CHAPTER 2
Polynomial and Rational Functions

C H A P T E R 2
Polynomial and Rational Functions

Section 2.1 Quadratic Functions

Solutions to Even-Numbered Exercises

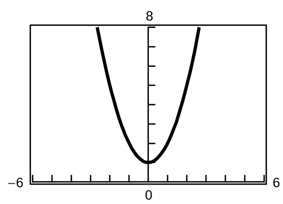
2. $f(x) = (x + 4)^2$ opens upward and has vertex $(-4, 0)$. Matches graph (c).

4. $f(x) = 3 - x^2$ opens downward and has vertex $(0, 3)$. Matches graph (h).

6. $f(x) = (x + 1)^2 - 2$ opens upward and has vertex $(-1, -2)$. Matches graph (a).

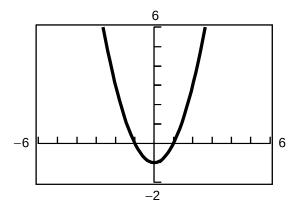
8. $f(x) = -(x - 4)^2$ opens downward and has vertex $(4, 0)$. Matches graph (d).

10. (a) $y = x^2 + 1$



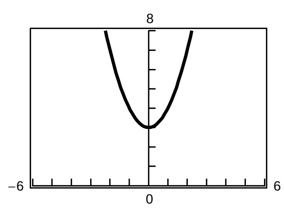
Vertical shift 1 unit upward

(b) $y = x^2 - 1$



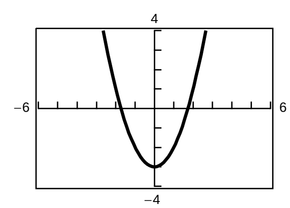
Vertical shift 1 unit downward

(c) $y = x^2 + 3$



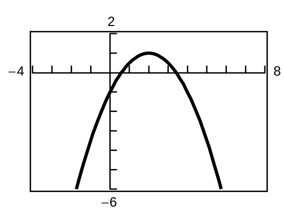
Vertical shift 3 units upward

(d) $y = x^2 - 3$



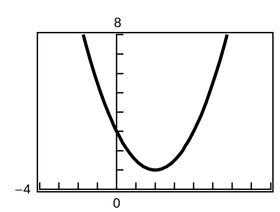
Vertical shift 3 units downward

12. (a) $y = -\frac{1}{2}(x - 2)^2 + 1$



Horizontal shift 2 units to right, vertical shrink by $\frac{1}{2}$, reflection in the x-axis, and vertical shift 1 unit upward

(b) $y = \frac{1}{2}(x - 2)^2 + 1$



Horizontal shift 2 units to the right, vertical shrink by $\frac{1}{2}$, vertical shift 1 unit upward

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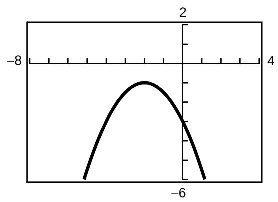
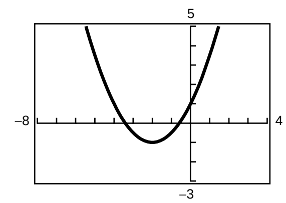
12. —CONTINUED—

(c) $y = -\frac{1}{2}(x + 2)^2 - 1$



Horizontal shift 2 units to left, vertical shrink by $\frac{1}{2}$, reflection in x-axis, and vertical shift 1 unit downward

(d) $y = \frac{1}{2}(x + 2)^2 - 1$

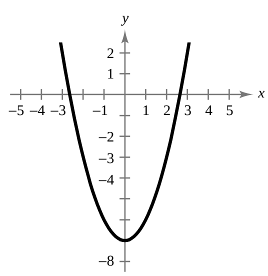


Horizontal shift 2 units to left, vertical shrink by $\frac{1}{2}$, and vertical shift 1 unit downward.

14. $f(x) = x^2 - 7$

Vertex: $(0, -7)$

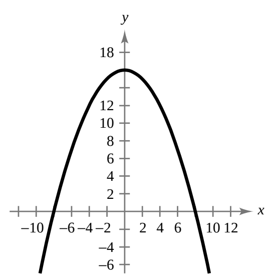
Intercepts: $(\pm\sqrt{7}, 0), (0, -7)$



16. $f(x) = 16 - \frac{1}{4}x^2$

Vertex: $(0, 16)$

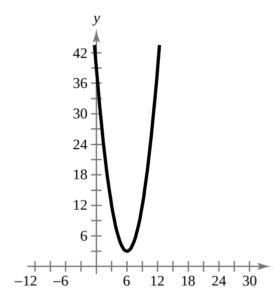
Intercepts: $(\pm 8, 0), (0, 16)$



18. $f(x) = (x - 6)^2 + 3$

Vertex: $(6, 3)$

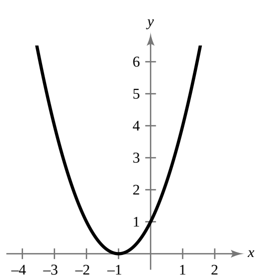
Intercepts: $(0, 39)$



20. $g(x) = x^2 + 2x + 1 = (x + 1)^2$

Vertex: $(-1, 0)$

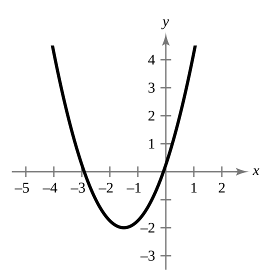
Intercepts: $(-1, 0), (0, 1)$



22. $f(x) = x^2 + 3x + \frac{1}{4} = \left(x + \frac{3}{2}\right)^2 - 2$

Vertex: $\left(-\frac{3}{2}, -2\right)$

Intercepts: $\left(-\frac{3}{2} \pm \sqrt{2}, 0\right), \left(0, \frac{1}{4}\right)$

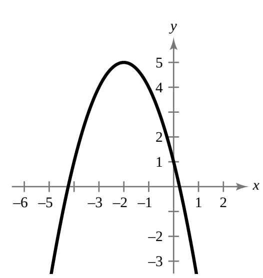


24.

$$\begin{aligned} f(x) = -x^2 - 4x + 1 &= -1(x^2 + 4x - 1) \\ &= -1[(x + 2)^2 - 5] \\ &= -(x + 2)^2 + 5 \end{aligned}$$

Vertex: $(-2, 5)$

Intercepts: $(-2 \pm \sqrt{5}, 0), (0, 1)$



26. $f(x) = 2x^2 - x + 1$

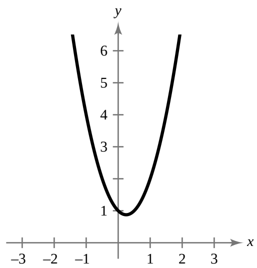
$= 2\left(x^2 - \frac{1}{2}x\right) + 1$

$= 2\left(x - \frac{1}{4}\right)^2 - \frac{1}{8} + 1$

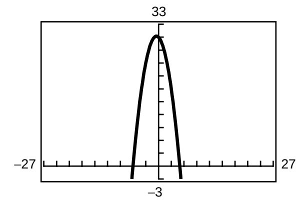
$= 2\left(x - \frac{1}{4}\right)^2 + \frac{7}{8}$

Vertex: $\left(\frac{1}{4}, \frac{7}{8}\right)$

Intercept: $(0, 1)$



28.



Vertex: $\left(-\frac{1}{2}, \frac{121}{4}\right)$

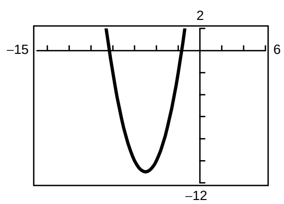
Intercepts: $(5, 0), (-6, 0), (0, 30)$

$f(x) = -(x^2 + x - 30)$

$= -\left(x^2 + x + \frac{1}{4}\right) + \frac{1}{4} + 30$

$= -\left(x + \frac{1}{2}\right)^2 + \frac{121}{4}$

30.



Vertex: $(-5, -11)$

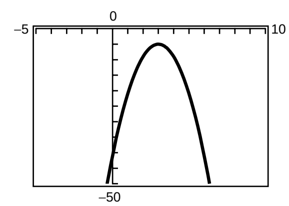
Intercepts: $(-1.683, 0), (-8.317, 0), (0, 14)$

$f(x) = x^2 + 10x + 14$

$= (x^2 + 10x + 25) - 11$

$= (x + 5)^2 - 11$

32.



Vertex: $(3, -5)$

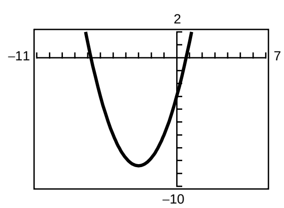
Intercept: $(0, -41)$

$f(x) = -4x^2 + 24x - 41$

$= -4(x^2 - 6x + 9) + 36 - 41$

$= -4(x - 3)^2 - 5$

34.



Vertex: $(-3, -8.4)$

Intercepts: $(0.742, 0), (-6.742, 0), (0, -3)$

$f(x) = \frac{3}{5}(x^2 + 6x - 5)$

$= \frac{3}{5}(x^2 + 6x + 9) - 3 - \frac{27}{5}$

$= \frac{3}{5}(x + 3)^2 - \frac{42}{5}$

36. $(0, 1)$ is the vertex.

$f(x) = a(x - 0)^2 + 1 = ax^2 + 1$

Since the graph passes through $(1, 0)$,

$0 = a(1)^2 + 1 = a + 1 \implies a = -1$

$f(x) = -x^2 + 1.$

38. $(-2, -1)$ is the vertex.

$f(x) = a(x + 2)^2 - 1$

Since the graph passes through $(0, 3)$,

$3 = a(0 + 2)^2 - 1$

$3 = 4a - 1$

$4 = 4a$

$1 = a.$

Thus, $y = (x + 2)^2 - 1.$

40. $(4, -1)$ is the vertex.

$f(x) = a(x - 4)^2 - 1$

Since the graph passes through $(2, 3)$,

$3 = a(2 - 4)^2 - 1$

$3 = 4a - 1$

$4 = 4a$

$1 = a.$

Thus, $f(x) = (x - 4)^2 - 1.$

42. $(2, 3)$ is the vertex.

$f(x) = a(x - 2)^2 + 3$

Since the graph passes through $(0, 2)$,

$2 = a(0 - 2)^2 + 3$

$2 = 4a + 3$

$-1 = 4a$

$-\frac{1}{4} = a.$

Thus, $f(x) = -\frac{1}{4}(x - 2)^2 + 3.$

44. $\left(-\frac{1}{4}, \frac{3}{2}\right)$ is the vertex.

$f(x) = a\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}$

Since the graph passes through $(-2, 0)$,

$0 = a\left(-2 + \frac{1}{4}\right)^2 + \frac{3}{2}$

$-\frac{3}{2} = a\left(-\frac{7}{4}\right)^2 = \frac{49a}{16}$

$-24 = 49a$

$a = -\frac{24}{49}$

Thus, $f(x) = \frac{-24}{49}\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}.$

46. $\left(-\frac{5}{2}, 0\right)$ is the vertex.

$f(x) = a\left(x + \frac{5}{2}\right)^2 + 0$

$= a\left(x + \frac{5}{2}\right)^2$

Since the graph passes through $\left(-\frac{7}{2}, -\frac{16}{3}\right)$

$-\frac{16}{3} = a\left(-\frac{7}{2} + \frac{5}{2}\right)^2$

$= a(-1)^2 = a$

Thus, $f(x) = -\frac{16}{3}\left(x + \frac{5}{2}\right)^2.$

48. $y = x^2 - 6x + 9$

x-intercept: $(3, 0)$

$0 = x^2 - 6x + 9$

$0 = (x - 3)^2$

$x = 3$

50. $y = 2x^2 + 5x - 3$

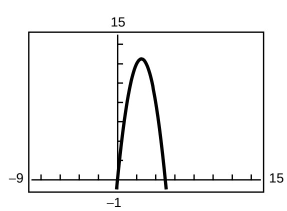
x-intercepts: $\left(\frac{1}{2}, 0\right), (-3, 0)$

$0 = 2x^2 + 5x - 3$

$0 = (2x - 1)(x + 3)$

$x = \frac{1}{2}, -3$

52. $y = -2x^2 + 10x$



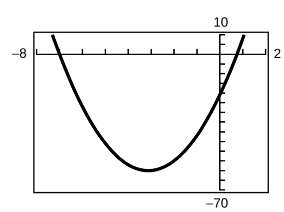
x-intercepts: $(0, 0), (5, 0)$

$0 = -2x^2 + 10x$

$0 = x(-2x + 10)$

$x = 0, x = 5$

54. $y = 4x^2 + 25x - 21$



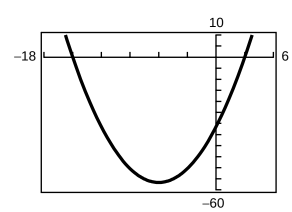
x-intercepts: $(-7, 0), (0.75, 0)$

$0 = 4x^2 + 25x - 21$

$= (x + 7)(4x - 3)$

$x = -7, \frac{3}{4}$

56. $y = \frac{7}{10}(x^2 + 12x - 45)$



x-intercepts: $(3, 0), (-15, 0)$

$0 = \frac{7}{10}(x^2 + 12x - 45)$

$0 = x^2 + 12x - 45$

$= (x - 3)(x + 15)$

$x = 3, -15$

58. $f(x) = a(x - 0)(x - 10) = ax(x - 10)$.

Many correct answers: $f(x) = x(x - 10) = x^2 - 10x$ opens upward, $f(x) = -x(x - 10) = -x^2 + 10x$ opens downward.

60. $f(x) = 2\left[x - \left(-\frac{5}{2}\right)\right](x - 2)$

$= 2\left(x + \frac{5}{2}\right)(x - 2)$

$= 2(x^2 + \frac{1}{2}x - 5)$

$= 2x^2 + x - 10$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -2x^2 - x + 10$

Many other answers possible.

62. Let x = first number and y = second number. Then, $x + y = S$, $y = S - x$. The product is $P(x) = xy = x(S - x)$.

$P(x) = Sx - x^2$

$= -x^2 + Sx$

$= -\left(x^2 - Sx + \frac{S^2}{4} - \frac{S^2}{4}\right)$

$= -\left(x - \frac{S}{2}\right)^2 + \frac{S^2}{4}$

The maximum value of the product occurs at the vertex of $P(x)$ and is $S^2/4$. This happens when $x = y = S/2$.

64. Let x = first number and y = second number. Then $x + 3y = 42$, $y = \frac{1}{3}(42 - x)$. The product is $P(x) = xy = x\frac{1}{3}(42 - x) = 14x - \frac{1}{3}x^2$.

$P(x) = -\frac{1}{3}x^2 + 14x$

$= -\frac{1}{3}(x^2 - 42x)$

$= -\frac{1}{3}(x^2 - 42x + 441) + 147$

$= -\frac{1}{3}(x - 21)^2 + 147$

The maximum value of the product is 147, and occurs when $x = 21$ and $y = \frac{1}{3}(42 - 21) = 7$.

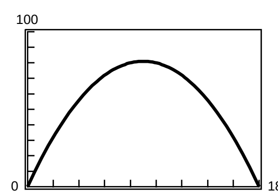
66. Let x = length of rectangle and y = width of rectangle.

$2x + 2y = 36$

$y = 18 - x$

(a) $A(x) = xy = x(18 - x)$
Domain: $0 < x < 18$

(b)



(c) The area is maximum (81 square meters) when $x = y = 9$ meters. The rectangle has dimensions 9 meters × 9 meters.

68. (a) $4x + 3y = 200 \implies y = \frac{1}{3}(200 - 4x)$

x	y	Area
2	$\frac{1}{3}[200 - 4(2)]$	$2xy = 256$
4	$\frac{1}{3}[200 - 4(4)]$	$2xy \approx 490$
6	$\frac{1}{3}[200 - 4(6)]$	$2xy = 704$
8	$\frac{1}{3}[200 - 4(8)]$	$2xy = 896$
10	$\frac{1}{3}[200 - 4(10)]$	$2xy \approx 1067$
12	$\frac{1}{3}[200 - 4(12)]$	$2xy = 1216$

(b)

x	y	Area
20	$\frac{1}{3}[200 - 4(20)]$	$2xy = 1600$
22	$\frac{1}{3}[200 - 4(22)]$	$2xy \approx 1643$
24	$\frac{1}{3}[200 - 4(24)]$	$2xy = 1664$
26	$\frac{1}{3}[200 - 4(26)]$	$2xy = 1664$
28	$\frac{1}{3}[200 - 4(28)]$	$2xy \approx 1643$
30	$\frac{1}{3}[200 - 4(30)]$	$2xy = 1600$

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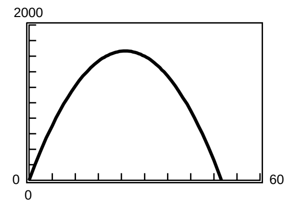
68. —CONTINUED—

(c) $A = 2xy = 2x\left(\dfrac{200 - 4x}{3}\right) = \dfrac{2x(4)(50 - x)}{3}$

$= \dfrac{8x(50 - x)}{3}$

(d)



(e) $A = \dfrac{8}{3}x(50 - x)$

$= -\dfrac{8}{3}(x^2 - 50x)$

$= -\dfrac{8}{3}(x^2 - 50x + 625 - 625)$

$= -\dfrac{8}{3}[(x - 25)^2 - 625]$

$= -\dfrac{8}{3}(x - 25)^2 + \dfrac{5000}{3}$

The maximum area occurs at the vertex and is $5000/3$ square feet. This happens when $x = 25$ feet and $y = (200 - 4(25))/3 = 100/3$ feet. The dimensions are $2x = 50$ feet by $33\frac{1}{3}$ feet.

70. Graphical Solution: Graph $C = 10{,}000 - 110x + 0.45x^2$ in the viewing window $[0, 250] \times [0, 10{,}000]$. Use the zoom and trace features to determine that the minimum is \$3277.78 at $x = 122.22 \approx 122$ units.

Analytic Solution:

$C = 0.45x^2 - 110x + 10{,}000$

$= 0.45\left(x^2 - \dfrac{2200}{9}x + \dfrac{1{,}210{,}000}{81}\right) + 10{,}000 - \dfrac{60{,}500}{9}$

$= 0.45\left(x - \dfrac{1100}{9}\right)^2 + \dfrac{29{,}500}{9}$

Thus, $\left(\dfrac{1100}{9}, \dfrac{29{,}500}{9}\right) \approx (122, 3277)$ is the vertex.

72. $P = -0.5x^2 + 20x + 230$

$= -0.5(x^2 - 40x + 400) + 230 + 200$

$= -0.5(x - 20)^2 + 430$

Since the vertex is $(20, 430)$, $x = 20$ yields the maximum profit of 430. Equivalently, the maximum profit occurs at the vertex,

$x = \dfrac{-b}{2a} = \dfrac{-20}{2(-0.5)} = 20$, or \$2000.

74. $y = -\dfrac{4}{9}x^2 + \dfrac{24}{9}x + 12$

The maximum height of the dive occurs at the vertex, $x = -\dfrac{b}{2a} = -\dfrac{\frac{24}{9}}{2\left(-\frac{4}{9}\right)} = 3$.

The height at $x = 3$ is $-\dfrac{4}{9}(3)^2 + \dfrac{24}{9}(3) + 12 = 16$.

The maximum height of the dive is 16 feet.