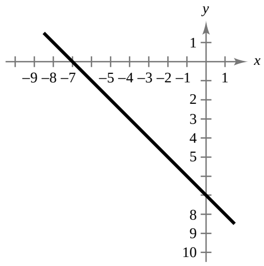
94.



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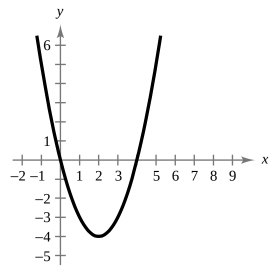

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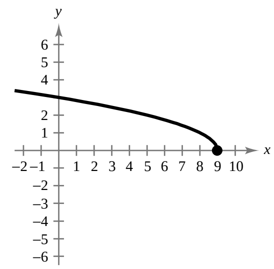


Section 1.4 Combinations of Functions

Solutions to Even-Numbered Exercises

2. $f(x) = 2x - 5,\ g(x) = 1 - x$

(a) $(f + g)(x) = 2x - 5 + 1 - x$
$= x - 4$

(b) $(f - g)(x) = 2x - 5 - (1 - x)$
$= 2x - 5 - 1 + x$
$= 3x - 6$

(c) $(fg)(x) = (2x - 5)(1 - x)$
$= 2x - 2x^2 - 5 + 5x$
$= -2x^2 + 7x - 5$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x - 5}{1 - x}$

(e) Domain: $1 - x \neq 0$
$x \neq 1$

4. $f(x) = 2x - 5,\ g(x) = 5$

(a) $(f + g)(x) = 2x - 5 + 5 = 2x$

(b) $(f - g)(x) = 2x - 5 - 5 = 2x - 10$

(c) $(fg)(x) = (2x - 5)(5) = 10x - 25$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x - 5}{5} = \dfrac{2}{5}x - 1$

(e) Domain: $-\infty < x < \infty$

6. $f(x) = \sqrt{x^2 - 4},\ g(x) = \dfrac{x^2}{x^2 + 1}$

(a) $(f + g)(x) = \sqrt{x^2 - 4} + \dfrac{x^2}{x^2 + 1}$

(b) $(f - g)(x) = \sqrt{x^2 - 4} - \dfrac{x^2}{x^2 + 1}$

(c) $(fg)(x) = \left(\sqrt{x^2 - 4}\right)\left(\dfrac{x^2}{x^2 + 1}\right) = \dfrac{x^2\sqrt{x^2 - 4}}{x^2 + 1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \sqrt{x^2 - 4} \div \dfrac{x^2}{x^2 + 1}$
$= \dfrac{(x^2 + 1)\sqrt{x^2 - 4}}{x^2}$

(e) Domain: $x^2 - 4 \geq 0$
$x^2 \geq 4 \Longrightarrow x \geq 2$ or $x \leq -2$
Domain: $|x| \geq 2$

8. $f(x) = \dfrac{x}{x + 1},\ g(x) = x^3$

(a) $(f + g)(x) = \dfrac{x}{x + 1} + x^3 = \dfrac{x + x^4 + x^3}{x + 1}$

(b) $(f - g)(x) = \dfrac{x}{x + 1} - x^3 = \dfrac{x - x^4 - x^3}{x + 1}$

(c) $(fg)(x) = \dfrac{x}{x + 1} \cdot x^3 = \dfrac{x^4}{x + 1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{x + 1} \div x^3$
$= \dfrac{x}{x + 1} \cdot \dfrac{1}{x^3} = \dfrac{1}{x^2(x + 1)}$

(e) Domain: $x \neq 0, x \neq -1$

10. $(f - g)(-2) = f(-2) - g(-2)$
$= (-2)^2 + 1 - (-2 - 4)$
$= 4 + 1 - (-6)$
$= 11$

12. $(f + g)(1) = f(1) + g(1)$
$= (1)^2 + 1 + (1) - 4$
$= -1$

14. $(fg)(-6) = f(-6) \cdot g(-6)$
$= [(-6)^2 + 1][(-6) - 4]$
$= (37)(-10)$
$= -370$

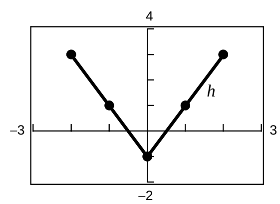
16. $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{0^2 + 1}{0 - 4} = -\frac{1}{4}$

18. $(f + g)(t - 4) = f(t - 4) + g(t - 4) = (t - 4)^2 + 1 + (t - 4) - 4$
$= t^2 - 8t + 16 + 1 + (t - 8)$
$= t^2 - 7t + 9$

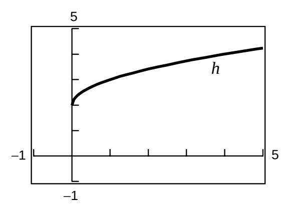
20. $(fg)(3t^2) = f(3t^2)g(3t^2) = [(3t^2)^2 + 1][3t^2 - 4]$
$= (9t^4 + 1)(3t^2 - 4)$
$= 27t^6 - 36t^4 + 3t^2 - 4$

22. $\left(\frac{f}{g}\right)(t + 2) = \frac{f(t + 2)}{g(t + 2)} = \frac{(t + 2)^2 + 1}{(t + 2) - 4}$
$= \frac{t^2 + 4t + 5}{t - 2}, \; t \neq 2$

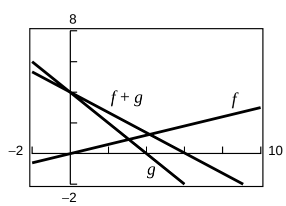
24.



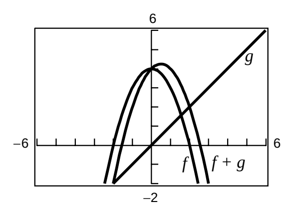
26.



28. $f(x) = \frac{1}{3}x,\; g(x) = -x + 4,$
$(f + g)(x) = \frac{1}{3}x + (-x + 4) = 4 - \frac{2}{3}x$

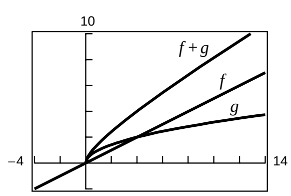


30. $f(x) = 4 - x^2,\; g(x) = x,$
$(f + g)(x) = (4 - x^2) + x = -x^2 + x + 4$



32. $f(x) = \frac{x}{2},\; g(x) = \sqrt{x}$

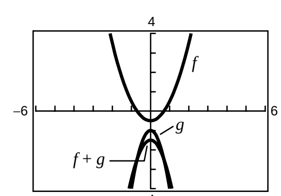
$(f + g)(x) = \frac{x}{2} + \sqrt{x}$



$g(x)$ contributes more to the magnitude of the sum for $0 \le x \le 2$. $f(x)$ contributes more to the magnitude of the sum for $x > 6$.

34. $f(x) = x^2 - \frac{1}{2},\; g(x) = -3x^2 - 1,$

$(f + g)(x) = \left(x^2 - \frac{1}{2}\right) + (-3x^2 - 1) = -2x^2 - \frac{3}{2}$



g contributes more on both intervals.

36. $f(x) = \sqrt[3]{x-1},\ g(x) = x^3 + 1$

(a) $(f \circ g)(x) = f(g(x))$
$= f(x^3 + 1)$
$= \sqrt[3]{(x^3+1) - 1}$
$= \sqrt[3]{x^3} = x$

(b) $(g \circ f)(x) = g(f(x))$
$= g\left(\sqrt[3]{x-1}\right)$
$= \left(\sqrt[3]{x-1}\right)^3 + 1$
$= (x - 1) + 1 = x$

38. $f(x) = x^3,\ g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x) = f(g(x))$
$= f\left(\dfrac{1}{x}\right)$
$= \left(\dfrac{1}{x}\right)^3 = \dfrac{1}{x^3}$

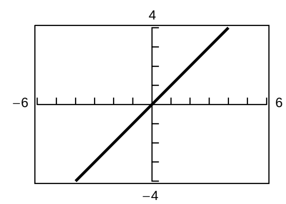
(b) $(g \circ f)(x) = g(f(x)) = g(x^3) = \dfrac{1}{x^3}$

40. (a) $(f \circ g)(x) = f(g(x)) = f(x^3 - 1)$
$= \sqrt[3]{(x^3 - 1) + 1}$
$= \sqrt[3]{x^3} = x$

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt[3]{x+1}\right)$
$= [\sqrt[3]{x+1}]^3 - 1$
$= (x + 1) - 1 = x$

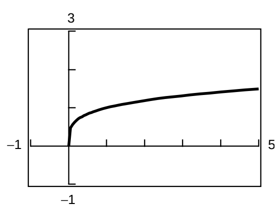
They are equal. $(f \circ g)(x) = (g \circ f)(x) = x$

(b)



42. (a) $(f \circ g)(x) = (g \circ f)(x) = \sqrt{\sqrt{x}} = x^{1/4}$

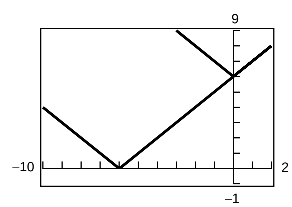
(b)



They are equal.

44. (a) $(f \circ g)(x) = f(g(x)) = f(x + 6) = |x + 6|$
$(g \circ f)(x) = g(f(x)) = g(|x|) = |x| + 6$

(b)



They are not equal. However, $|x + 6| = |x| + 6$ for $x \geq 0$.

46. (a) $(f \circ g)(x) = f(4x + 1) = \frac{1}{4}[(4x + 1) - 1]$
$= \frac{1}{4}[4x] = x$
$(g \circ f)(x) = g\left(\frac{1}{4}(x - 1)\right) = 4\left[\frac{1}{4}(x - 1)\right] + 1$
$= (x - 1) + 1 = x$

(b) They are equal because $x = x$.

(c)

x	$f(g(x))$	$g(f(x))$
-1	-1	-1
0	0	0
1	1	1
2	2	2
3	3	3

48. (a) $(f \circ g)(x) = f\left(\sqrt[3]{x + 10}\right) = \left[\sqrt[3]{x+10}\right]^3 - 4$
$= (x + 10) - 4 = x + 6$
$(g \circ f)(x) = g(x^3 - 4) = \sqrt[3]{(x^3 - 4) + 10}$
$= \sqrt[3]{x^3 + 6}$

(b) They are not equal because $x + 6 \neq \sqrt[3]{x^3 + 6}$.

(c)

x	$f(g(x))$	$g(f(x))$
-2	4	$\sqrt[3]{-2}$
0	6	$\sqrt[3]{6}$
1	7	$\sqrt[3]{7}$
2	8	$\sqrt[3]{14}$
3	9	$\sqrt[3]{33}$

50. (a) $(f \circ g)(x) = f(g(x)) = f(-x) = \dfrac{6}{3(-x) - 5}$

$= \dfrac{6}{-3x - 5}$

$(g \circ f)(x) = g\left(\dfrac{6}{3x - 5}\right) = -\left(\dfrac{6}{3x - 5}\right)$

$= \dfrac{-6}{3x - 5}$

(b) They are not equal because $\dfrac{6}{-3x - 5} \neq \dfrac{-6}{3x - 5}$.

(c)

x	$f(g(x))$	$g(f(x))$
0	$-\frac{6}{5}$	$\frac{6}{5}$
1	$-\frac{3}{4}$	3
2	$-\frac{6}{11}$	-6
3	$-\frac{3}{7}$	$-\frac{3}{2}$

52. (a) $(f - g)(1) = f(1) - g(1)$

$= 2 - 3 = -1$

(b) $(fg)(4) = f(4) \cdot g(4)$

$= 4 \cdot 0 = 0$

54. (a) $(f \circ g)(1) = f(g(1))$

$= f(3) = 2$

(b) $(g \circ f)(3) = g(f(3))$

$= g(2) = 2$

56. (a) $(g \circ g)(1) = g(g(1)) = g(3) = 1$

(b) $(g \circ g)(0) = g(g(0)) = g(4) = 0$

58. $h(x) = (1 - x)^3$

One possibility: Let $g(x) = 1 - x$ and $f(x) = x^3$.

$(f \circ g)(x) = f(1 - x) = (1 - x)^3 = h(x)$

60. $h(x) = \sqrt{9 - x}$

One possibility: Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$.

$(f \circ g)(x) = f(9 - x) = \sqrt{9 - x} = h(x)$

62. $h(x) = \dfrac{4}{(5x + 2)^2}$

One possibility:

Let $g(x) = 5x + 2$ and $f(x) = \dfrac{4}{x^2}$.

$(f \circ g)(x) = f(5x + 2) = \dfrac{4}{(5x + 2)^2}$

64. $h(x) = (x + 3)^{3/2}$

One possibility:

Let $g(x) = x + 3$ and $f(x) = x^{3/2}$.

$(f \circ g) = f(x + 3)$

$= (x + 3)^{3/2} = h(x)$

66. (a) Domain of f: $x + 3 \geq 0 \implies x \geq -3$

(b) Domain of g: all real numbers

(c) Domain of $(f \circ g)(x) = f\left(\dfrac{x}{2}\right) = \sqrt{\dfrac{x}{2} + 3}$:

$\dfrac{x}{2} + 3 \geq 0 \implies x \geq -6$

68. (a) Domain of f: all $x \neq 0$

(b) Domain of g: all $x \neq 0$

(c) Domain of $(f \circ g)(x) = f\left(\frac{1}{2x}\right) = 2x,\ x \neq 0,$

is all $x \neq 0$.

70. (a) Domain of f: all $x \neq \pm 1$

(b) Domain of g: all real numbers

(c) Domain of $(f \circ g)(x) = f(x+1) = \dfrac{3}{(x+1)^2 - 1}$

$$= \frac{3}{x^2 + 2x} = \frac{3}{x(x+2)}$$

is all real numbers $\neq 0, -2$.

72. $$\frac{f(x+h) - f(x)}{h} = \frac{[5(x+h) + 1] - (5x + 1)}{h}$$

$$= \frac{5h}{h} = 5,\ h \neq 0$$

74. $$\frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^2 + 4] - (x^2 + 4)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 4 - x^2 - 4}{h}$$

$$= \frac{2xh + h^2}{h} = \frac{h(2x + h)}{h}$$

$$= 2x + h,\ h \neq 0$$

76. $$\frac{f(x+h) - f(x)}{h} = \frac{\dfrac{2}{(x+h)^2} - \dfrac{2}{x^2}}{h} = \frac{2x^2 - 2(x+h)^2}{h(x+h)^2x^2}$$

$$= \frac{2x^2 - 2(x^2 + 2xh + h^2)}{h(x+h)^2x^2}$$

$$= \frac{-4xh - 2h^2}{h(x+h)^2x^2}$$

$$= \frac{h(-4x - 2h)}{h(x+h)^2x^2} = \frac{-4x - 2h}{(x+h)^2x^2},\ h \neq 0$$

78. $$\frac{f(x+h) - f(x)}{h} = \frac{-\sqrt{4(x+h)} + \sqrt{4x}}{h} \cdot \frac{-\sqrt{4(x+h)} - \sqrt{4x}}{-\sqrt{4(x+h)} - \sqrt{4x}}$$

$$= \frac{4(x+h) - 4x}{h[-\sqrt{4(x+h)} - \sqrt{4x}]}$$

$$= \frac{4h}{-h[\sqrt{4x + 4h} + \sqrt{4x}]} = \frac{-4}{\sqrt{4x + 4h} + \sqrt{4x}}$$

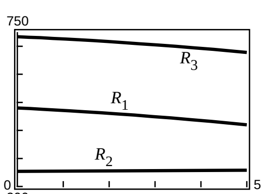
$$= \frac{-2}{\sqrt{x+h} + \sqrt{x}},\ h \neq 0$$

80. (a) Total sales $= R_1 + R_2$

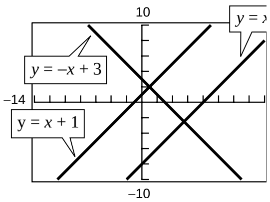
$= (480 - 8t - 0.8t^2) + (254 + 0.78t)$

$= 734 - 7.22t - 0.8t^2$

(b)



82.



For 2000, $t = 10$ and $(y_1 + y_2 + y_3)(10) \approx 613.95$
≈ 614 billion.

84. (a) $r(x) = \dfrac{x}{2}$

(b) $A(r) = \pi r^2$

(c) $(A \circ r)(x) = A(r(x))$

$$= A\left(\frac{x}{2}\right) = \pi\left(\frac{x}{2}\right)^2 = \frac{1}{4}\pi x^2$$

$A \circ r$ represents the area of the circular base of the tank with radius $x/2$.

86. $x = 150$ miles $-$ (450 mph)(t hours)

$y = 200$ miles $-$ (450 mph)(t hours)

$$s = \sqrt{x^2 + y^2}$$
$$= \sqrt{(150 - 450t)^2 + (200 - 450t)^2}$$
$$= 50\sqrt{162t^2 - 126t + 25}$$

88. (a) $R = p - 1200$

(b) $S = 0.92p$

(c) $(R \circ S)(p) = 0.92p - 1200$

$(S \circ R)(p) = 0.92(p - 1200)$

(d) $(R \circ S)(18{,}400) = 15{,}728$

$(S \circ R)(18{,}400) = 15{,}824$

The discount first yields a lower cost.

90. False. $(f \circ g)(x) = f(6x) = 6x + 1$, but $(g \circ f)(x) = g(x + 1) = 6(x + 1)$

92. Let $f(x)$ and $g(x)$ be odd functions, and define $h(x) = f(x)g(x)$. Then,

$$h(-x) = f(-x)g(-x)$$
$$= [-f(x)][-g(x)] \quad \text{since } f \text{ and } g \text{ are both odd}$$
$$= f(x)g(x) = h(x).$$

Thus, h is even.

Let $f(x)$ and $g(x)$ be even functions, and define $h(x) = f(x)g(x)$. Then,

$$h(-x) = f(-x)g(-x)$$
$$= f(x)g(x) \qquad \text{since } f \text{ and } g \text{ are both even}$$
$$= h(x).$$

Thus, h is even.

94. $g(-x) = \frac{1}{2}[f(-x) + f(-(-x))] = \frac{1}{2}[f(-x) + f(x)] = g(x)$,

which shows that g is even.

$$h(-x) = \tfrac{1}{2}[f(-x) - f(-(-x))] = \tfrac{1}{2}[f(-x) - f(x)]$$
$$= -\tfrac{1}{2}[f(x) - f(-x)] = -h(x),$$

which shows that h is odd.

96. (a) $f(x) = g(x) + h(x)$

$$= \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$$

$$= \frac{1}{2}[(x^2 - 2x + 1) + (x^2 + 2x + 1)] + \frac{1}{2}[(x^2 - 2x + 1) - (x^2 + 2x + 1)]$$

$$= (x^2 + 1) + (-2x) = (\text{even}) + (\text{odd})$$

(b) $f(x) = g(x) + h(x)$

$$= \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$$

$$= \frac{1}{2}\left[\left(\frac{1}{x+1} + \frac{1}{-x+1}\right) + \frac{1}{2}\left[\left(\frac{1}{x+1} - \frac{1}{-x+1}\right) = \frac{1}{1-x^2} + \frac{-x}{1-x^2}\right.\right.$$

$$= \frac{-1}{x^2 - 1} + \frac{x}{x^2 - 1} = (\text{even}) + (\text{odd})$$

98. Three points on the graph of $y = \frac{1}{5}x^3 - 4x^2 + 1$ are $(0, 1)$, $(1, -2.8)$ and $(-1, -3.2)$.

100. Three points on the graph of $y = \frac{x}{x^2 - 5}$ are $(0, 0)$, $\left(1, -\frac{1}{4}\right)$ and $\left(-1, \frac{1}{4}\right)$.

102.

$$y - 5 = \frac{2 - 5}{-8 - 1}(x - 1)$$

$$y - 5 = \frac{1}{3}(x - 1)$$

$$3y - x - 14 = 0$$

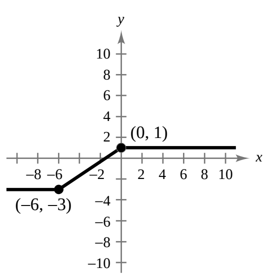
104.

$$y - 1.1 = \frac{3.1 - 1.1}{-4 - 0}(x - 0)$$

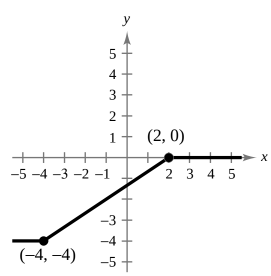
$$y - 1.1 = -\frac{1}{2}x$$

$$2y + x - 2.2 = 0$$

106.



108.



110.