90. $5x - 4y = 8$

$5x - 8 = 4y$

$y = \frac{5}{4}x - 2 \quad \text{slope: } \frac{5}{4}$

(a) Parallel line:
$$y - (-2) = \frac{5}{4}(x - 3)$$
$$4y + 8 = 5x - 15$$
$$4y - 5x + 23 = 0$$

(b) Perpendicular line:
$$y - (-2) = -\frac{4}{5}(x - 3)$$
$$5y + 10 = -4x + 12$$
$$5y + 4x = 2$$

92. $y = -x^2 + 6$

Let $y = 0$: $x^2 = 6 \implies x = \pm\sqrt{6}$.

x-intercepts: $(\sqrt{6}, 0), (-\sqrt{6}, 0)$

Let $x = 0$: $y = 6$. y-intercept: $(0, 6)$

94. $y = |x - 4| + 1$

Let $y = 0$: $|x - 4| + 1 = 0$ impossible. No x-intercepts.

Let $x = 0$: $y = |-4| + 1 = 5$. y-intercept: $(0, 5)$

96. Let $x =$ the amount withdrawn and then replaced.

$$(1.0)(x) + (0.50)(5 - x) = (0.60)5$$
$$x + 2.50 - .5x = 3.0$$
$$.5x = .50$$
$$x = 1 \text{ liter}$$

Section 2.5 The Fundamental Theorem of Algebra

Solutions to Even-Numbered Exercises

2. $f(x) = x^2(x + 3)(x^2 - 1)$

$= x^2(x + 3)(x + 1)(x - 1)$

The five zeros are $x = 0, 0, -3, -1, 1$.

4. $f(x) = (x + 5)(x - 8)^2$

The three zeros are $x = -5$, 8 and 8

6. $h(t) = (t - 3)(t - 2)(t - 3i)(t + 3i)$

The four zeros are $t = 3, 2, 3i, -3i$

8. $h(m) = (m - 4)^2(m - 2 + 4i)(m - 2 - 4i)$

$= (m - 4)(m - 4)(m - 2 + 4i)(m - 2 - 4i)$

The four zeros are: $4, 4, 2 - 4i, 2 + 4i$

10. $f(x) = x^3 - 4x^2 - 4x + 16$

$= x^2(x - 4) - 4(x - 4)$

$= (x^2 - 4)(x - 4)$

$= (x + 2)(x - 2)(x - 4)$

The zeros are: $x = 2, -2$, and 4. This corresponds to the x-intercepts of $(-2, 0)$, $(2, 0)$, and $(4, 0)$ on the graph.

12. $f(x) = x^4 - 3x^2 - 4$

$= (x^2 - 4)(x^2 + 1)$

$= (x + 2)(x - 2)(x^2 + 1)$

Zeros: $\pm 2, \pm i$

The only real zeros are $x = -2, 2$. This corresponds to the x-intercepts of $(-2, 0)$ and $(2, 0)$ on the graph.

14. $g(x) = x^2 + 10x + 23$

Zeros: $x = \dfrac{-10 \pm \sqrt{8}}{2} = -5 \pm \sqrt{2}$

$g(x) = \left(x + 5 + \sqrt{2}\right)\left(x + 5 - \sqrt{2}\right)$

16. $f(x) = x^2 + 6x - 2$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-2)}}{2} = -3 \pm \sqrt{11}$$

$$f(x) = \left(x - \left(-3 + \sqrt{11}\right)\right)\left(x - \left(-3 - \sqrt{11}\right)\right)$$
$$= \left(x + 3 - \sqrt{11}\right)\left(x + 3 + \sqrt{11}\right)$$

18. $f(x) = x^2 - x + 56$

Zeros: $x = \dfrac{1 \pm \sqrt{223}i}{2}$

$f(x) = \left(x - \dfrac{1 - \sqrt{223}i}{2}\right)\left(x - \dfrac{1 + \sqrt{223}i}{2}\right)$

20. $f(y) = y^4 - 625$

Zeros: $x = \pm 5, \pm 5i$

$f(y) = (y + 5)(y - 5)(y + 5i)(y - 5i)$

22. $h(x) = x^3 - 3x^2 + 4x - 2$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 4 & -2 \\ & & 1 & -2 & 2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

Zeros: $x = 1, \dfrac{2 \pm \sqrt{4}i}{2} = 1 \pm i$

$h(x) = (x - 1)(x - 1 - i)(x - 1 + i)$

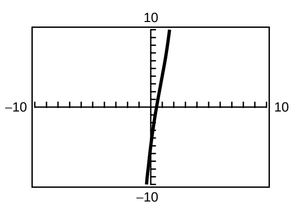
24. $f(x) = x^3 + 11x^2 + 39x + 29$

$$\begin{array}{r|rrrr} -1 & 1 & 11 & 39 & 29 \\ & & -1 & -10 & -29 \\ \hline & 1 & 10 & 29 & 0 \end{array}$$

Zeros: $x = -1, \dfrac{-10 \pm \sqrt{16}i}{2} = -5 \pm 2i$

$f(x) = (x + 1)(x + 5 + 2i)(x + 5 - 2i)$

26. $f(s) = 2s^3 - 5s^2 + 12s - 5$



The graph reveals one zero at $x = \frac{1}{2}$.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -5 & 12 & -5 \\ & & 1 & -2 & 5 \\ \hline & 2 & -4 & 10 & 0 \end{array}$$

Zeros: $s = \dfrac{1}{2}, \dfrac{4 \pm \sqrt{64}i}{4} = 1 \pm 2i$

$f(s) = (2s - 1)(s - 1 + 2i)(s - 1 - 2i)$

28. $f(x) = x^4 + 29x^2 + 100$

$= (x^2 + 25)(x^2 + 4)$

Zeros: $x = \pm 2i, \pm 5i$

$f(x) = (x + 2i)(x - 2i)(x + 5i)(x - 5i)$

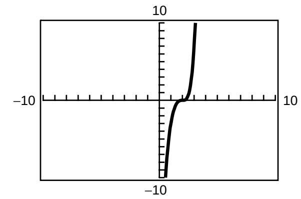
30. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

-3	1	6	10	6	9
		-3	-9	-3	-9
-3	1	3	1	3	0
		-3	0	-3	
	1	0	1	0	

Zeros: $x = -3, \pm i$

$h(x) = (x + 3)^2(x + i)(x - i)$

32. $g(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$



The graph reveals one zero at $x = 2$.

2	1	-8	28	-56	64	-32
		2	-12	32	-48	32
2	1	-6	16	-24	16	0
		2	-8	16	-16	
2	1	-4	8	-8	0	
		2	-4	8		
	1	-2	4	0		

Zeros: $x = 2, 2, 2, \dfrac{2 \pm \sqrt{12}\,i}{2} = 1 \pm \sqrt{3}\,i$

$g(x) = (x - 2)^3\left(x - 1 + \sqrt{3}\,i\right)\left(x - 1 - \sqrt{3}\,i\right)$

34. (a) $f(x) = x^2 - 12x + 34$. By the Quadratic Formula,

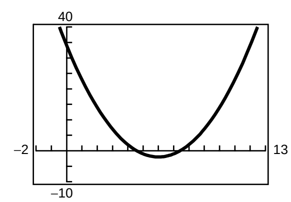
$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(34)}}{2} = 6 \pm \sqrt{2}$$

The zeros are $6 + \sqrt{2}$ and $6 - \sqrt{2}$.

(b) $f(x) = \left(x - \left(6 + \sqrt{2}\right)\right)\left(x - \left(6 - \sqrt{2}\right)\right)$
$= \left(x - 6 - \sqrt{2}\right)\left(x - 6 + \sqrt{2}\right)$

(c) x-intercepts: $\left(6 + \sqrt{2}, 0\right)\left(6 - \sqrt{2}, 0\right)$

(d)



36. (a) $f(x) = x^2 - 16x + 62$. By the Quadratic Formula,

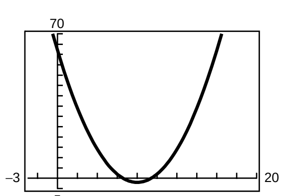
$$x = \frac{16 \pm \sqrt{16^2 - 4(62)}}{2} = 8 \pm \sqrt{2}$$

The zeros are $8 + \sqrt{2}$ and $8 - \sqrt{2}$.

(b) $f(x) = \left(x - \left(8 + \sqrt{2}\right)\right)\left(x - \left(8 - \sqrt{2}\right)\right)$
$= \left(x - 8 - \sqrt{2}\right)\left(x - 8 + \sqrt{2}\right)$

(c) x-intercepts: $\left(8 + \sqrt{2}, 0\right), \left(8 - \sqrt{2}, 0\right)$

(d)



38. (a) $f(x) = x^3 + 10x^2 + 33x + 34$
$= (x + 2)(x^2 + 8x + 17)$

Use the Quadratic formula to find the zeros of $x^2 + 8x + 17$.

$$x = \frac{-8 \pm \sqrt{8^2 - 4(17)}}{2}$$

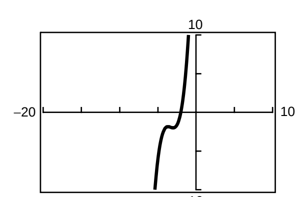
$$= \frac{-8 \pm \sqrt{-4}}{2} = -4 + i$$

The zeros are -2, $-4 + i$, and $-4 - i$.

(b) $f(x) = (x + 2)(x + 4 + i)(x + 4 - i)$

(c) x-intercept: $(-2, 0)$

(d)



40. (a) $f(x) = x^4 - 8x^3 + 17x^2 - 8x + 16$

$= (x^2 + 1)(x^2 - 8x + 16)$

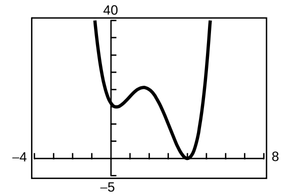
$= (x^2 + 1)(x - 4)^2$

The zeros are i, $-i$, 4 and 4.

(b) $f(x) = (x^2 + 1)^2(x - 4)^2$

(c) x-intercept: $(4, 0)$

(d)



42. $f(x) = (x - 4)(x - 3i)(x + 3i)$

$= (x - 4)(x^2 + 9)$

$= x^3 - 4x^2 + 9x - 36$

Note: $f(x) = a(x^3 - 4x^2 + 9x - 36)$, where a is any nonzero real number, has the zeros 4, $3i$ and $-3i$.

44. $f(x) = (x - 6)(x - (-5 + 2i))(x - (-5 - 2i))$

$= (x - 6)(x^2 + 10x + 29)$

$= x^3 + 4x^2 - 31x - 174$

46. $f(x) = (x - 2)^3(x - 4i)(x + 4i)$

$= (x^3 - 6x^2 + 12x - 8)(x^2 + 16)$

$= x^5 - 6x^4 + 28x^3 - 104x^2 + 192x - 128$

Note: $f(x) = a(x^5 - 6x^4 + 28x^3 - 104x^2 + 192x - 128)$,where a is any nonzero real number, has the zeros 2, 2, 2, $4i$, $-4i$.

48. Since $1 + \sqrt{2}i$ is a zero, so is $1 - \sqrt{2}i$.

$f(x) = (x - 0)^2(x - 4)\left(x - 1 - \sqrt{2}i\right)\left(x - 1 + \sqrt{2}i\right)$

$= x^2(x - 4)((x - 1)^2 + 2)$

$= (x^3 - 4x^2)(x^2 - 2x + 3)$

$= x^5 - 6x^4 + 11x^3 - 12x^2$

50. $f(x) = x^4 + 6x^2 - 27$

(a) $f(x) = (x^2 + 9)(x^2 - 3)$

(b) $f(x) = (x^2 + 9)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

(c) $f(x) = (x + 3i)(x - 3i)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

52. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$

(a) $f(x) = (x^2 + 4)(x^2 - 3x - 5)$

(b) $f(x) = (x^2 + 4)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

(c) $f(x) = (x + 2i)(x - 2i)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

54. $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

Since $2i$ is a zero, so is $-2i$.

$$\begin{array}{c|ccccc} 2i & 2 & -1 & 7 & -4 & -4 \\ & & 4i & -8-2i & 4-2i & 4 \\ \hline & 2 & -1+4i & -1-2i & -2 & 0 \end{array}$$

$$\begin{array}{c|cccc} -2i & 2 & -1+4i & -1-2i & -2i \\ & & -4i & 2i & 2i \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

The zeros of $2x^2 - x - 1 = (2x + 1)(x - 1)$ are $x = -\frac{1}{2}$ and $x = 1$. The zeros of f are $x = \pm 2i,\ x = -\frac{1}{2},$ and $x = 1$.

Alternate Solution

Since $x = \pm 2i$ are zeros of $f(x)$, $(x + 2i)(x - 2i) = x^2 + 4$ is a factor of $f(x)$. By long division we have:

$$\begin{array}{r} 2x^2 - x - 1 \\ x^2 + 0x + 4 \overline{\smash{)}\, 2x^4 - x^3 + 7x^2 - 4x - 4} \\ \underline{2x^4 + 0x^3 + 8x^2} \qquad\qquad \\ -x^3 - x^2 - 4x \qquad \\ \underline{-x^3 + 0x^2 - 4x} \qquad \\ -x^2 + 0x - 4 \\ \underline{-x^2 + 0x - 4} \\ 0 \end{array}$$

Thus, $f(x) = (x^2 + 4)(2x^2 - x - 1)$

$= (x + 2i)(x - 2i)(2x + 1)(x - 1)$

and the zeros of f are $x = \pm 2i,\ x = -\frac{1}{2},$ and $x = 1$.

56. $g(x) = 4x^3 + 23x^2 + 34x - 10$

Since $-3 + i$ is a zero, so is $-3 - i$.

$$\begin{array}{c|cccc} -3+i & 4 & 23 & 34 & -10 \\ & & -12+4i & -37-i & 10 \\ \hline & 4 & 11+4i & -3-i & 0 \end{array}$$

$$\begin{array}{c|ccc} -3-i & 4 & 11+4i & -3-i \\ & & -12-4i & 3+i \\ \hline & 4 & -1 & 0 \end{array}$$

The zeros of $4x - 1$ is $x = \frac{1}{4}$. The zeros of $g(x)$ are $x = -3 \pm i$ and $x = \frac{1}{4}$.

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Alternate Solution

Since $-3 \pm i$ are zeros of

$$g(x),\ [x-(-3+i)][x-(-3-i)] = [(x-3)-i][(x+3)+i]$$
$$= (x+3)^2 - i^2 = x^2 + 6x + 10$$

is a factor of $g(x)$. By long division we have:

$$\begin{array}{r|l} & 4x - 1 \\ \hline x^2 + 6x + 10 & 4x^3 + 23x^2 + 34x - 10 \\ & \underline{4x^3 + 24x^2 + 40x} \\ & -x^2 - 6x - 10 \\ & \underline{-x^2 - 6x - 10} \\ & 0 \end{array}$$

Thus, $g(x) = (x^2 + 6x + 10)(4x - 1)$ and the zeros of g are $x = -3 \pm i$ and $x = \frac{1}{4}$.

58. $f(x) = x^3 + 4x^2 + 14x + 20$

Since $-1 - 3i$ is a zero, so is $-1 + 3i$.

$$\begin{array}{r|rrrr} -1-3i & 1 & 4 & 14 & 20 \\ & & -1-3i & -12-6i & -20 \\ \hline & 1 & 3-3i & 2-6i & 0 \end{array}$$

$$\begin{array}{r|rrr} -1+3i & 1 & 3-3i & 2-6i \\ & & -1+3i & -2+6i \\ \hline & 1 & 2 & 0 \end{array}$$

The zero of $x + 2$ is $x = -2$. The zeros of f are $x = -2, -1 \pm 3i$.

60. $f(x) = 25x^3 - 55x^2 - 54x - 18$

Since $\frac{1}{5}\left(-2 + \sqrt{2}i\right) = \frac{-2 + \sqrt{2}i}{5}$ is a zero, so is $\frac{-2 - \sqrt{2}i}{5}$.

$$\begin{array}{r|rrrr} \frac{-2+\sqrt{2}i}{5} & 25 & -55 & -54 & -18 \\ & & -10 + 5\sqrt{2}i & 24 - 15\sqrt{2}i & 18 \\ \hline & 25 & -65 + 5\sqrt{2}i & -30 - 15\sqrt{2}i & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{-2-\sqrt{2}i}{5} & 25 & -65 + 5\sqrt{2}i & -30 - 15\sqrt{2}i \\ & & -10 - 5\sqrt{2}i & 30 + 15\sqrt{2}i \\ \hline & 25 & -75 & 0 \end{array}$$

The zero of $25x - 75$ is $x = 3$. The zeros of f are $x = 3, \frac{-2 \pm \sqrt{2}i}{5}$.

62. $f(x) = x^3 + 4x^2 + 14x + 20$

(a) Zeros: $-2, -1 \pm 3i$

(b) $x = -2$

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 14 & 20 \\ & & -2 & -4 & -20 \\ \hline & 1 & 2 & 10 & 0 \end{array}$$

$x^2 + 2x + 10$ has zeros $-1 \pm 3i$

64. $f(x) = 25x^3 - 55x^2 - 54x - 18$

(a) Zeros: $3, -0.4 \pm 0.2828i$

(b)
$$\begin{array}{r|rrrr} 3 & 25 & -55 & -54 & -18 \\ & & 75 & 60 & 18 \\ \hline & 25 & 20 & 6 & 0 \end{array}$$

$25x^2 + 20x + 6$ has zeros $\dfrac{-2 \pm \sqrt{2}i}{5} \approx -0.4 \pm 0.2828i$

66. No. Setting $P = R - C = xp - C = x(140 - 0.0001x) - (80x + 150{,}000) = 9{,}000{,}000$ yields a quadratic with no real roots:

$-0.0001x^2 + 60x - 9{,}150{,}000 = 0$

68. True. The complex conjugate of the zero $4 + 3i$ is also a zero.

70. (a) No, the answers will not change if the graph is shifted to the right 2 units.

(b) No, the answer will not change.

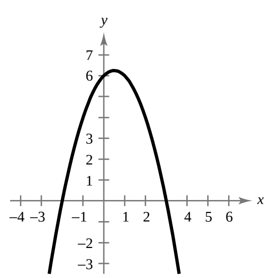
72.
$$\begin{aligned} f(x) &= [x - (a + bi)][x - (a - bi)] \\ &= [(x - a) - bi][(x - a) + bi] \\ &= (x - a)^2 - (bi)^2 \\ &= x^2 - 2ax + a^2 + b^2 \end{aligned}$$

74.
$$\begin{aligned} f(x) &= -x^2 + x + 6 \\ &= -\left(x^2 - x + \tfrac{1}{4}\right) + 6 + \tfrac{1}{4} \\ &= -\left(x - \tfrac{1}{2}\right)^2 + \tfrac{25}{4} \end{aligned}$$

Vertex: $\left(\frac{1}{2}, \frac{25}{4}\right)$

$f(x) = -(x^2 - x - 6) = -(x - 3)(x + 2)$

Intercepts: $(3, 0), (-2, 0), (0, 6)$

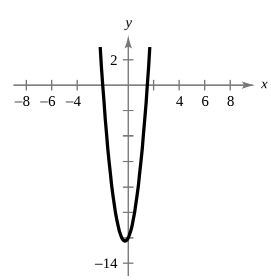


76.
$$\begin{aligned} f(x) &= 4x^2 + 2x - 12 \\ &= 4\left(x^2 + \tfrac{1}{2}x + \tfrac{1}{16}\right) - 12 - \tfrac{1}{4} \\ &= 4\left(x + \tfrac{1}{4}\right)^2 - \tfrac{49}{4} \end{aligned}$$

Vertex: $\left(-\frac{1}{4}, -\frac{49}{4}\right)$

$f(x) = (2x - 3)(2x + 4)$

Intercepts: $\left(\frac{3}{2}, 0\right), (-2, 0), (0, -12)$



78.
$$\begin{aligned} -8i - i(2 + 3i) &= -8i - 2i + 3 \\ &= 3 - 10i \end{aligned}$$

80.
$$\begin{aligned} \frac{6 - i}{1 - 4i} &= \frac{6 - i}{1 - 4i} \cdot \frac{1 + 4i}{1 + 4i} \\ &= \frac{6 + 23i + 4}{1 + 16} \\ &= \frac{10}{17} + \frac{23}{17}i \end{aligned}$$