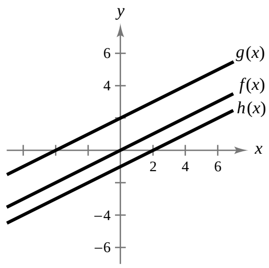
Section 1.3 Shifting, Reflecting, and Stretching Graphs

Solutions to Even-Numbered Exercises

2.



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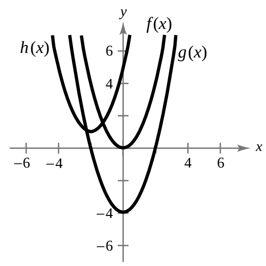

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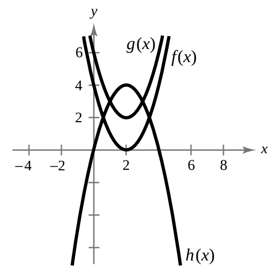

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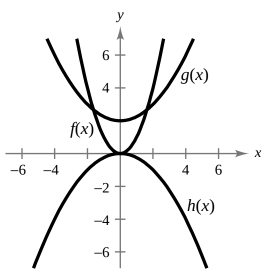

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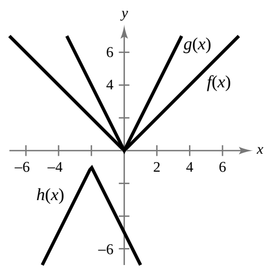

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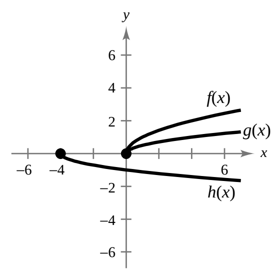
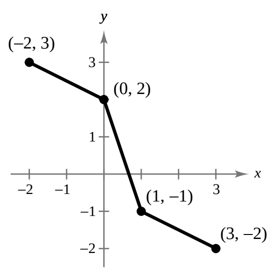
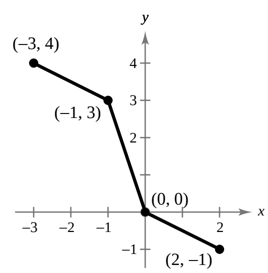


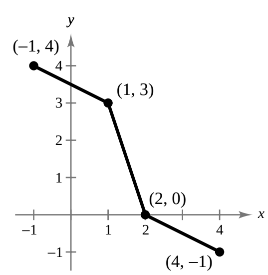
14. (a)



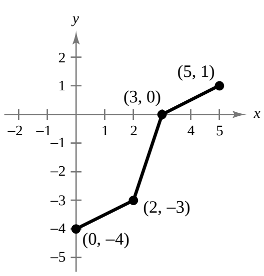
(b)



(c)



(d)



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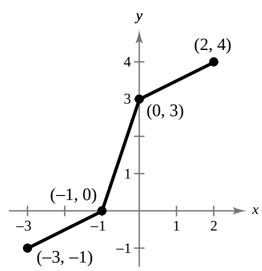
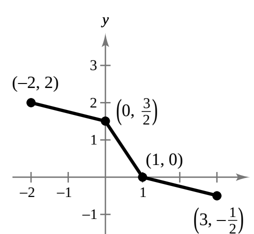


(f)



16. Horizontal shift 3 units to left of $y = x$: $y = x + 3$ (or vertical shift 3 units upward)

18. Constant function: $y = -8$

20. Horizontal shift 3 units to the right of $y = \sqrt{x}$, followed by reflection in the y-axis:
$y = \sqrt{-(x - 3)} = \sqrt{3 - x}$

22. Horizontal shift 4 units to the right of $y = |x|$, followed by reflection in the x-axis followed by vertical shift 3 units downward.

$y = -|x - 4| - 3$

24. Reflection in the x-axis of $y = x^2$ followed by vertical and horizontal shifts

$y = 1 - (x + 1)^2$

26. Horizontal and vertical shifts of $y = x^3$

$y = (x - 1)^3 + 1$

28. $y = -\sqrt{x} - 1$ is $f(x)$ reflected in the x-axis, followed by a vertical shift downward 1 unit.

30. $y = \sqrt{x + 3}$ is $f(x)$ shifted left three units.

32. $y = \sqrt{-x + 3}$ is $f(x)$ reflected in the y-axis, followed by a horizontal shift to the right 3 units.

34. $y = |x| - 3$ is $f(x) = |x|$ shifted down three units.

36. $y = |-x|$ is a reflection in the y-axis. In fact $y = |-x| = |x|$.

38. $y = \frac{1}{2}|x|$ is a vertical shrink.

40. $g(x) = -(x - 4)^3$ is obtained by a horizontal shift of four units to the right, followed by a reflection in the x-axis.

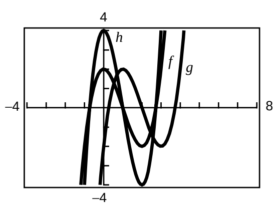
42. $h(x) = -2(x - 1)^3 + 3$ is obtained from $f(x)$ by a right shift of one unit, a vertical stretch by a factor of two, a reflection in the x-axis, and a vertical shift three units upward.

44. $p(x) = [3(x - 2)]^3$ is obtained from $f(x)$ by a right shift of two units, followed by a vertical stretch.

46. $f(x) = x^3 - 3x^2 + 2$

$g(x) = f(x - 1) = (x - 1)^3 - 3(x - 1)^2 + 2$ is a horizontal shift one unit to the right.

$h(x) = 2f(x) = 2(x^3 - 3x^2 + 2)$ is a vertical stretch.



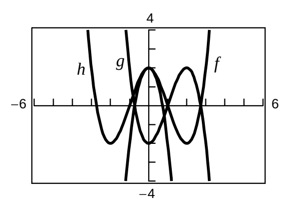
48. $f(x) = x^3 - 3x^2 + 2$

$g(x) = -f(x) = -(x^3 - 3x^2 + 2)$

(reflection in the x-axis)

$h(x) = f(-x) = (-x)^3 - 3(-x)^2 + 2$

(reflection in the y-axis)

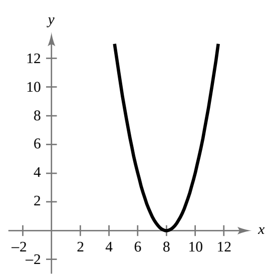


50. The graph of g is obtained from that of f by first shifting horizontally two units to the right, and then vertically upward one unit. Hence, $g(x) = (x - 2)^3 - 3(x - 2)^2 + 1$.

52. (a) $f(x) = x^2$

(b) $g(x) = (x - 8)^2$ is obtained from f by a horizontal shift 8 units to the right.

(c)

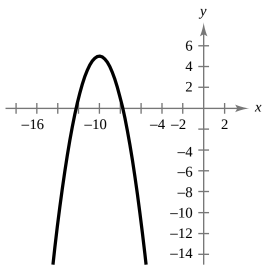


(d) $g(x) = f(x - 8)$

54. (a) $f(x) = x^2$

(b) $g(x) = -(x + 10)^2 + 5$ is obtained from f by a horizontal shift 10 units to the left, a reflection in the x-axis, and a vertical shift 5 units upward.

(c)

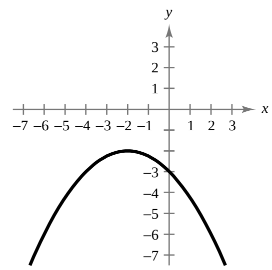


(d) $g(x) = -f(x + 10) + 5$

56. (a) $f(x) = x^2$

(b) $g(x) = -\frac{1}{4}(x + 2)^2 - 2$ is obtained from f by a horizontal shift 2 units to the left, a vertical shrink of $\frac{1}{4}$, a reflection in the x-axis, and a vertical shift 2 units downward.

(c)

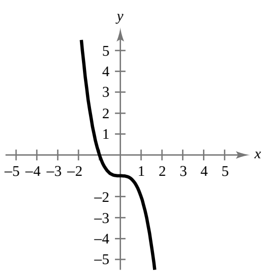


(d) $g(x) = -\frac{1}{4}f(x + 2) - 2$

58. (a) $f(x) = x^3$

(b) $g(x) = -x^3 - 1$ is obtained from f by a reflection in the x-axis followed by a vertical shift 1 unit downward.

(c)

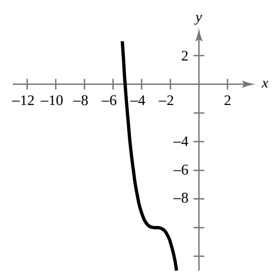


(d) $g(x) = -f(x) - 1$

60. (a) $f(x) = x^3$

(b) $g(x) = -(x + 3)^3 - 10$ is obtained from f by a horizontal shift 3 units to the left, a reflection in the x-axis, and a vertical shift 10 units downward.

(c)

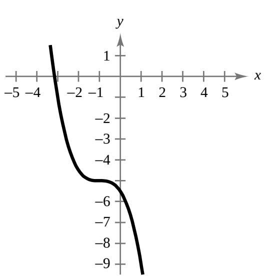


(d) $g(x) = -f(x + 3) - 10$

62. (a) $f(x) = x^3$

(b) $g(x) = -\frac{1}{2}(x + 1)^3 - 5$ is obtained from f by a horizontal shift 1 unit to the left, a vertical shrink, a reflection in the x-axis, and a vertical shift 5 units downward.

(c)

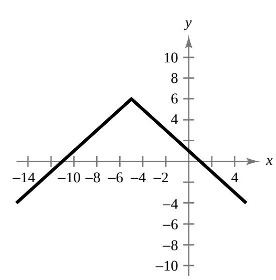


(d) $g(x) = -\frac{1}{2}f(x + 1) - 5$

64. (a) $f(x) = |x|$

(b) $g(x) = 6 - |x + 5|$ is obtained from f by a horizontal shift of 5 units to the left, a reflection in the x-axis, and a vertical shift 6 units upward.

(c)

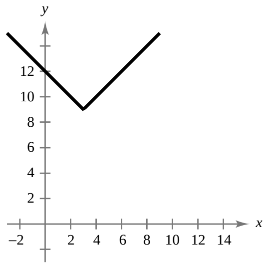


(d) $g(x) = 6 - f(x + 5)$

66. (a) $f(x) = |x|$

(b) $g(x) = |-x + 3| + 9 = |-(x - 3)| + 9 = |x - 3| + 9$ is obtained from f by a horizontal shift 3 units to the right, followed by a vertical shift 9 units upward.

(c)

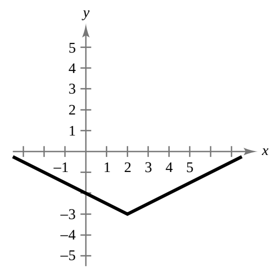


(d) $g(x) = f(x - 3) + 9 = f(-x + 3) + 9$

68. (a) $f(x) = |x|$

(b) $g(x) = \frac{1}{2}|x - 2| - 3$ is obtained from f by a horizontal shift 2 units to the right, a vertical shrink, and a vertical shift 3 units downward.

(c)

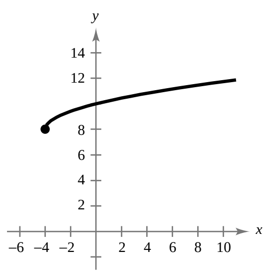


(d) $g(x) = \frac{1}{2}f(x - 2) - 3$

70. (a) $f(x) = \sqrt{x}$

(b) $g(x) = \sqrt{x + 4} + 8$ is obtained from f by a horizontal shift 4 units to the left, and a vertical shift 8 units upward.

(c)

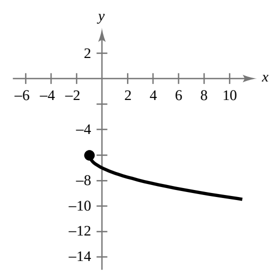


(d) $g(x) = f(x + 4) + 8$

72. (a) $f(x) = \sqrt{x}$

(b) $g(x) = -\sqrt{x + 1} - 6$ is obtained from f by a horizontal shift 1 unit to the left, a reflection in the x-axis, and a vertical shift 6 units downward.

(c)

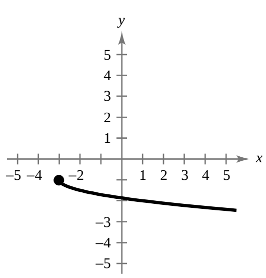


(d) $g(x) = -f(x + 1) - 6$

74. (a) $f(x) = \sqrt{x}$

(b) $g(x) = -\frac{1}{2}\sqrt{x + 3} - 1$ is obtained from f by a horizontal shift 3 units to the left, a vertical shrink, a reflection in the x-axis, and a vertical shift 1 unit downward.

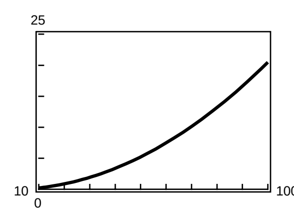
(c)



(d) $g(x) = -\frac{1}{2}f(x + 3) - 1$

76. $H(x) = 0.002x^2 + 0.005x - 0.029,\ 10 \le x \le 100$

(a)

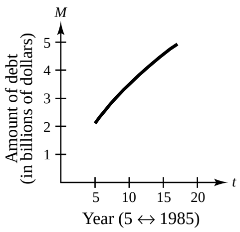


(b) $K(x) = H\left(\frac{x}{1.6}\right) = 0.002\left(\frac{x}{1.6}\right)^2 + 0.005\left(\frac{x}{1.6}\right) - 0.029$

$= 0.00078125x^2 + 0.003125x - 0.029$

where x is in kilometers/hour. This is a vertical shrink.

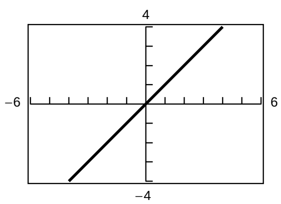
78. (a) $M = 1.5\sqrt{t} - 1.25$, $5 \le t \le 17$ is obtained from $f(x) = \sqrt{x}$ by a vertical stretch of 1.5 followed by a vertical shift 1.25 units downward.



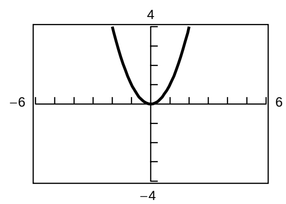
(b) $g(t) = M(t + 10) = 1.5\sqrt{t + 10} - 1.25$,
$-5 \le t \le 7$

80. True. $|x| = |-x|$ implies $f(x) = |x| - 5 = |-x| - 5 = g(x)$

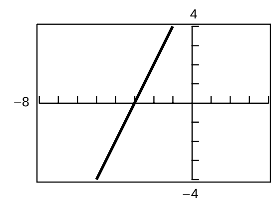
82. (a)



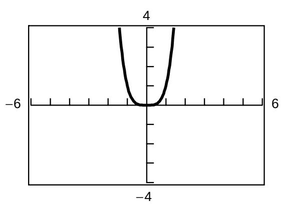
(b)



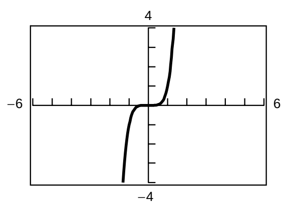
(c)



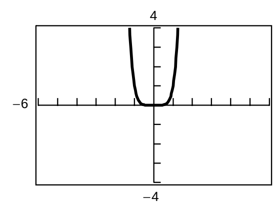
(d)



(e)

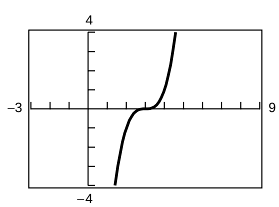


(f)



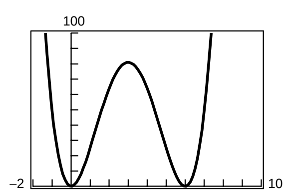
All the graphs pass through the origin. The graphs of the odd powers of x are symmetric to the origin and the graphs of the even powers are symmetric to the y-axis. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

84.

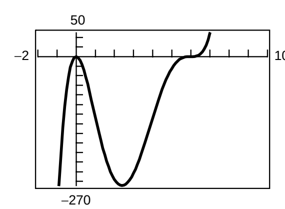


The graph of $y = (x - 3)^3$ is a horizontal shift of $f(x) = x^3$.

86. $f(x) = x^2(x - 6)^2$



88. $f(x) = x^2(x - 6)^3$



90. Domain: All $x \ne 9$

92. Domain: $100 - x^2 \ge 0 \implies x^2 \le 100$
$\implies -10 \le x \le 10$