	4%	6%	8%	10%	12%
$t = \frac{\ln 3}{r}$	54.93	27.47	18.31	13.73	10.99	9.16

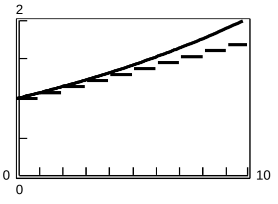
(b) $t = 60.89(3.613 \times 10^{-8})^r$



29. Continuous compounding results in faster growth.

$A = 1 + 0.075[\![t]\!]$

and $A = e^{0.07t}$



31. $\frac{1}{2}C = Ce^{k(1620)}$

$$k = \frac{\ln 0.5}{1620}$$

Given $C = 10$ grams, after 1000 years, we have

$y = Ce^{[(\ln 0.5)/1620](1000)}$

$C \approx 6.52$ grams.

33. $\frac{1}{2}C = Ce^{k(5730)}$

$$k = \frac{\ln 0.5}{5730}$$

Given $y = 3$ grams, after 1000 years, we have

$3 = Ce^{[(\ln 0.5)/5730](1000)}$

$C \approx 2.66$ grams.

35.

$$P = 105{,}300e^{0.015t}$$
$$150{,}000 = 105{,}300e^{0.015t}$$
$$\ln \tfrac{1500}{1053} = 0.015t$$
$$t \approx 23.59$$

The population will reach 150,000 during 2023.
[Note: 2000 + 13.59.]

37. $P = 2500e^{kt}$, $P(0) = 2500$ represents year 2000

For 1945, $t = -55$ and

$$1350 = 2500e^{k(-55)}$$
$$\ln\frac{1350}{2500} = -55k$$
$$k \approx 0.0112$$

For 2010, $t = 10$ and

$P \approx 2500e^{0.0112(10)} \approx 2796$ people.

39.

$$N = 100e^{kt}$$
$$300 = 100e^{5k}$$
$$k = \frac{\ln 3}{5} \approx 0.2197$$
$$N = 100e^{0.2197t}$$
$$200 = 100e^{0.2197t}$$
$$t = \frac{\ln 2}{0.2197} \approx 3.15 \text{ hours}$$

41.

$$y = Ce^{kt}$$
$$\frac{1}{2}C = Ce^{(1620)k}$$
$$\ln\frac{1}{2} = 1620k$$
$$k = \frac{\ln(1/2)}{1620}$$

When $t = 100$, we have

$y = Ce^{[\ln(1/2)/1620](100)} \approx 0.958C = 95.8\%C.$

After 100 years, approximately 95.8% of the radioactive radium will remain.

43. (a) $V = mt + b$; $V(0) = 22{,}000 \Longrightarrow b = 22{,}000$

$V(2) = 13{,}000 \Longrightarrow 13{,}000 = 2m + 22{,}000 \Longrightarrow m = -4500$

$V(t) = -4500t + 22{,}000$

(b) $V = ae^{kt}$; $V(0) = 22{,}000 \Longrightarrow a = 22{,}000$

$V(2) = 13{,}000 \Longrightarrow 13{,}000 = 22{,}000e^{2k}$

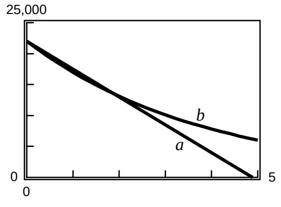
$\frac{13}{22} = e^{2k}$

$\ln \frac{13}{22} = 2k$

$k = \frac{1}{2} \ln \frac{13}{22} \approx -0.263$

$V = 22{,}000e^{-0.263t}$

(c)



The exponential model depreciates faster in the first two years.

(d) Straight line: $V(1) = \$17{,}500$

$V(3) = \$8500$

Exponential: $V(1) = \$16{,}912$

$V(3) = \$9993$

(e) The negative slope means the car depreciates \$4500 per year.

45. $S(t) = 100(1 - e^{kt})$

(a) $15 = 100(1 - e^{k(1)})$

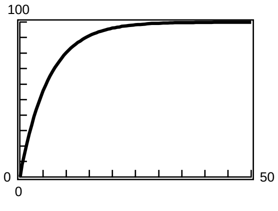
$-85 = -100e^{k}$

$k = \ln 0.85$

$k \approx -0.1625$

$S(t) = 100(1 - e^{-0.1625t})$

(b)



(c) $S(5) = 100(1 - e^{-0.1625(5)})$

$\approx 55.625 = 55{,}625$ units

47. $N = 30(1 - e^{kt})$

(a) $N = 19,\ t = 20$

$19 = 30(1 - e^{20k})$

$20k = \ln\frac{11}{30}$

$k \approx -0.050$

$N = 30(1 - e^{-0.050t})$

(b) $N = 25$

$25 = 30(1 - e^{-0.05t})$

$\frac{5}{30} = e^{-0.05t}$

$t = -\frac{1}{0.05}\ln\frac{5}{30} \approx 36$ days

(c) No, this is not a linear function.

49. $R = \log_{10}\left(\frac{I}{I_0}\right) = \log_{10} I$, since $I_0 = 1$

(a) $R = \log_{10} 39{,}811{,}000 \approx 7.6$

(b) $R = \log_{10} 12{,}589{,}000 \approx 7.1$

51. $\beta(I) = 10\log_{10}(I/I_0)$, where $I_0 = 10^{-12}$ watts per sq meter

(a) $\beta(10^{-10}) = 10 \cdot \log_{10}\left(\dfrac{10^{-10}}{10^{-12}}\right) = 10\log_{10}10^2 = 20$ decibels

(b) $\beta(10^{-5}) = 10 \cdot \log_{10}\left(\dfrac{10^{-5}}{10^{-12}}\right) = 10\log_{10}10^7 = 70$ decibels

(c) $\beta(10^{0}) = 10 \cdot \log_{10}\left(\dfrac{10^{0}}{10^{-12}}\right) = 10\log_{10}10^{12} = 120$ decibels

53. $\beta = 10\log_{10}\dfrac{I}{\mathrm{I}_0}$

$10^{\beta/10} = \dfrac{I}{I_0}$

$I = I_0 10^{\beta/10}$

$\%\text{ decrease} = \dfrac{I_0 10^{9.3} - I_0 10^{8.0}}{I_0 10^{9.3}} \times 100 \approx 95\%$

55. $\text{pH} = -\log_{10}[\text{H}^+] = -\log_{10}[2.3 \times 10^{-5}] \approx 4.64$

57. $\text{pH} = -\log_{10}[\text{H}^+]$

$-\text{pH} = \log_{10}[\text{H}^+]$

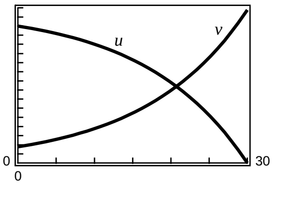
$10^{-\text{pH}} = [\text{H}^+]$

$$\frac{\text{Hydrogen ion concentration of fruit}}{\text{Hydrogen ion concentration of tablet}} = \frac{10^{-2.5}}{10^{-9.5}} = 10^7$$

59. (a) $P = 120{,}000,\ t = 30,\ r = 0.075,\ M = 839.06$

$u = M - \left(M - \dfrac{Pr}{12}\right)\left(1 + \dfrac{r}{12}\right) = 839.06 - (839.06 - 750)(1 + 0.00625)^{12t}$

$v = (839.06 - 750)(1.00625)^{12t}$

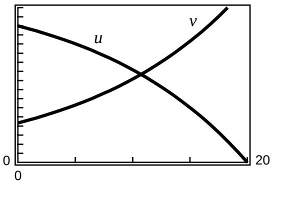


(b) In the early years, the majority of the monthly payment goes toward interest. The interest and principle are equal when $t \approx 20.729 \approx 21$ years.

(c) $P = 120{,}000,\ t = 20,\ r = 0.075,\ M = 966.71$

$u = 966.71 - (966.71 - 750)(1.00625)^{12t}$

$v = (966.71 - 750)(1.00625)^{12t}$



$u = v$ when $t \approx 10.73$ years

61.
$$y = ae^{bx}$$
$$1 = ae^{b(0)} \implies 1 = a$$
$$10 = e^{b(3)}$$
$$\ln 10 = 3b$$
$$\frac{\ln 10}{3} = b \implies b \approx 0.7675$$
Thus, $y = e^{0.7675x}$.

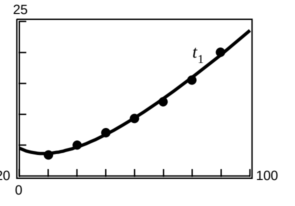
63.
$$y = ae^{bx}$$
$$\frac{1}{2} = ae^{b(0)} \implies a = \frac{1}{2}$$
$$5 = \frac{1}{2}e^{b(4)}$$
$$10 = e^{4b}$$
$$\ln 10 = 4b$$
$$\frac{\ln 10}{4} = b \implies b \approx 0.5756$$
Thus, $y = \frac{1}{2}e^{0.5756x}$.

65. $t_1 = 40.757 + 0.556s - 15.817 \ln s$

$t_2 = 1.2259 + 0.0023s^2$

(a) Linear Model: $t_3 \approx 0.2729s - 6.0143$
Exponential Model: $t_4 \approx 1.5385e^{1.0291s}$

(b)



(c)

s	30	40	50	60	70	80	90
t_1	3.6	4.7	6.7	9.4	12.5	15.9	19.6
t_2	3.3	4.9	7.0	9.5	12.5	15.9	19.9
t_3	2.2	4.9	7.6	10.4	13.1	15.8	18.5
t_4	3.7	4.9	6.6	8.8	11.8	15.8	21.1

(d) Model t_1: $S_1 = |3.4 - 3.6| + |5 - 4.7| + |7 - 6.7| + |9.3 - 9.4| + |12 - 12.5| + |15.8 - 15.9| + |20 - 19.6| = 1.9$

Model t_2: $S_2 = |3.4 - 3.3| + |5 - 4.9| + |7 - 7| + |9.3 - 9.5| + |12 - 12.5| + |15.8 - 15.9| + |20 - 19.9| = 1.1$

Model t_3: $S_3 = |3.4 - 2.2| + |5 - 4.9| + |7 - 7.6| + |9.3 - 10.4| + |12 - 13.1| + |15.8 - 15.8| + |20 - 18.5| = 5.6$

Model t_4: $S_4 = |3.4 - 3.7| + |5 - 4.9| + |7 - 6.6| + |9.3 - 8.8| + |12 - 11.8| + |15.8 - 15.8| + |20 - 21.1| = 2.6$

t_2, the Quadratic model, is the best fit with the data.

67. $t = -2.5 \ln\left(\frac{T - 70}{98.6 - 70}\right)$

At 9:00 A.M. we have: $t = -2.5 \ln\left(\frac{85.7 - 70}{98.6 - 70}\right) \approx 1.5$ hours.

From this we can conclude that the person died at 7:30 A.M.

69. (a) $y = 0.08245x + 4.45274$

(b) $y = 4.5355(1.01519)^x$

(c) The models are nearly identical.

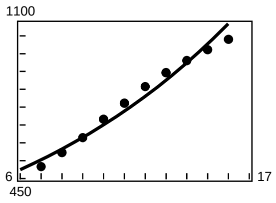
(d) For 2005, $x = 25$

Linear model: $y = 0.08245(25) + 4.45274 \approx 6.514$ billion

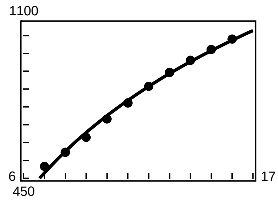
Exponential model: $y = 4.5355(1.01519)^{25} \approx 6.61$ billion

(Answers will vary.)

71. (a) $y = 298.794(1.0851)$



(b) $y = -837.735 + 673.619 \ln(x)$



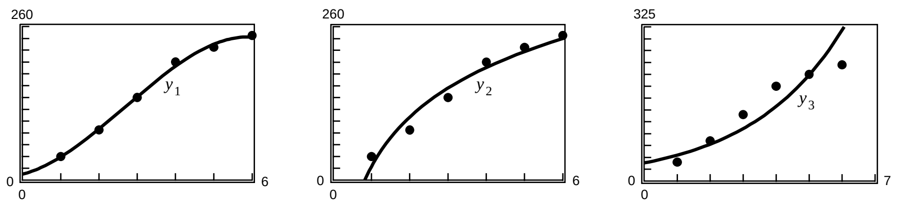
(c) The logarithmic model is more accurate. If the rate of growth of health costs is slowed, then the logarithmic model would be better.

73. (a) $y_1 = -1.81x^3 + 14.58x^2 + 16.39x + 10.00$

$y_2 = 23.07 + 121.08 \ln x$

$y_3 = 38.38(1.4227)^x$

(b)



(c) Cubic model

x	y	$y - y_1$	$(y - y_1)^2$	$y - y_2$	$(y - y_2)^2$	$y - y_3$	$(y - y_3)^2$
1	40	0.84	0.71	16.93	286.62	−14.60	213.25
2	85	−1.62	2.62	−22.00	483.84	7.32	53.52
3	140	−1.52	2.31	−16.09	258.89	29.48	869.01
4	200	7.00	49.00	9.08	82.40	42.76	1828.56
5	225	5.20	27.04	7.06	49.83	1.30	1.68
6	245	2.74	7.51	4.98	24.84	−73.26	5367.34

(d) Cubic model
y_1: 89.19; y_2: 1186.42; y_3: 8333.36;

(e) The sums represent the sum of the squares of the errors.

75. False. See Example 5, page 263.

77. True. See page 262.