Section 1.5 Inverse Functions

Solutions to Even-Numbered Exercises

2. The inverse is a line through (0, 6) and (6, 0). Matches graph (b).

4. The inverse is a reflection in $y = x$ of a third-degree equation through (0, 0). Matches graph (d).

6.
$$f(x) = \tfrac{1}{5}x$$
$$f^{-1}(x) = 5x$$
$$f(f^{-1}(x)) = f(5x) = \tfrac{1}{5}(5x) = x$$
$$f^{-1}(f(x)) = f^{-1}\left(\tfrac{1}{5}x\right) = 5\left(\tfrac{1}{5}x\right) = x$$

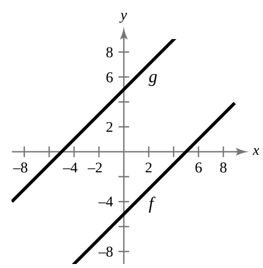
8.
$$f(x) = x - 5$$
$$f^{-1}(x) = x + 5$$
$$f(f^{-1}(x)) = f(x + 5) = (x + 5) - 5 = x$$
$$f^{-1}(f(x)) = f^{-1}(x - 5) = (x - 5) + 5 = x$$

10. $f(x) = \dfrac{x - 1}{4}$

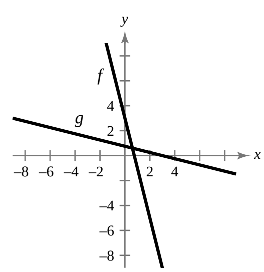
$$f^{-1}(x) = 4x + 1$$
$$f(f^{-1}(x)) = f(4x + 1) = \frac{(4x + 1) - 1}{4} = \frac{4x}{4} = x$$

12.
$$f(x) = x^5$$
$$f^{-1}(x) = \sqrt[5]{x}$$
$$f(f^{-1}(x)) = f\left(\sqrt[5]{x}\right) = \left(\sqrt[5]{x}\right)^5 = x$$
$$f^{-1}(f(x)) = f^{-1}(x^5) = \sqrt[5]{x^5} = x$$

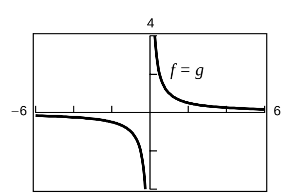
14. $f(x) = x - 5,\ g(x) = x + 5$
$$f(g(x)) = f(x + 5) = (x + 5) - 5 = x$$
$$g(f(x)) = g(x - 5) = (x - 5) + 5 = x$$



16. $f(x) = 3 - 4x,\ g(x) = \dfrac{3 - x}{4}$
$$f(g(x)) = f\left(\frac{3 - x}{4}\right) = 3 - 4\left(\frac{3 - x}{4}\right) = 3 - (3 - x) = x$$
$$g(f(x)) = g(3 - 4x) = \frac{3 - (3 - 4x)}{4} = \frac{4x}{4} = x$$



18. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x}$
$$f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = 1 \div \frac{1}{x} = 1 \cdot \frac{x}{1} = x$$
$$g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{1/x} = 1 \div \frac{1}{x} = 1 \cdot \frac{x}{1} = x$$



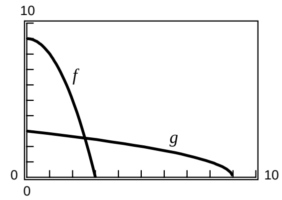
Reflections in the line $y = x$

20. $f(x) = 9 - x^2, x \geq 0$

$g(x) = \sqrt{9 - x}, x \leq 9$

$f(g(x)) = f\left(\sqrt{9 - x}\right) = 9 - \left(\sqrt{9 - x}\right)^2 = 9 - (9 - x) = x$

$g(f(x)) = g(9 - x^2) = \sqrt{9 - (9 - x^2)} = \sqrt{x^2} = x$

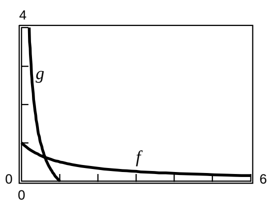


Reflections in the line $y = x$

22. $f(x) = \dfrac{1}{1 + x},\ x \geq 0;\ g(x) = \dfrac{1 - x}{x},\ 0 < x \leq 1$

$$f(g(x)) = f\left(\frac{1 - x}{x}\right) = \frac{1}{1 + \left(\dfrac{1 - x}{x}\right)} = \frac{1}{\dfrac{x}{x} + \dfrac{1 - x}{x}} = \frac{1}{\dfrac{1}{x}} = x$$

$$g(f(x)) = g\left(\frac{1}{1 + x}\right) = \frac{1 - \left(\dfrac{1}{1 + x}\right)}{\left(\dfrac{1}{1 + x}\right)} = \frac{\dfrac{1 + x}{1 + x} - \dfrac{1}{1 + x}}{\dfrac{1}{1 + x}} = \frac{\dfrac{x}{1 + x}}{\dfrac{1}{1 + x}} = \frac{x}{1 + x} \cdot \frac{x + 1}{1} = x$$



Reflections in the line $y = x$

24. (a) $f(g(x)) = f(3x - 8) = \dfrac{(3x - 8) + 8}{3} = \dfrac{3x}{3} = x$

$$g(f(x)) = g\left(\frac{x + 8}{3}\right) = 3\left(\frac{x + 8}{3}\right) - 8 = (x + 8) - 8 = x$$

(b)

x	-8	-5	-2	1	4
$f(x)$	0	1	2	3	4

x	0	1	2	3	4
$g(x)$	-8	-5	-2	1	4

Note that the entries in the tables are the same except that the rows are interchanged.

26. (a) $f(g(x)) = f\left(\sqrt[3]{5x}\right) = \dfrac{\left[\sqrt[3]{5x}\right]^3}{5} = \dfrac{5x}{5} = x$

$$g(f(x)) = g\left(\frac{x^3}{5}\right) = \sqrt[3]{5\left(\frac{x^3}{5}\right)} = \sqrt[3]{x^3} = x$$

(b)

x	-5	-1	0	1	5
$f(x)$	-25	$-\frac{1}{5}$	0	$\frac{1}{5}$	25

x	-25	$-\frac{1}{5}$	0	$\frac{1}{5}$	25
$g(x)$	-5	-1	0	1	5

The entries in the table are the same except that the rows are interchanged.

28. (a) $f(g(x)) = f\left(\dfrac{x^3 + 10}{3}\right) = \sqrt[3]{3\left(\dfrac{x^3 + 10}{3}\right) - 10} = \sqrt[3]{(x^3 + 10) - 10} = \sqrt[3]{x^3} = x$

$g(f(x)) = g(\sqrt[3]{3x - 10}) = \dfrac{\left[\sqrt[3]{3x - 10}^3 + 10\right.}{3} = \dfrac{(3x - 10 =) + 10}{3} = \dfrac{3x}{3} = x$

(b)

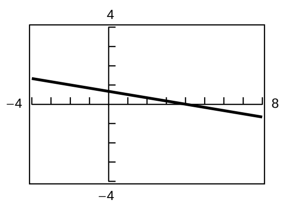
x	$\frac{2}{3}$	3	$\frac{10}{3}$	$\frac{11}{3}$	6
$f(x)$	-2	-1	0	1	2

x	-2	-1	0	1	2
$g(x)$	$\frac{2}{3}$	3	$\frac{10}{3}$	$\frac{11}{3}$	6

The entries in the table are the same except that the rows are interchanged.

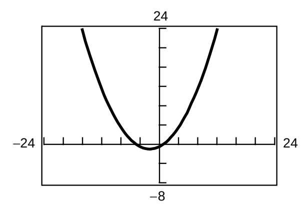
30. $g(x) = \dfrac{4 - x}{6} = \dfrac{2}{3} - \dfrac{x}{6}$

g is one-to-one because it passes the Horizontal Line Test.



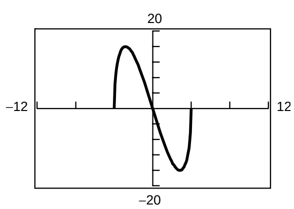
32. $f(x) = \frac{1}{8}(x + 2)^2 - 1$

f does not pass the horizontal test, so f is not one-to-one.



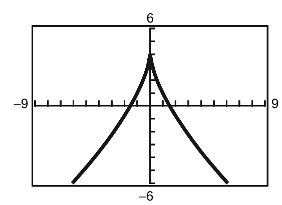
34. $f(x) = -2x\sqrt{16 - x^2}$

is not one-to-one because it does not pass the Horizontal Line Test.



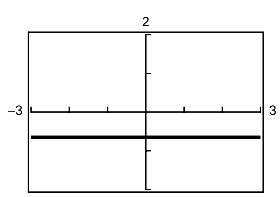
36. $f(x) = 4 - 3x^{2/3}$

is not one-to-one because it does not pass the Horizontal Line Test.



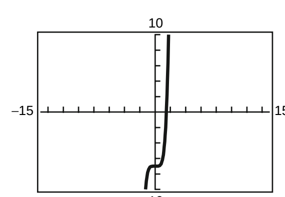
38. $f(x) = -0.65$

is not one-to-one because it does not pass the Horizontal Line Test.



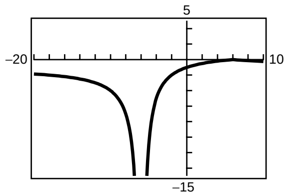
40. $f(x) = x^5 - 7$

is one-to-one because it passes the Horizontal Line Test.

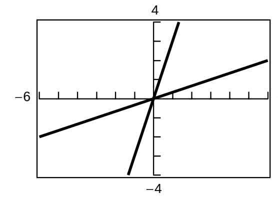


42. $f(x) = -\dfrac{|x-6|}{|x+6|}$

is not one-to-one because it does not pass the Horizontal Line Test.

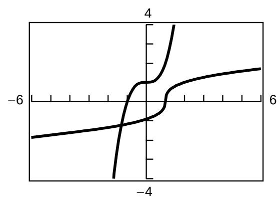


44.
$$f(x) = 3x$$
$$y = 3x$$
$$x = 3y$$
$$\frac{x}{3} = y$$
$$f^{-1}(x) = \frac{x}{3}$$



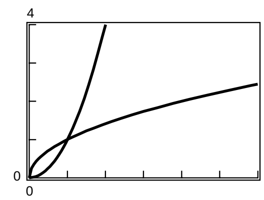
Reflections in the line $y = x$

46.
$$f(x) = x^3 + 1$$
$$y = x^3 + 1$$
$$x = y^3 + 1$$
$$x - 1 = y^3$$
$$\sqrt[3]{x-1} = y$$
$$f^{-1}(x) = \sqrt[3]{x-1}$$



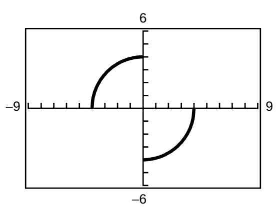
Reflections in the line $y = x$

48.
$$f(x) = x^2, x \geq 0$$
$$y = x^2$$
$$x = y^2$$
$$\sqrt{x} = y$$
$$f^{-1}(x) = \sqrt{x}$$

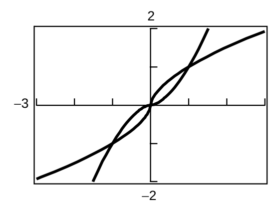


Reflections in the line $y = x$

50.
$$f(x) = \sqrt{16 - x^2},\ -4 \leq x \leq 0$$
$$y = \sqrt{16 - x^2}$$
$$x = \sqrt{16 - y^2}, -4 \leq y \leq 0$$
$$x^2 = 16 - y^2$$
$$y^2 = 16 - x^2$$
$$y = -\sqrt{16 - x^2},\ 0 \leq x \leq 4$$

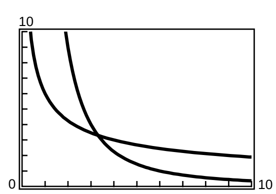


52.
$$f(x) = x^{3/5}$$
$$y = x^{3/5}$$
$$x = y^{3/5}$$
$$x^{5/3} = (y^{3/5})^{5/3}$$
$$x^{5/3} = y$$
$$f^{-1}(x) = x^{5/3}$$



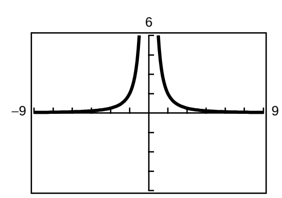
Reflections in the line $y = x$

54.
$$f(x) = \frac{6}{\sqrt{x}}$$
$$y = \frac{6}{\sqrt{x}}$$
$$x = \frac{6}{\sqrt{y}}$$
$$x^2 = \frac{36}{y}$$
$$y = \frac{36}{x^2},\ x > 0$$
$$f^{-1}(x) = \frac{36}{x^2},\ x > 0$$



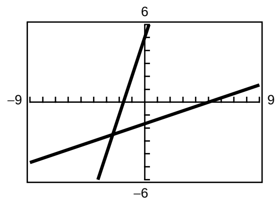
56. $f(x) = \dfrac{1}{x^2}$

f is not one-to-one.
For instance, $f(1) = f(-1)$.
f does not have an inverse.



58. $f(x) = 3x + 5$
f is one-to-one.

$$y = 3x + 5$$
$$x = 3y + 5$$
$$x - 5 = 3y$$
$$\frac{x - 5}{3} = y$$



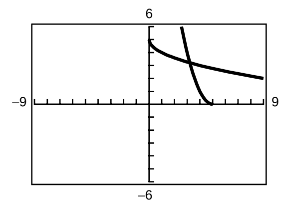
This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{x - 5}{3}.$$

60. $q(x) = (x - 5)^2,\ x \le 5$, is one-to-one.

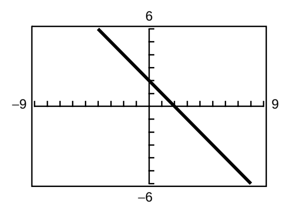
$$y = (x - 5)^2$$
$$x = (y - 5)^2$$
$$-\sqrt{x} = y - 5,\ y \le 5$$
$$y = -\sqrt{x} + 5$$

The inverse is $q^{-1}(x) = -\sqrt{x} + 5$



62. $f(x) = |x - 2|,\ x \le 2 \Longrightarrow y \ge 0$

$$y = |x - 2|,\ x \le 2,\ y \ge 0$$
$$x = |y - 2|,\ y \le 2,\ x \ge 0$$
$$-x = y - 2$$
$$2 - x = y$$
$$f^{-1}(x) = 2 - x,\ x \ge 0$$

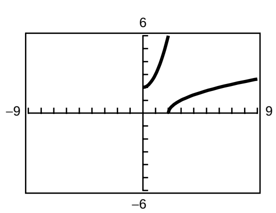


64. $f(x) = \sqrt{x - 2} \Longrightarrow x \ge 2,\ y \ge 0$

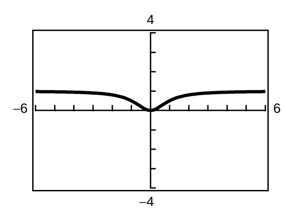
$$y = \sqrt{x - 2},\ x \ge 2,\ y \ge 0$$
$$x = \sqrt{y - 2},\ y \ge 2,\ x \ge 0$$
$$x^2 = y - 2,\ x \ge 0,\ y \ge 2$$
$$x^2 + 2 = y,\ x \ge 0,\ y \ge 2$$

f is one-to-one, so f has an inverse.

$$f^{-1}(x) = x^2 + 2,\ x \ge 0$$



66. $f(x) = \dfrac{x^2}{x^2 + 1}$
f is not one-to-one.
For instance $f(1) = f(-1)$.
Hence, f does not have an inverse.



68. $f(x) = c$ is not one-to-one. For instance, $f(0) = f(1) = c$. Hence, f does not have an inverse.

70. If we let $f(x) = 1 - x^4,\ x \ge 0$, then f has an inverse. [Note: we could also let $x \le 0$.]

$$f(x) = 1 - x^4,\ x \ge 0 \Longrightarrow y \le 1$$
$$y = 1 - x^4,\ x \ge 0,\ \ y \le 1$$
$$x = 1 - y^4,\ y \ge 0,\ x \le 1$$
$$y^4 = 1 - x,\ y \ge 0,\ x \le 1$$
$$y = \sqrt[4]{1 - x},\ x \le 1,\ y \ge 0$$

Thus, $f^{-1}(x) = \sqrt[4]{1 - x},\ x \le 1.$

72. If we let $f(x) = |x - 2|,\ x \geq 2$, then f has an inverse. [Note: we could also let $x \leq 2$.]

$f(x) = |x - 2|,\ x \geq 2$

$f(x) = x - 2$ when $x \geq 2$.

$y = x - 2,\ x \geq 2,\ y \geq 0$

$x = y - 2,\ x \geq 0,\ y \geq 2$

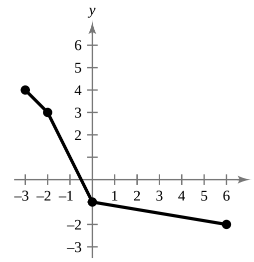
$x + 2 = y,\ x \geq 0,\ y \geq 2$

Thus, $f^{-1}(x) = x + 2,\ x \geq 0$.

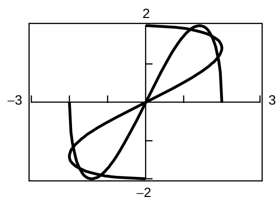
74.

x	$f(x)$
4	−3
3	−2
−1	0
−2	6

x	$f^{-1}(x)$
−3	4
−2	3
0	−1
6	−2

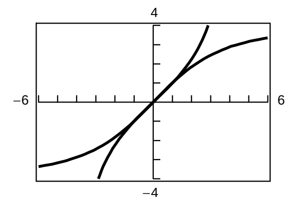


76. (a) and (b)



(c) Not an inverse function since it does not satisfy the Vertical Line Test.

78. (a) and (b)



(c) Inverse function since it satisfies the Vertical Line Test.

In Exercises 80, 82, and 84, $f(x) = \frac{1}{8}x - 3,\ g(x) = x^3,\ f^{-1}(x) = 8(x + 3),\ g^{-1}(x) = \sqrt[3]{x}$.

80. $(g^{-1} \circ f^{-1})(-3) = g^{-1}(f^{-1}(-3))$

$= g^{-1}(8(-3 + 3))$

$= g^{-1}(0) = \sqrt[3]{0} = 0$

82. $(g^{-1} \circ g^{-1})(-4) = g^{-1}(g^{-1}(-4))$

$= g^{-1}\left(\sqrt[3]{-4}\right)$

$= \sqrt[3]{\sqrt[3]{-4}} = -\sqrt[9]{4}$

84. $(g^{-1} \circ f^{-1}x) = g^{-1}(f^{-1}(x))$

$= g^{-1}(8(x + 3))$

$= \sqrt[3]{8(x + 3)}$

$= 2\sqrt[3]{x + 3}$

In Exercises 86 and 88, $f(x) = x + 4,\ g(x) = 2x - 5,\ f^{-1}(x) = x - 4,\ g^{-1}(x) = \dfrac{x + 5}{2}$.

86. $f^{-1} \circ g^{-1}(x) = f^{-1}(g^{-1}(x))$

$= f^{-1}\left(\dfrac{x + 5}{2}\right)$

$= \dfrac{x + 5}{2} - 4$

$= \dfrac{x + 5 - 8}{2}$

$= \dfrac{x - 3}{2}$

88. $(g \circ f)(x) = g(f(x))$

$= g(x + 4)$

$= 2(x + 4) - 5$

$= 2x + 8 - 5$

$= 2x + 3$. Now find inverse:

$y = 2x + 3$

$x = 2y + 3$

$x - 3 = 2y$

$\dfrac{x - 3}{2} = y$

$(g \circ f)^{-1}(x) = \dfrac{x - 3}{2}$

Note that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

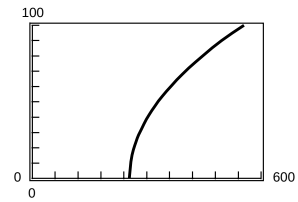
90. (a)

$$y = 0.03x^2 + 254.50, \quad 0 < x < 100$$
$$x = 0.03y^2 + 254.50$$
$$x - 254.50 = 0.03y^2$$
$$\frac{x - 254.50}{0.03} = y^2$$
$$\sqrt{\frac{x - 254.50}{0.03}} = y, \quad x > 254.50$$
$$f^{-1}(x) = \sqrt{\frac{x - 254.50}{0.03}}$$

$x =$ temperature in degrees Fahrenheit

$y =$ percent load for a diesel engine

(b)



(c) $0.03x^2 + 254.50 < 500$

$$0.03x^2 < 245.5$$
$$x^2 < 8183\tfrac{1}{3}$$
$$x < 90.46$$

Thus, $0 < x < 90.46$.

92. False. Consider $f(x) = x^2$ which is even, but does not have an inverse.

94. Answers will vary.

96. If $f(x) = k(2 - x - x^3)$ has an inverse and $f^{-1}(3) = -2$, then $f(-2) = 3$. Thus,

$$f(-2) = k(2 - (-2) - (-2)^3) = 3$$
$$k(2 + 2 + 8) = 3$$
$$12k = 3$$
$$k = \frac{3}{12} = \frac{1}{4}.$$

Thus, $k = \frac{1}{4}$.

98. $(f - g)(4) = f(4) - g(4) = 27 - 1 = 26$

100. $\left(\frac{f}{g}\right)\left(\frac{3}{2}\right) = \dfrac{f\left(\frac{3}{2}\right)}{g\left(\frac{3}{2}\right)} = \dfrac{\frac{9}{2} - 5}{\frac{3}{2} - 3} = \dfrac{-\frac{1}{2}}{-\frac{3}{2}} = \dfrac{1}{3}$

102. $y = 5x + 8$

$$x = 5y + 8$$
$$y = \frac{x - 8}{5}$$
$$f^{-1}(x) = \frac{1}{5}(x - 8)$$

104. $y = \sqrt[3]{x - 7}$

$$x = \sqrt[3]{y - 7}$$
$$x^3 = y - 7$$
$$y = x^3 + 7$$
$$f^{-1}(x) = x^3 + 7$$