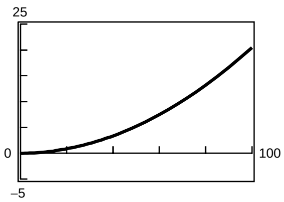
76. (a)



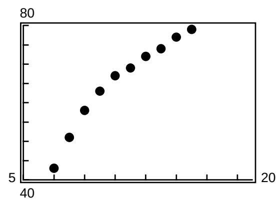
(b) The parabola intersects $y = 10$ at $s \approx 69.6$. Thus, the maximum speed is 69.6 mph.

Analytically,

$$0.002s^2 + 0.005s - 0.029 = 10$$
$$2s^2 + 5s - 29 = 10{,}000$$
$$2s^2 + 5s - 10{,}029 = 0$$
$$a = 2,\ b = 5,\ c = -10{,}029$$
$$s = \frac{-5 \pm \sqrt{5^2 - 4(2)(-10{,}029)}}{2(2)}$$
$$s = \frac{-5 \pm \sqrt{80{,}257}}{4}$$
$$s \approx -72.1,\ 69.6$$

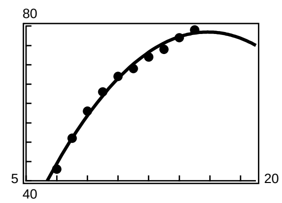
The maximum speed if power is not to exceed 10 horsepower is 69.6 miles per hour.

78. (a)



(b) $y = -0.352t^2 + 11.830t - 21.245$

(c)



(d) No. The model begins to decrease at a rapid rate.

80. True. For $f(x)$, $\dfrac{-b}{2a} = -\dfrac{-10}{2(-4)} = -\dfrac{10}{8} = -\dfrac{5}{4}$

For $g(x)$, $\dfrac{-b}{2a} = \dfrac{-30}{2(12)} = \dfrac{-30}{24} = \dfrac{-5}{4}$

In both cases, $x = -\frac{5}{4}$ is the axis of symmetry.

82. $x + y = 8 \implies y = 8 - x$. Then, $-\frac{2}{3}x + y = -\frac{2}{3}x + (8 - x) = 6$
$\implies -\frac{5}{3}x = -2 \implies x = \frac{6}{5}$ and $y = 8 - \frac{6}{5} = \frac{34}{5}$

(1.2, 6.8)

84. $y = x + 3 = 9 - x^2$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3,\ x = 2$

Thus, $(-3, 0)$ and $(2, 5)$ are the points of intersection.

86. $3x - 4y = 12$

$-4y = 12 - 3x$

$y = -\frac{1}{4}(12 - 3x)$

$y = \frac{3}{4}x - 3$

Yes, y is a function of x.

88. $y = \sqrt{x + 3}$

Yes, y is a function of x.

Section 2.2 Polynomial Functions of Higher Degree

Solutions to Even-Numbered Exercises

2. $f(x) = x^2 - 4x$ is a parabola with intercepts (0, 0) and (4, 0) and opens upward. Matches graph (h).

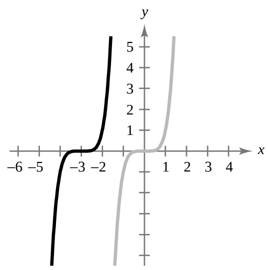
4. $f(x) = 2x^3 - 3x + 1$ has intercepts (0, 1), (1, 0), $\left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}, 0\right)$ and $\left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}, 0\right)$. Matches graph (a).

6. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$ has y-intercept $\left(0, -\frac{4}{3}\right)$. Matches graph (d).

8. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ has intercepts $(0, 0)$, $(1, 0)$, $(-1, 0)$, $(3, 0)$, $(-3, 0)$. Matches (b).

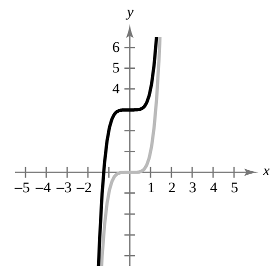
10. $y = x^5$

(a) $f(x) = (x + 3)^5$



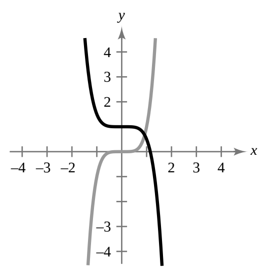
Horizontal shift three units to the left

(b) $f(x) = x^5 + 3$



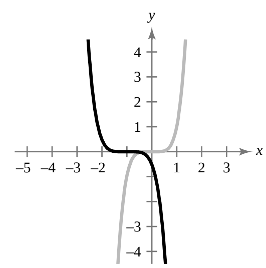
Vertical shift three units upward

(c) $f(x) = 1 - \frac{1}{2}x^5$



Reflection in the x-axis, vertical shrink and vertical shift one unit upward

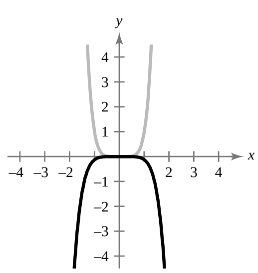
(d) $f(x) = -\frac{1}{2}(x + 1)^5$



Reflection in the x-axis, vertical shrink and horizontal shift one unit to the left

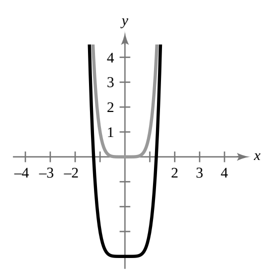
12. $y = x^6$

(a) $f(x) = -\frac{1}{8}x^6$



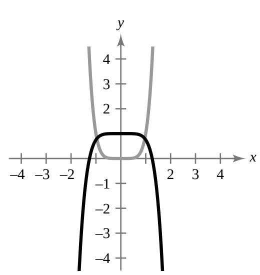
Vertical shrink and reflection in the x-axis

(b) $f(x) = x^6 - 4$



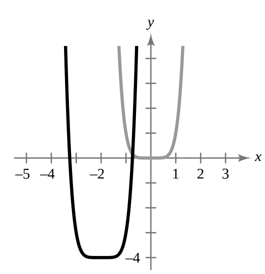
Vertical shift 4 units downward

(c) $f(x) = -\frac{1}{4}x^6 + 1$



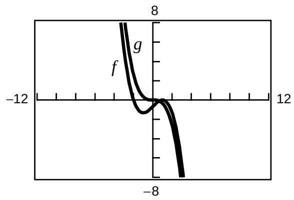
Vertical shrink, vertical shift upward one unit, and reflection in the x-axis

(d) $f(x) = (x + 2)^6 - 4$

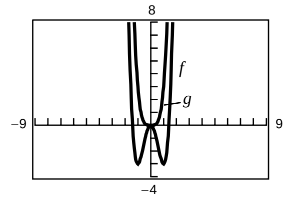


Horizontal shift two units to the left and vertical shift 4 units downward

14. $f(x) = -\frac{1}{3}(x^3 - 3x + 2),\ g(x) = -\frac{1}{3}x^3$



16. $f(x) = 3x^4 - 6x^2,\ g(x) = 3x^4$



18. $f(x) = \frac{1}{3}x^3 + 5x$

Degree: 3

Leading coefficient: $\frac{1}{3}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

20. $h(x) = 1 - x^6$

Degree: 6

Leading coefficient: -1

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

22. $f(x) = 2x^5 - 5x + 7.5$

Degree: 5

Leading coefficient: 2

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

24. $f(x) = \dfrac{3x^4 - 2x + 5}{4}$

Degree: 4

Leading coefficient: $\frac{3}{4}$

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

26. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$

Degree: 3

Leading coefficient: $-\frac{7}{8}$

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

28. $f(x) = 49 - x^2$

$= (7 - x)(7 + x)$

$x = \pm 7$

30. $f(x) = x^2 + 10x + 25$

$= (x + 5)^2$

$x = -5$

32. $f(x) = 2x^2 - 14x + 24$

$= 2(x^2 - 7x + 12)$

$= 2(x - 3)(x - 4)$

$x = 3, 4$

34. $f(x) = x^4 - x^3 - 20x^2$

$= x^2(x^2 - x - 20)$

$= x^2(x + 4)(x - 5)$

$x = 0, -4, 5$

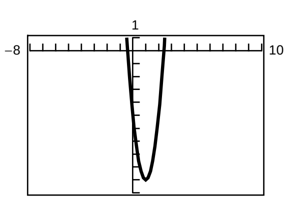
36. $f(x) = \frac{5}{3}x^2 + \frac{8}{3}x - \frac{4}{3}$

$= \frac{1}{3}(5x^2 + 8x - 4)$

$= \frac{1}{3}(5x - 2)(x + 2)$

$x = \frac{2}{5}, -2$

38. (a)



(b) zeros: $-0.414, 2.414$

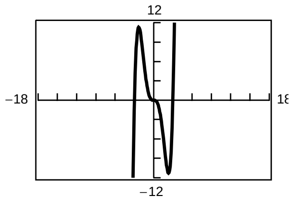
(c) $g(x) = 5(x^2 - 2x - 1)$

$$x = \frac{2 \pm \sqrt{4 - 4(-1)}}{2} = 1 \pm \sqrt{2} \quad (\approx -0.414, 2.414)$$

$\left(1 \pm \sqrt{2}, 0\right)$

40. $y = \frac{1}{4}x^3(x^2 - 9)$

(a)



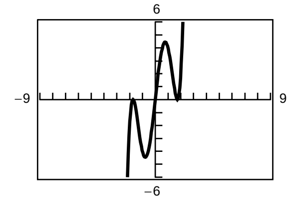
(b) Zeros: $0, \pm 3$

(c) $0 = \frac{1}{4}x^3(x^2 - 9)$

$x = 0, \pm 3$

x-intercepts: $(0, 0)$, $(\pm 3, 0)$

42. (a)



(b) Zeros: $0, \pm 1.732$

(c) $g(t) = t^5 - 6t^3 + 9t$

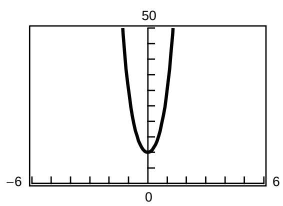
$= t(t^4 - 6t^2 + 9)$

$= t(t^2 - 3)^2$

$t = 0, \pm\sqrt{3} (\approx 0, \pm 1.732)$

$(0, 0)$, $(\pm\sqrt{3}, 0)$

44. (a)



(b) No real zeros

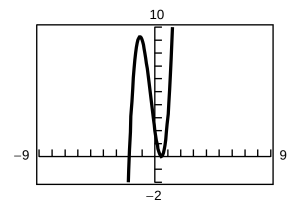
(c) $f(x) = 5(x^4 + 3x^2 + 2)$

$= 5(x^2 + 1)(x^2 + 2) > 0$

no real zeros

46. $y = 4x^3 + 4x^2 - 7x + 2$

(a)



(b) Zeros: $-2, \frac{1}{2}$

(c) $0 = 4x^3 + 4x^2 - 7x + 2$

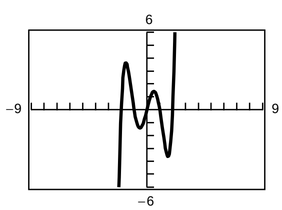
$= (2x - 1)(2x^2 + 3x - 2)$

$= (2x - 1)(2x - 1)(x + 2)$

$x = -2, \frac{1}{2}$

x-intercepts: $(-2, 0)$, $\left(\frac{1}{2}, 0\right)$

48. (a)



(b) Zeros: $0, \pm 1, \pm 2$

(c) $y = x^5 - 5x^3 + 4x$

$= x(x^4 - 5x^2 + 4)$

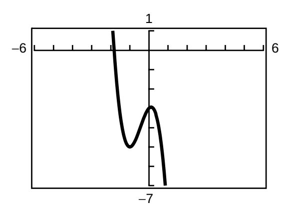
$= x(x^2 - 4)(x^2 - 1)$

$= x(x - 2)(x + 2)(x - 1)(x + 1)$

Zeros: $0, \pm 1, \pm 2$

$(0, 0)$, $(\pm 1, 0)$, $(\pm 2, 0)$

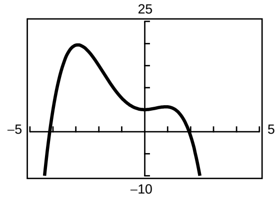
50.



Relative maximum: $(0.111, -2.942)$

Relative minimum: $(-1, -5)$

52.



Relative maximums: $(0.915, 5.646)$, $(-2.915, 19.688)$

Relative minimum: $(0, 5)$

54. $f(x) = (x - 0)(x - (-8))$

$= x(x + 8)$

$= x^2 + 8x$

Note: $f(x) = ax(x + 8)$ has zeros 0 and -8 for all nonzero real numbers a.

56. $f(x) = (x - (-4))(x - 5)$

$= (x + 4)(x - 5)$

$= x^2 - x - 20$

Note: $f(x) = a(x + 4)(x - 5)$ has zeros -4 and 5 for all nonzero real numbers a.

58. $f(x) = (x - 0)(x - 2)(x - 7)$

$= x(x - 2)(x - 7)$

$= x^3 - 9x^2 + 14x$

Note: $f(x) = ax(x - 2)(x - 7)$ has zeros 0, 2, 7 for all nonzero real numbers a.

60. $f(x) = (x - (-2))(x - (-1))(x - 0)(x - 1)(x - 2)$

$= x(x + 2)(x + 1)(x - 1)(x - 2)$

$= x(x^2 - 4)(x^2 - 1)$

$= x(x^4 - 5x^2 + 4)$

$= x^5 - 5x^3 + 4x$

Note: $f(x) = a\,x(x + 2)(x + 1)(x - 1)(x - 2)$ has zeros $-2, -1, 0, 1, 2$ for all nonzero real numbers a.

62. $f(x) = \left(x - \left(6 + \sqrt{3}\right)\right)\left(x - \left(6 - \sqrt{3}\right)\right)$

$= \left((x - 6) - \sqrt{3}\right)\left((x - 6) + \sqrt{3}\right)$

$= (x - 6)^2 - 3$

$= x^2 - 12x + 36 - 3$

$= x^2 - 12x + 33$

Note: $f(x) = a\left(x - \left(6 + \sqrt{3}\right)\right)\left(x - \left(6 - \sqrt{3}\right)\right)$ has zeros $6 + \sqrt{3}$ and $6 - \sqrt{3}$ for all nonzero real numbers a.

64. $f(x) = (x - 4)\left(x - \left(2 + \sqrt{7}\right)\right)\left(x - \left(2 - \sqrt{7}\right)\right)$

$= (x - 4)\left((x - 2) - \sqrt{7}\right)\left((x - 2) + \sqrt{7}\right)$

$= (x - 4)\left((x - 2)^2 - 7\right)$

$= (x - 4)(x^2 - 4x - 3)$

$= x^3 - 8x^2 + 13x + 12$

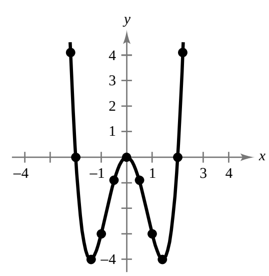
66. (a) The degree of g is even and the leading coefficient is 2. The graph rises to the left and rises to the right.

(b) $g(x) = x^4 - 4x^2 = x^2(x^2 - 4)$

$= x^2(x - 2)(x + 2)$

zeros: $0, 2, -2$: $(0, 0)$, $(\pm 2, 0)$

(c), (d)



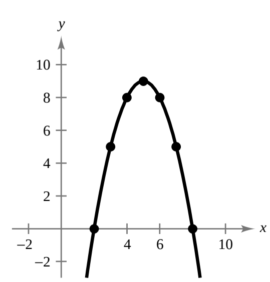
68. (a) The degree of g is even and the leading coefficient is -1. The graph falls to the left and to the right.

(b) $g(x) = -x^2 + 10x - 16 = -(x^2 - 10x + 16)$

$= -(x - 8)(x - 2)$

Zeros: 2, 8: $(2, 0)$, $(8, 0)$

(c), (d)

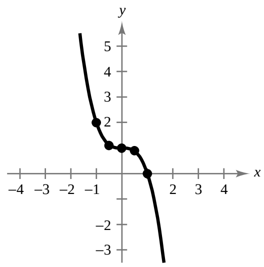


70. (a) The degree of f is odd and the leading coefficient is -1. The graph rises to the left and falls to the right.

(b) $f(x) = 1 - x^3 = (1 - x)(1 + x + x^2)$

Zero: 1: $(1, 0)$

(c), (d)



72. (a) The degree of f is odd and the leading coefficient is -4. The graph rises to the left and falls to the right.

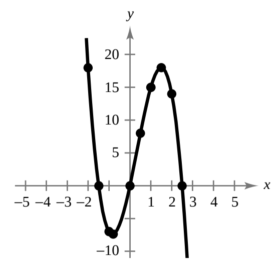
(b) $f(x) = -4x^3 + 4x^2 + 15x$

$= -x(4x^2 - 4x - 15)$

$= -x(2x + 3)(2x - 5)$

zeros: $0, -\dfrac{3}{2}, \dfrac{5}{2}$: $(-1.5, 0), (0, 0), (2.5, 0)$

(c), (d)



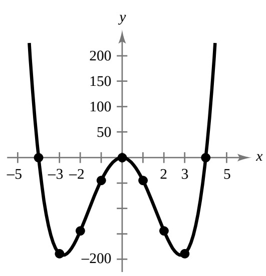
74. (a) The degree of f is even and the leading coefficient is 3. The graph of f rises to the right and to the left.

(b) $f(x) = 3x^4 - 48x^2 = 3x^2(x^2 - 16)$

$= 3x^2(x - 4)(x + 4)$

Zeros: $0, 4, -4$: $(0, 0), (\pm 4, 0)$

(c), (d)

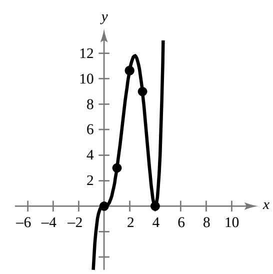


76. (a) The degree of h is odd and the leading coefficient is $\frac{1}{3}$. The graph falls to the left and rises to the right.

(b) $h(x) = \frac{1}{3}x^3(x - 4)^2$

Zeros: $0, 4$: $(0, 0), (4, 0)$

(c), (d)

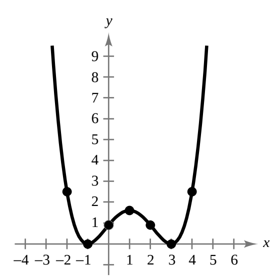


78. (a) The degree of g is even and the leading coefficient is $\frac{1}{10}$. The graph rises to the left and to the right.

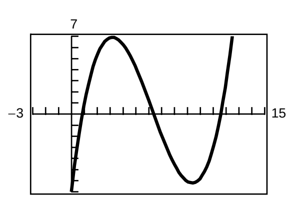
(b) $g(x) = \frac{1}{10}(x + 1)^2(x - 3)^2$

Zeros: $-1, 3$: $(-1, 0), (3, 0)$

(c), (d)



80. (a)



The function has three zeros. They are in the intervals $(0, 1)$, $(6, 7)$ and $(11, 12)$.

(b) 0.845, 6.385, 11.588

—CONTINUED—

80. —CONTINUED—

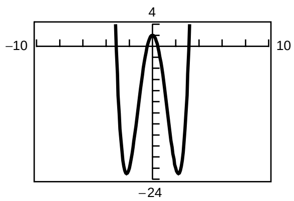
(c)

x	y_1
0.81	−0.2336
0.82	−0.167
0.83	−0.1008
0.84	−0.035
0.85	0.03048
0.86	0.09559
0.87	0.16035

x	y_1
6.36	0.07947
6.37	0.04775
6.38	0.01604
6.39	−0.0157
6.40	−0.0474
6.41	−0.079
6.42	−0.1107

x	y_1
11.55	−0.2298
11.56	−0.1695
11.57	−0.1088
11.58	−0.0478
11.59	0.01363
11.60	0.07536
11.61	0.13744

82. (a)



The function has four zeros. They are in the intervals $(0, 1)$, $(3, 4)$, $(-1, 0)$ and $(-4, -3)$.

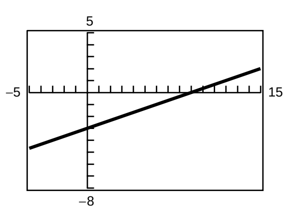
(b) Notice that f is even. Hence, the zeros come in symmetric pairs. Zeros: $\pm 0.452, \pm 3.130$

(c) Because the function is even, we only need to verify the positive zeros.

x	y_1
0.42	0.26712
0.43	0.18519
0.44	0.10148
0.45	0.01601
0.46	−0.0712
0.47	−0.1602
0.48	−0.2509

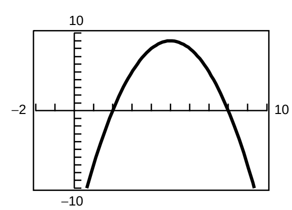
x	y_1
3.09	−2.315
3.10	−1.748
3.11	−1.171
3.12	−0.5855
3.13	0.01025
3.14	0.61571
3.15	1.231

84. $h(x) = \frac{1}{3}x - 3$



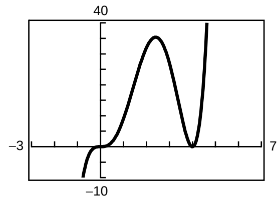
Xmin = -5
Xmax = 15
Xscl = 1
Ymin = -8
Ymax = 5
Yscl = 1

86 $g(x) = -x^2 + 9x - 14$



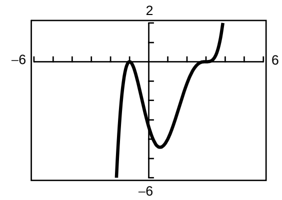
Xmin = -2
Xmax = 10
Xscl = 1
Ymin = -10
Ymax = 10
Yscl = 1

88. $h(x) = x^3(x - 4)^2$



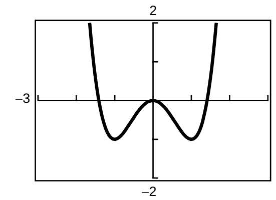
No symmetry. Two x-intercepts $(0, 0)$, $(4, 0)$

90. $g(x) = \frac{1}{8}(x + 1)^2(x - 3)^3$



No symmetry. Two x-intercepts $(-1, 0)$, $(3, 0)$

92. $f(x) = x^4 - 2x^2$

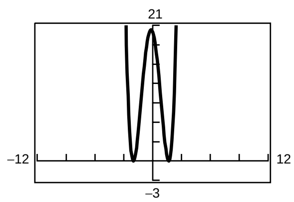


Symmetric with respect to y-axis

Three x-intercepts $(0, 0)$, $\left(\pm\sqrt{2}, 0\right)$

94. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$

No symmetry; two x-intercepts



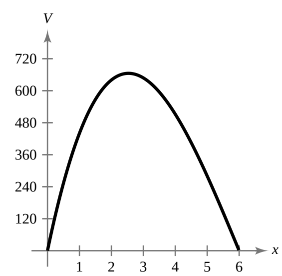
96. (a) $V(x) = \text{length} \times \text{width} \times \text{height}$

$$= (24 - 2x)(24 - 4x)x$$

$$= 8x(12 - x)(6 - x)$$

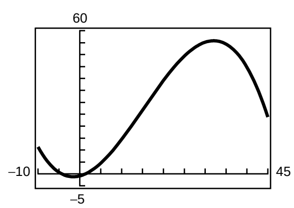
(b) Domain: $0 < x < 6$

(c)



Maximum occurs at $x \approx 2.54$

98.

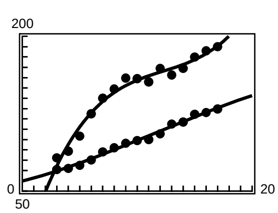


Point of Diminishing Returns: $(15.2, 27.3)$
15.2 years.

100. (a) $y_1 = 0.07501t^3 - 2.7605t^2 + 37.3522t - 15.0200$

(b) $y_2 = -0.004619t^3 + 0.1576t^2 + 2.8238t + 59.6796$

(c)



The median price of homes in the South is less than the median price of homes in the Northeast.

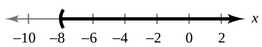
102. True. The degree is odd and the leading coefficient is -1.

104. $(f + g)(-4) = f(-4) + g(-4)$
$$= -59 + 128 = 69$$

106. $(fg)\left(-\frac{4}{7}\right) = f\left(-\frac{4}{7}\right)g\left(-\frac{4}{7}\right)$
$$= (-11)\left(\frac{8 \cdot 16}{49}\right)$$
$$= -\frac{1408}{49} \approx -28.7347$$

108. $(f \circ g)(-1) = f(g(-11) = f(8) = 109$

110. $3(x - 5) < 4x - 7$
$$3x - 15 < 4x - 7$$
$$-8 < x$$

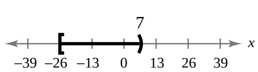


112. $$\frac{5x - 2}{x - 7} \le 4$$
$$\frac{5x - 2}{x - 7} - 4 \le 0$$
$$\frac{5x - 2 - 4(x - 7)}{x - 7} \le 0$$
$$\frac{x + 26}{x - 7} \le 0$$
$[x + 26 \ge 0 \text{ and } x - 7 < 0]$ or $[x + 26 \le 0 \text{ and } x - 7 > 0]$

$[x \ge -26 \text{ and } x < 7]$ or $[x \le -26 \text{ and } x > 7]$

impossible

$-26 \le x < 7$



114. Vertex: $(3, -6)$

$f(x) = a(x - 3)^2 - 6$

Point: $(-1, 2) \implies 2 = a(-1 - 3)^2 - 6$
$$8 = 16a$$
$$a = \tfrac{1}{2}$$
$f(x) = \frac{1}{2}(x - 3)^2 - 6$

116. Vertex: $(4, -4)$

$f(x) = a(x - 4)^2 - 4$

Point: $(1, 10) \implies 10 = a(1 - 4)^2 - 4$
$$14 = 9a$$
$$a = \tfrac{14}{9}$$
$f(x) = \frac{14}{9}(x - 4)^2 - 4$