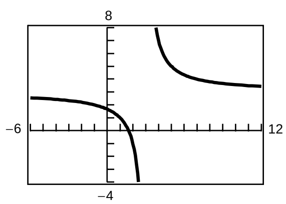
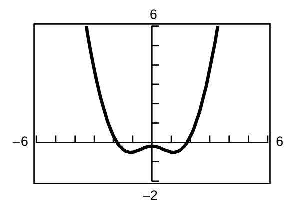
Section 2.3 Real Zeros of Polynomial Functions

Solutions to Even-Numbered Exercises

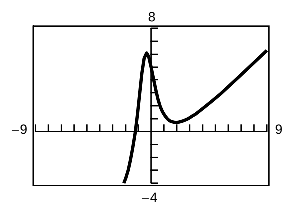
2.
$$\begin{aligned} y_2 &= 3 + \frac{4}{x-3} \\ &= \frac{3(x-3)+4}{x-3} \\ &= \frac{3x-9+4}{x-3} \\ &= \frac{3x-5}{x-3} \\ &= y_1 \end{aligned}$$



4.
$$\begin{aligned} y_2 &= x^2 - 8 + \frac{39}{x^2+5} \\ &= \frac{(x^2-8)(x^2+5)+39}{x^2+5} \\ &= \frac{x^4 - 8x^2 + 5x^2 - 40 + 39}{x^2+5} \\ &= \frac{x^4 - 3x^2 - 1}{x^2+5} \\ &= y_1 \end{aligned}$$



6.
$$\begin{aligned} y_2 &= x - 3 + \frac{2(x+4)}{x^2+x+1} \\ &= \frac{x(x^2+x+1) - 3(x^2+x+1) + 2(x+4)}{x^2+x+1} \\ &= \frac{x^3 + x^2 + x - 3x^2 - 3x - 3 + 2x + 8}{x^2+x+1} \\ &= \frac{x^3 - 2x^2 + 5}{x^2+x+1} \\ &= y_1 \end{aligned}$$



8.
$$\begin{array}{r} 5x + 3 \\ x - 4 \overline{\smash{)}\,5x^2 - 17x - 12} \\ \underline{5x^2 - 20x} \quad\quad \\ 3x - 12 \\ \underline{3x - 12} \\ 0 \end{array}$$

$$\frac{5x^2 - 17x - 12}{x - 4} = 5x + 3$$

10.
$$\begin{array}{r} x^3 + 3x^2 \quad\quad - 1 \\ x + 2 \overline{\smash{)}\,x^4 + 5x^3 + 6x^2 - x - 2} \\ \underline{x^4 + 2x^3} \quad\quad\quad\quad\quad\quad \\ 3x^3 + 6x^2 \quad\quad\quad \\ \underline{3x^3 + 6x^2} \quad\quad\quad \\ -x - 2 \\ \underline{-x - 2} \\ 0 \end{array}$$

$$\frac{x^4 + 5x^3 + 6x^2 - x - 2}{x + 2} = x^3 + 3x^2 - 1$$

12.
$$\begin{array}{r} 4 \\ 2x+1\,\overline{)\,8x-5} \\ \underline{8x+4} \\ -9 \end{array}$$
$$\frac{8x-5}{2x+1} = 4 - \frac{9}{2x+1}$$

14.
$$\begin{array}{r} x \\ x^2+1\,\overline{)\,x^3+0x^2+0x-9} \\ \underline{x^3 \qquad\quad +x\quad\;} \\ -x-9 \end{array}$$
$$\frac{x^3-9}{x^2+1} = x - \frac{x+9}{x^2+1}$$

16.
$$\begin{array}{r} x^2 \\ x^3-1\,\overline{)\,x^5+0x^4+0x^3+0x^2+0x+7} \\ \underline{x^5 \qquad\qquad -x^2 \qquad\quad} \\ x^2+7 \end{array}$$
$$\frac{x^5+7}{x^3-1} = x^2 + \frac{x^2+7}{x^3-1}$$

18. $(x-1)^3 = x^3 - 3x^2 + 3x - 1$
$$\begin{array}{r} x+3 \\ x^3-3x^2+3x-1\,\overline{)\,x^4 \qquad\qquad\qquad} \\ \underline{x^4-3x^3+3x^2-x\quad} \\ 3x^3-3x^2+x\quad \\ \underline{3x^3-9x^2+9x-3} \\ 6x^2-8x+3 \end{array}$$
$$\frac{x^4}{(x-1)^3} = x + 3 + \frac{6x^2-8x+3}{(x-1)^3}$$

20.
$$\begin{array}{r|rrrr} 1 & 2 & 6 & -14 & 9 \\ & & 2 & 8 & -6 \\ \hline & 2 & 8 & -6 & 3 \end{array}$$
$$\frac{2x^3+6x^2-14x+9}{x-1} = 2x^2 + 8x - 6 + \frac{3}{x-1}$$

22.
$$\begin{array}{r|rrrr} -6 & 2 & 14 & -20 & 7 \\ & & -12 & -12 & 192 \\ \hline & 2 & 2 & -32 & 199 \end{array}$$
$$\frac{2x^3+14x^2-20x+7}{x+6} = 2x^2 + 2x - 32 + \frac{199}{x+6}$$

24.
$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$
$$\frac{5x^3+6x+8}{x+2} = 5x^2 - 10x + 26 - \frac{44}{x+2}$$

26.
$$\begin{array}{r|rrrr} 9 & 1 & 0 & 0 & -729 \\ & & 9 & 81 & 729 \\ \hline & 1 & 9 & 81 & 0 \end{array}$$
$$\frac{x^3-729}{x-9} = x^2 + 9x + 81$$

28.
$$\begin{array}{r|rrrr} \frac{3}{2} & 3 & -4 & 0 & 5 \\ & & \frac{9}{2} & \frac{3}{4} & \frac{9}{8} \\ \hline & 3 & \frac{1}{2} & \frac{3}{4} & \frac{49}{8} \end{array}$$
$$\frac{3x^3-4x^2+5}{x-\frac{3}{2}} = 3x^2 + \frac{1}{2}x + \frac{3}{4} + \frac{49}{8x-12}$$

30. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14, \quad k = -\dfrac{2}{3}$
$$\begin{array}{r|rrrrr} -\frac{2}{3} & 15 & 10 & -6 & 0 & 14 \\ & & -10 & 0 & 4 & -\frac{8}{3} \\ \hline & 15 & 0 & -6 & 4 & \frac{34}{3} \end{array}$$
$$f(x) = \left(x + \frac{2}{3}\right)(15x^3 - 6x + 4) + \frac{34}{3}$$
$$f\left(-\frac{2}{3}\right) = \frac{34}{3}$$

32.
$$\begin{array}{r|rrrr} -\sqrt{5} & 1 & 2 & -5 & -4 \\ & & -\sqrt{5} & 5-2\sqrt{5} & 10 \\ \hline & 1 & 2-\sqrt{5} & -2\sqrt{5} & 6 \end{array}$$

$f(x) = \left(x + \sqrt{5}\right)\left(x^2 + \left(2 - \sqrt{5}\right)x - 2\sqrt{5}\right) + 6$

$f\left(-\sqrt{5}\right) = 6$

34.
$$\begin{array}{r|rrrr} 2+\sqrt{2} & -3 & 8 & 10 & -8 \\ & & -6-3\sqrt{2} & -2-4\sqrt{2} & 8 \\ \hline & -3 & 2-3\sqrt{2} & 8-4\sqrt{2} & 0 \end{array}$$

$f(x) = \left(x - \left(2 + \sqrt{2}\right)\right)\left(-3x^2 + \left(2 - 3\sqrt{2}\right)x + 8 - 4\sqrt{2}\right)$

$f\left(2 + \sqrt{2}\right) = 0$

36. $g(x) = x^6 - 4x^4 + 3x^2 + 2$

(a)
$$\begin{array}{r|rrrrrrl} 2 & 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ & & 2 & 4 & 0 & 0 & 6 & 12 \\ \hline & 1 & 2 & 0 & 0 & 3 & 6 & 14 = g(2) \end{array}$$

(b)
$$\begin{array}{r|rrrrrrl} -4 & 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ & & -4 & 16 & -48 & 192 & -780 & 3120 \\ \hline & 1 & -4 & 12 & -48 & 195 & -780 & 3122 = g(-4) \end{array}$$

(c)
$$\begin{array}{r|rrrrrrl} 3 & 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ & & 3 & 9 & 15 & 45 & 144 & 432 \\ \hline & 1 & 3 & 5 & 15 & 48 & 144 & 434 = g(3) \end{array}$$

(d)
$$\begin{array}{r|rrrrrrl} -1 & 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ & & -1 & 1 & 3 & -3 & 0 & 0 \\ \hline & 1 & -1 & -3 & 3 & 0 & 0 & 2 = g(-1) \end{array}$$

38. $f(x) = 0.4x^4 - 1.6x^3 + 0.7x^2 - 2$

(a)
$$\begin{array}{r|rrrrl} 1 & 0.4 & -1.6 & 0.7 & 0 & -2 \\ & & 0.4 & -1.2 & -0.5 & -0.5 \\ \hline & 0.4 & -1.2 & -0.5 & -0.5 & -2.5 = f(1) \end{array}$$

(b)
$$\begin{array}{r|rrrrl} -2 & 0.4 & -1.6 & 0.7 & 0 & -2 \\ & & -0.8 & 4.8 & -11 & 22 \\ \hline & 0.4 & -2.4 & 5.5 & -11 & 20 = f(-2) \end{array}$$

(c)
$$\begin{array}{r|rrrrl} 5 & 0.4 & -1.6 & 0.7 & 0 & -2 \\ & & 2.0 & 2.0 & 13.5 & 67.5 \\ \hline & 0.4 & 0.4 & 2.7 & 13.5 & 65.5 = f(5) \end{array}$$

(d)
$$\begin{array}{r|rrrrl} -10 & 0.4 & -1.6 & 0.7 & 0 & -2 \\ & & -4.0 & 56.0 & -567 & 5670 \\ \hline & 0.4 & -5.6 & 56.7 & -567 & 5668 = f(-10) \end{array}$$

40.

-4	1	0	-28	-48
		-4	16	48
	1	-4	-12	0

Zeros: $-4, -2, 6$

$x^3 - 28x - 48 = (x + 4)(x^2 - 4x - 12)$
$= (x + 4)(x - 6)(x + 2)$

42.

-2	1	2	-3	-6
		-2	0	6
	1	0	-3	0

$x^3 + 2x^2 - 3x - 6 = (x + 2)(x^2 - 3)$
$= (x + 2)(x + \sqrt{3})(x - \sqrt{3})$

Zeros: $-2, -\sqrt{3}, \sqrt{3}$

44.

-3	1	-1	-13	-3
		-3	12	3
	1	-4	-1	0

$x^3 - x^2 - 13x - 3 = (x + 3)(x^2 - 4x - 1)$
$= (x + 3)(x - (2 - \sqrt{5}))(x - (2 + \sqrt{5}))$

Zeros: $2 - \sqrt{5}, 2 + \sqrt{5}, -3$

46. (a)

-3	3	2	-19	6
		-9	21	-6
	3	-7	2	0

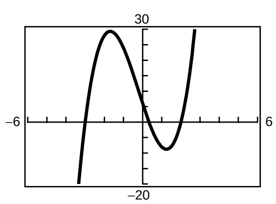
2	3	-7	2
		6	-2
	3	-1	0

(b) Remaining factor: $(3x - 1)$

(c) $f(x) = (x + 3)(x - 2)(3x - 1)$

(d) Real zeros: $-3, 2, \frac{1}{3}$

(e)



48. (a)

-2	8	-14	-71	-10	24
		-16	60	22	-24
	8	-30	-11	12	0

4	8	-30	-11	12
		32	8	-12
	8	2	-3	0

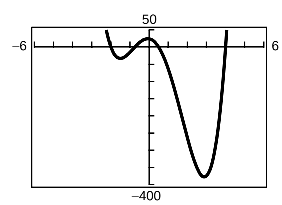
(b) $8x^2 + 2x - 3 = (4x + 3)(2x - 1)$.

Remaining factors: $(4x + 3), (2x - 1)$

(c) $f(x) = (x + 2)(x - 4)(4x + 3)(2x - 1)$

(d) Real zeros: $-2, 4, -\frac{3}{4}, \frac{1}{2}$

(e)



50. (a)

$\frac{1}{2}$	2	-1	-10	5
		1	0	-5
	2	0	-10	0

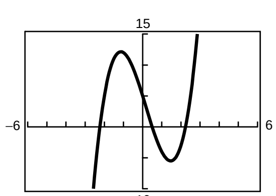
$-\sqrt{5}$	2	0	-10
		$-2\sqrt{5}$	10
	2	$-2\sqrt{5}$	0

(b) Remaining factor: $x - \sqrt{5}$

(c) $f(x) = (2x - 1)(x + \sqrt{5})(x - \sqrt{5})$
$= (x - \frac{1}{2})(x + \sqrt{5})(2x - 2\sqrt{5})$

(d) Real zeros: $\frac{1}{2}, -\sqrt{5}, \sqrt{5}$

(e)



52. $f(x) = x^3 - 4x^2 - 4x + 16$

$p =$ factor of 16

$q =$ factor of 1

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

Zeros shown on graph: $-2, 2, 4$

54. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

Possible rational zeros: $\pm 2, \pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}$

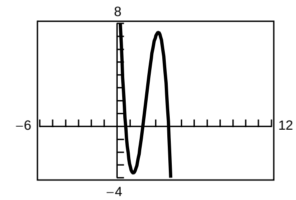
Zeros shown on graph: $-1, -\frac{1}{2}, 1, 2.$

56. $f(x) = -3x^3 + 20x^2 - 36x + 16$

(a) Possible rational zeros:

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}$

(b)



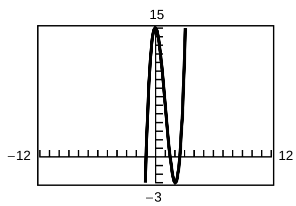
(c) Real zeros: $\frac{2}{3}, 2, 4$

58. $f(x) = 4x^3 - 12x^2 - x + 15$

(a) Possible rational zeros:

$\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2},$
$\pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}$

(b)

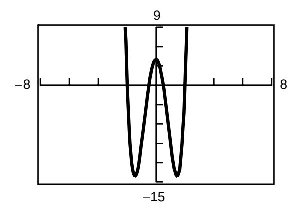


(c) Real zeros: $-1, \frac{3}{2}, \frac{5}{2}$

60. $f(x) = 4x^4 - 17x^2 + 4$

(a) Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}, \pm\frac{1}{4}$

(b)

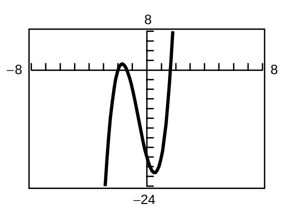


(c) Real zeros: $\pm 2, \pm\frac{1}{2}$

62. $f(x) = 4x^3 + 7x^2 - 11x - 18$

(a) Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9,$
$\pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}$

(b)



(c)
$$\begin{array}{r|rrrr} -2 & 4 & 7 & -11 & -18 \\ & & -8 & 2 & 18 \\ \hline & 4 & -1 & -9 & 0 \end{array}$$

The zeros of $4x^2 - x - 9$ are

$$\frac{1 \pm \sqrt{1 + 4(4)(9)}}{8} = \frac{1}{8} \pm \frac{\sqrt{145}}{8}$$

Real zeros: $-2, \dfrac{1}{8} \pm \dfrac{\sqrt{145}}{8}.$

64. $x^4 - x^3 - 29x^2 - x - 30 = 0$. Using a graphing utility and synthetic division, $x = 6$ and $x = -5$ are rational zeros. Hence,
$(x - 6)(x + 5)(x^2 + 1) = 0 \Longrightarrow x = -5, 6.$

66. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

Using a graphing utility and synthetic division, $1/2$, 1, and -6 are rational zeros. Hence,
$(y + 6)(y - 1)^2\left(y - \frac{1}{2}\right) = 0 \Longrightarrow y = -6, 1, \frac{1}{2}.$

68. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$

$x(x^4 - x^3 - 3x^2 + 5x - 2) = 0$

1	1	−1	−3	5	−2
		1	0	−3	2
	1	0	−3	2	0

−2	1	0	−3	2
		−2	4	−2
	1	−2	1	0

$x(x - 1)(x + 2)(x^2 - 2x + 1) = 0$

$x(x - 1)(x + 2)(x - 1)(x - 1) = 0$

The real zeros are $-2, 0, 1$.

70. $g(x) = x^3 - 4x^2 - 2x + 8$

(a) Zeros: 4, 1.414, −1.414

(b)

4	1	−4	−2	8
		4	0	−8
	1	0	−2	0

$x = 4$ is a zero

$g(x) = (x - 4)(x^2 - 2)$

$= (x - 4)(x + \sqrt{2})(x - \sqrt{2})$

zeros: $4, \pm\sqrt{2}$

72. $f(s) = s^3 - 12s^2 + 40s - 24$

(a) Zeros: 6, 5.236, 0.764

(b)

6	1	−12	40	−24
		6	−36	24
	1	−6	4	0

$f(s) = (s - 6)(s^2 - 6s + 4)$

$= (s - 6)(s - 3 - \sqrt{5})(5 - 3 + \sqrt{5})$

74. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) $x = \pm 3, \frac{3}{2}, \frac{1}{3}$

(b)

3	6	−11	−51	99	−27
		18	21	−90	27
	6	7	−30	9	0

−3	6	7	−30	9
		−18	33	−9
	6	−11	3	0

$g(x) = (x - 3)(x + 3)(6x^2 - 11x + 3)$

$= (x - 3)(x + 3)(3x - 1)(2x - 3)$

76. $f(x) = 2x^3 - 3x^2 - 12x + 8$

(a)

4	2	−3	−12	8
		8	20	32
	2	5	8	40

4 is an upper bound.

(b)

−3	2	−3	−12	8
		−6	27	−45
	2	−9	15	−37

−3 is a lower bound.

78. $f(x) = 2x^4 - 8x + 3$

(a)

3	2	0	0	−8	3
		6	18	54	138
	2	6	18	46	141

3 is an upper bound.

(b)

−4	2	0	0	−8	3
		−8	32	−128	544
	2	−8	32	−136	547

−4 is a lower bound.

80. $f(x) = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}$

4	2	−3	−23	12
		8	20	−12
	2	5	−3	0

$f(x) = \frac{1}{2}(x - 4)(2x^2 + 5x - 3)$

$= \frac{1}{2}(x - 4)(2x - 1)(x + 3)$

Rational zeros: $-3, \frac{1}{2}, 4$

82. $f(x) = \frac{1}{6}(6z^3 + 11z^2 - 3z - 2)$

Possible rational zeros: $\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

−2	6	11	−3	−2
		−12	2	2
	6	−1	−1	0

$f(x) = \frac{1}{6}(x + 2)(6x^2 - x - 1)$

$= \frac{1}{6}(x + 2)(3x + 1)(2x - 1)$

Rational zeros: $-2, -\frac{1}{3}, \frac{1}{2}$

84. $f(x) = x^3 - 2$

$= \left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + \sqrt[3]{4}\right)$

Rational zeros: 0

Irrational zeros: 1 $\left(x = \sqrt[3]{2}\right)$

Matches (a).

86. $f(x) = x^3 - 2x$

$= x(x^2 - 2)$

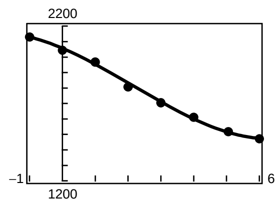
$= x\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$

Rational zeros: 1 $(x = 0)$

Irrational zeros: 2 $\left(x = \pm\sqrt{2}\right)$

Matches (c).

88. (a) and (b)



$M = 2.4116t^3 - 15.5574t^2 - 90.4885t + 2056.6169$

(c)

t	-1	0	1	2	3	4	5	6
M	2130	2044	1986	1807	1705	1611	1518	1472
Model	2129	2057	1953	1833	1710	1600	1517	1475

(d)

$$\begin{array}{r|rrrr} 11 & 2.4116 & -15.5574 & -90.4885 & 2056.6169 \\ & & 26.5276 & 120.6722 & 332.0207 \\ \hline & 2.4116 & 10.9702 & 30.1837 & 2388.6376 \end{array}$$

For 2001 $(t = 11)$, $M \approx 2{,}389$ thousand

No, the model predicts that M decreases indefinitely.

90. (a) Combined length and width:

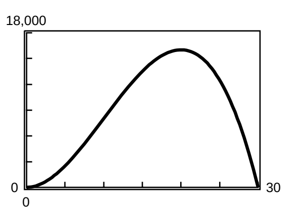
$4x + y = 120 \Longrightarrow y = 120 - 4x$

Volume $= l \cdot w \cdot h = x^2y$

$= x^2(120 - 4x)$

$= 4x^2(30 - x)$

(b)



Dimensions with maximum volume:
$20 \times 20 \times 40$

(c) $13{,}500 = 4x^2(30 - x)$

$4x^3 - 120x^2 + 13{,}500 = 0$

$x^3 - 30x^2 + 3375 = 0$

$$\begin{array}{r|rrrr} 15 & 1 & -30 & 0 & 3375 \\ & & 15 & -225 & -3375 \\ \hline & 1 & -15 & -225 & 0 \end{array}$$

$(x - 15)(x^2 - 15x - 225) = 0$

Using the Quadratic equation,

$x = 15, \dfrac{15 \pm 15\sqrt{5}}{2}$

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

92.

$P = -45x^3 + 2500x^2 - 275{,}000$

$800{,}000 = -45x^3 + 2500x^2 - 275{,}000$

$0 = 45x^3 - 2500x^2 + 1{,}075{,}000$

$0 = 9x^3 - 500x^2 + 215{,}000$

The zeros of this equation are $x \approx -18.0$, $x \approx 31.5$,and $x \approx 42.0$. Because $0 \le x \le 50$, disregard $x \approx -18.02$. The smaller remaining solution is $x \approx 31.5$, or \$315,000.