CHAPTER 1
Functions and Their Graphs

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CHAPTER 1
Functions and Their Graphs

Section 1.1 Functions

Solutions to Even-Numbered Exercises

2. No, it is not a function. The domain value of -1 is matched with two output values.

4. Yes, it is a function. Each domain value is matched with only one range value.

6. No, the table does not represent a function. The input values of 0 and 1 are each matched with two different output values.

8. Yes, the table does represent a function. Each input value is matched with only one output value.

10. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.
(b) Each element of A is matched with exactly one element of B, so it does represent a function.
(c) This is not a function from A to B (it represents a function from B to A instead).
(d) Each element in A is matched with exactly one element of B, so it does represent a function.

12. Reading from the graph, $f(1994)$ is approximately 16 million.

14. $x = y^2 \implies y = \pm\sqrt{x}$
Thus, y *is not* a function of x.

16. $x + y^2 = 4 \implies y = \pm\sqrt{4 - x}$
Thus, y *is not* a function of x.

18. $x = -y + 5 \implies y = -x + 5.$
This is a function of x.

20. $y = \sqrt{x + 5}$
This is a function of x.

22. $|y| = 4 - x \implies y = 4 - x$ or $y = -(4 - x)$
Thus, y *is not* a function of x.

24. $y = -2$ is a function of x, a constant function.

26. $g(x) = x^2 - 2x$
(a) $g(2) = (2)^2 - 2(2) = 0$
(b) $g(-3) = (-3)^2 - 2(-3) = 15$
(c) $g(t + 1) = (t + 1)^2 - 2(t + 1) = t^2 - 1$
(d) $g(x + c) = (x + c)^2 - 2(x + c)$
$= x^2 + 2cx + c^2 - 2x - 2c$

28. $g(y) = 7 - 3y$
(a) $g(0) = 7 - 3(0) = 7$
(b) $g\left(\frac{7}{3}\right) = 7 - 3\left(\frac{7}{3}\right) = 0$
(c) $g(s + 2) = 7 - 3(s + 2)$
$= 7 - 3s - 6 = 1 - 3s$

30. $V(r) = \dfrac{4}{3}\pi r^3$
(a) $V(3) = \dfrac{4}{3}\pi(3)^3 = 36\pi$
(b) $V\left(\dfrac{3}{2}\right) = \dfrac{4}{3}\pi\left(\dfrac{3}{2}\right)^3 = \dfrac{4}{3} \cdot \dfrac{27}{8}\pi = \dfrac{9\pi}{2}$
(c) $V(2r) = \dfrac{4}{3}\pi(2r)^3 = \dfrac{32\pi r^3}{3}$

32. $f(x) = \sqrt{x + 8} + 2$
(a) $f(-8) = \sqrt{(-8) + 8} + 2 = 2$
(b) $f(1) = \sqrt{(1) + 8} + 2 = 5$
(c) $f(x - 8) = \sqrt{(x - 8) + 8} + 2 = \sqrt{x} + 2$

34. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2) = \dfrac{2(2)^2 + 3}{(2)^2} = \dfrac{8 + 3}{4} = \dfrac{11}{4}$

(b) $q(0) = \dfrac{2(0)^2 + 3}{(0)^2}$ Division by zero is undefined.

(c) $q(-x) = \dfrac{2(-x)^2 + 3}{(-x)^2} = \dfrac{2x^2 + 3}{x^2}$

36. $f(x) = |x| + 4$

(a) $f(2) = |2| + 4 = 6$

(b) $f(-2) = |-2| + 4 = 6$

(c) $f(x^2) = |x^2| + 4 = x^2 + 4$

38. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(1) = (1)^2 + 2 = 3$

(c) $f(2) = 2(2)^2 + 2 = 10$

40. $g(x) = \sqrt{x - 3}$

$g(3) = \sqrt{3 - 3} = 0$

$g(4) = \sqrt{4 - 3} = 1$

$g(5) = \sqrt{5 - 3} = \sqrt{2}$

$g(6) = \sqrt{6 - 3} = \sqrt{3}$

$g(7) = \sqrt{7 - 3} = 2$

x	3	4	5	6	7
$g(x)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2

42. $f(s) = \dfrac{|s - 2|}{s - 2}$

$f(0) = \dfrac{|0 - 2|}{0 - 2} = \dfrac{2}{-2} = -1$

$f(1) = \dfrac{|1 - 2|}{1 - 2} = \dfrac{1}{-1} = -1$

$f\left(\dfrac{3}{2}\right) = \dfrac{\left|\frac{3}{2} - 2\right|}{\frac{3}{2} - 2} = \dfrac{\frac{1}{2}}{-\frac{1}{2}} = -1$

$f\left(\dfrac{5}{2}\right) = \dfrac{\left|\frac{5}{2} - 2\right|}{\frac{5}{2} - 2} = \dfrac{\frac{1}{2}}{\frac{1}{2}} = 1$

$f(4) = \dfrac{|4 - 2|}{4 - 2} = \dfrac{2}{2} = 1$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$	-1	-1	-1	1	1

44. $h(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \ge 3 \end{cases}$

$h(1) = 9 - (1)^2 = 8$

$h(2) = 9 - (2)^2 = 5$

$h(3) = (3) - 3 = 0$

$h(4) = (4) - 3 = 1$

$h(5) = (5) - 3 = 2$

x	1	2	3	4	5
$h(x)$	8	5	0	1	2

46. $f(x) = 5x + 1 = 0$

$$5x = -1$$

$$x = -\frac{1}{5}$$

48. $f(x) = \dfrac{12 - x^2}{5} = 0$

$$12 - x^2 = 0$$

$$x^2 = 12$$

$$x = \pm\sqrt{12} = \pm 2\sqrt{3}$$

50.

$$\begin{aligned} f(x) &= 0 \\ x^3 - x &= 0 \\ x(x^2 - 1) &= 0 \\ x(x + 1)(x - 1) &= 0 \\ x &= 0 \\ x + 1 &= 0 \Longrightarrow x = -1 \\ x - 1 &= 0 \Longrightarrow x = 1 \end{aligned}$$

52. $f(x) = \sqrt{4x^2 - x} = 0$

$$\begin{aligned} 4x^2 - x &= 0 \\ x(4x - 1) &= 0 \\ x &= 0, x = \tfrac{1}{4} \end{aligned}$$

54.

$$\begin{aligned} f(x) &= g(x) \\ x^2 + 2x + 1 &= 3x + 3 \\ x^2 - x - 2 &= 0 \\ (x - 2)(x + 1) &= 0 \\ x - 2 &= 0 \Longrightarrow x = 2 \\ x + 1 &= 0 \Longrightarrow x = -1 \end{aligned}$$

56.

$$\begin{aligned} f(x) &= g(x) \\ x^4 - 2x^2 &= 2x^2 \\ x^4 - 4x^2 &= 0 \\ x^2(x^2 - 4) &= 0 \\ x^2(x + 2)(x - 2) &= 0 \\ x^2 &= 0 \Longrightarrow x = 0 \\ x + 2 &= 0 \Longrightarrow x = -2 \\ x - 2 &= 0 \Longrightarrow x = 2 \end{aligned}$$

58. $g(x) = 1 - 2x^2$

Because $g(x)$ is a polynomial, the domain is all real numbers x.

60. $s(y) = \dfrac{3y}{y + 5}$

$y + 5 \neq 0$

$y \neq -5$

The domain is all real numbers $y \neq 5$.

62. $f(t) = \sqrt[3]{t + 4}$

Because $f(t)$ is a cube root, the domain is all real numbers t.

64. $f(x) = \sqrt[4]{x^2 + 3x}$. $x^2 + 3x = x(x + 3) \geq 0$

Domain: $x \leq -3$ or $x \geq 0$

66. $h(x) = \dfrac{10}{x^2 - 2x}$

$x^2 - 2x \neq 0$

$x(x - 2) \neq 0$

The domain is all real numbers $x \neq 0$ and $x \neq 2$.

68. $f(x) = \dfrac{\sqrt{x + 6}}{6 + x}$. $x + 6 \geq 0$ for numerator, and $x \neq -6$ for denominator. Domain: $x > -6$.

70. $f(x) = \dfrac{x - 5}{\sqrt[4]{x^2 - 9}}$. $x^2 - 9 = (x - 3)(x + 3) > 0$ for denominator. Domain: $x < -3$ or $x > 3$.

72. $f(x) = \dfrac{2x}{x^2 + 1}$

$$\left\{\left(-2, -\frac{4}{5}\right), (-1, -1), (0, 0), (1, 1), \left(2, \frac{4}{5}\right)\right\}$$

74. $f(x) = |x + 1|$

$\{(-2, 1), (-1, 0), (0, 1), (1, 2), (2, 3)\}$

76. By plotting the data, you can see that it represents a line, or $f(x) = cx$. Because $(0, 0)$ and $\left(1, \frac{1}{4}\right)$ are on the line, the slope is $\frac{1}{4}$. Thus, $f(x) = \frac{1}{4}x$.

78. By plotting the data, you can see that it represents $h(x) = c\sqrt{|x|}$. Because $\sqrt{|-4|} = 2$ and $\sqrt{|-1|} = 1$, but the corresponding y values are 6 and 3, you have $c = 3$ and $h(x) = 3\sqrt{|x|}$.

80.

$$\begin{aligned} g(x) &= 3x - 1 \\ g(x + h) &= 3(x + h) - 1 = 3x + 3h - 1 \\ g(x + h) - g(x) &= (3x + 3h - 1) - (3x - 1) = 3h \\ \frac{g(x + h) - g(x)}{h} &= \frac{3h}{h} = 3,\ h \neq 0 \end{aligned}$$

82.

$$\begin{aligned} f(x) &= 5x - x^2 \\ f(5 + h) &= 5(5 + h) - (5 + h)^2 \\ &= 25 + 5h - (25 + 10h + h^2) \\ &= 25 + 5h - 25 - 10h - h^2 \\ &= -h^2 - 5h \\ f(5) &= 5(5) - (5)^2 \\ &= 25 - 25 = 0 \\ \frac{f(5 + h) - f(5)}{h} &= \frac{-h^2 - 5h}{h} \\ &= \frac{-h(h + 5)}{h} = -(h + 5),\ h \neq 0 \end{aligned}$$

84. $f(x) = x^3 + x$

$$\begin{aligned} f(x + h) &= (x + h)^3 + (x + h) = x^3 + 3x^2h + 3xh^2 + h^3 + x + h \\ f(x + h) - f(x) &= (x^3 + 3x^2h + 3xh^2 + h^3 + x + h) - (x^3 + x) \\ &= 3x^2h + 3xh^2 + h^3 + h \\ &= h(3x^2 + 3xh + h^2 + 1) \\ \frac{f(x + h) - f(x)}{h} &= \frac{h(3x^2 + 3xh + h^2 + 1)}{h} = 3x^2 + 3xh + h^2 + 1,\ h \neq 0 \end{aligned}$$

86. $f(x) = \dfrac{4}{x + 1}$

$$f(7) = \frac{4}{7 + 1} = \frac{1}{2}$$

$$\begin{aligned} \frac{f(x) - f(7)}{x - 7} &= \frac{\dfrac{4}{x + 1} - \dfrac{1}{2}}{x - 7} = \frac{8 - (x + 1)}{2(x + 1)(x - 7)} = \frac{7 - x}{2(x + 1)(x - 7)} \\ &= \frac{-1}{2(x + 1)},\ x \neq 7 \end{aligned}$$

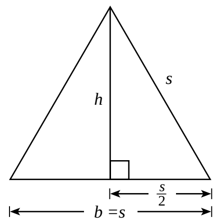
88. $A = \frac{1}{2}bh$, in an equilateral triangle $b = s$ and:

$$s^2 = h^2 + \left(\frac{s}{2}\right)^2$$

$$h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$$

$$h = \sqrt{\frac{4s^2}{4} - \frac{s^2}{4}} = \frac{\sqrt{3}s}{2}$$

$$A = \frac{1}{2}s \cdot \frac{\sqrt{3}s}{2} = \frac{\sqrt{3}s^2}{4}$$



90. Let x be the length of the sides, and h the height. Then

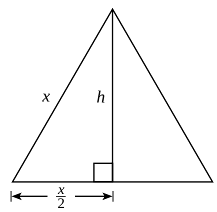
$$\left(\frac{1}{2}x\right)^2 + h^2 = x^2$$

$$h^2 = x^2 - \frac{1}{4}x^2$$

$$h^2 = \frac{3}{4}x^2$$

$$h = \frac{\sqrt{3}}{2}x \text{ and } x = \frac{2h}{\sqrt{3}}$$

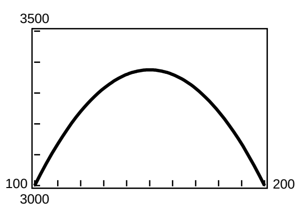
$$\text{Area} = A = \frac{1}{2}b \cdot h = \frac{1}{2}\left(\frac{2h}{\sqrt{3}}\right)h = \frac{1}{\sqrt{3}}h^2 = \frac{\sqrt{3}}{3}h^2$$



92. (a)

Units x	*Price*	*Profit P*
102	$90 - 2(0.15)$	$102[90 - 2(0.15)] - 102(60) = 3029.40$
104	$90 - 4(0.15)$	$104[90 - 4(0.15)] - 104(60) = 3057.60$
106	$90 - 6(0.15)$	$106[90 - 6(0.15)] - 106(60) = 3084.60$
108	$90 - 8(0.15)$	$108[90 - 8(0.15)] - 108(60) = 3110.40$
110	$90 - 10(0.15)$	$110[90 - 10(0.15)] - 110(60) = 3135.00$
112	$90 - 12(0.15)$	$112[90 - 112(0.15)] - 12(60) = 3158.40$

(b)



Yes, P is a function.

$$\begin{aligned}
\text{Profit} &= \text{Revenue} - \text{Cost} \\
&= (\text{price per unit})(\text{number of units}) - (\text{cost})(\text{number of units}) \\
&= [90 - (x - 100)(0.15)]x - 60x, \ x > 100 \\
&= (90 - 0.15x + 15)x - 60x \\
&= (105 - 0.15x)x - 60x \\
&= 105x - 0.15x^2 - 60x \\
&= 45x - 0.15x^2, \ x > 100
\end{aligned}$$

(c) $P(120) = 3240$, $P(130) = 3315$, $P(140) = 3360$

(d) $P(120) = 45(120) - .15(120)^2 = 3240$

$P(130) = 45(130) - .15(130)^2 = 3315$

$P(140) = 45(140) - .15(140)^2 = 3360$

94. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$, $0 < x < 6$.

96. For 1978, $t = -2$ and $p \approx \$15{,}200$

For 1988, $t = 8$ and $p \approx \$25{,}600$

For 1993, $t = 13$ and $p \approx \$34{,}500$

For 1997, $t = 17$ and $p \approx \$47{,}400$

98. (a) $C(x) = 0.95x + 6000$

(b) $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{0.95x + 6000}{x} = 0.95 + \dfrac{6000}{x}$

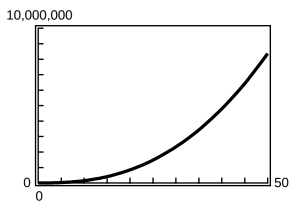
100. $F(y) = 149.76\sqrt{10}y^{5/2}$

y	5	10	20	30	40
$F(y)$	2.65×10^4	1.50×10^5	8.47×10^5	2.33×10^6	4.79×10^6

(Answers will vary.)

(a) F increases very rapidly as y increases.

(b)



(c) From the table, $y \approx 22$ ft (slightly above 20). You could obtain a better approximation by completing the table for values of y between 20 and 30.

(d) By graphing $F(y)$ together with the horizontal line $y_2 = 1{,}000{,}000$, you obtain $y \approx 21.37$ feet.

102. (a) $f(1992) = 28$ lynx

(b) $\dfrac{f(1994) - f(1991)}{1994 - 1991} = \dfrac{9 - 60}{3} = -\dfrac{51}{3} = -17$

This represents the average loss per year of lynx.

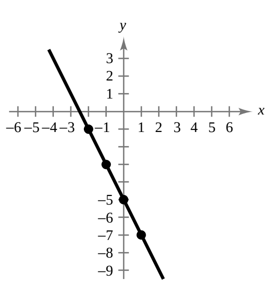
(c) $N(t) = \dfrac{434t + 4387}{45t^2 - 55t + 100}$

t	1988	1989	1990	1991	1992	1993	1994	1995
N	9	20	44	54	31	17	10	7

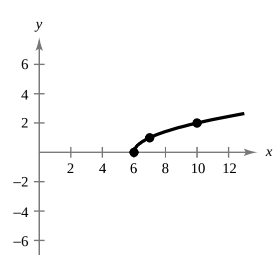
104. True. The first number in each ordered pair corresponds to exactly one second number.

106. The domain is the set of inputs of the function and the range is the set of corresponding outputs.

108.



110.



112. Center: $(0, 0)$

Radius: 9

$$(x - 0)^2 + (y - 0)^2 = 9^2$$
$$x^2 + y^2 = 81$$

114. Center: $(10, 1)$

Solution Point: $(-2, -4)$

$$(x - 10)^2 + (y - 1)^2 = r^2$$
$$(-2 - 10)^2 + (-4 - 1)^2 = r^2$$
$$144 + 25 = r^2$$
$$r = 13$$
$$(x - 10)^2 + (y - 1)^2 = 169$$

Section 1.2 Graphs of Functions

Solutions to Even-Numbered Exercises

2. $f(x) = x^3 - 3x + 2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

4. $h(x) = \sqrt{16 - x^2}$

Domain: $[-4, 4]$

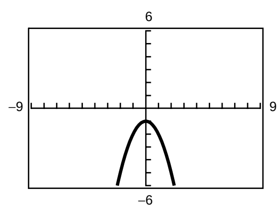
Range: $[0, 4]$

6. $g(x) = -|x - 1|$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

8. $f(x) = -x^2 - 1$



Domain: $(-\infty, \infty)$

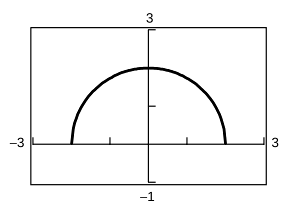
Range: $(-\infty, -1]$

10. $h(t) = \sqrt{4 - t^2}$

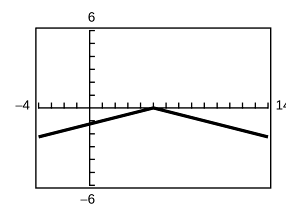
$4 - t^2 \geq 0 \implies t^2 \leq 4$

Domain: $[-2, 2]$

Range: $[0, 2]$



12. $f(x) = -\frac{1}{4}|x - 5|$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

14. $y = \frac{1}{4}x^3$

A vertical line intersects the graph no more than once, so y is a function of x.

16. $x^2 + y^2 = 25$

A vertical line intersects the graph more than once, so y is not a function of x. Graph the circle as

$$y_1 = \sqrt{25 - x^2}$$
$$y_2 = -\sqrt{25 - x^2}$$

18. $x = |y + 2|$

A vertical line intersects the graph more than once, so y is not a function of x. Graph as

$$y_1 = x - 2, x \geq 0$$
$$y_2 = -x - 2, x \geq 0$$

20. $f(x) = x^2 - 4x$

(a) The graph is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

(b) $f(-x) = (-x)^2 - 4(-x) = x^2 + 4x$

$x^2 + 4x \neq f(x)$

$x^2 + 4x \neq -f(x)$

The function is neither odd nor even.

22. $f(x) = \sqrt{x^2 - 1}$

(a) The graph is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

(b) $f(-x) = \sqrt{(-x)^2 - 1} = \sqrt{x^2 - 1} = f(x)$

The function is even.