Section 2.6 Rational Functions and Asymptotes

Solutions to Even-Numbered Exercises

2. $f(x) = \dfrac{5x}{x-1}$

(a)

x	$f(x)$
0.5	-5
0.9	-45
0.99	-495
0.999	-4995

x	$f(x)$
1.5	15
1.1	55
1.01	505
1.001	5005

x	$f(x)$
5	6.25
10	$5.\overline{55}$
100	$5.\overline{05}$
1000	$5.\overline{005}$

x	$f(x)$
-5	4.167
-10	4.545
-100	4.950
-1000	4.995

(b) The zero of the denominator is $x = 1$, so $x = 1$ is a vertical asymptote. The degree of the numerator is equal to the degree of the denominator, so the line $y = \frac{5}{1} = 5$ is a horizontal asymptote.

(c) The domain is all real numbers except $x = 1$.

4. $f(x) = \dfrac{3}{|x-1|}$

(a)

x	$f(x)$
0.5	6
0.9	30
0.99	300
0.999	3000

x	$f(x)$
1.5	6
1.1	30
1.01	300
1.001	3000

x	$f(x)$
5	0.75
10	$0.\overline{33}$
100	$0.\overline{03}$
1000	$0.\overline{003}$

x	$f(x)$
-5	0.5
-10	0.273
-100	0.0297
-1000	0.003

(b) The zero of the denominator is $x = 1$, so $x = 1$ is a vertical asymptote. Because the degree of the numerator is less than the degree of the denominator, the x-axis or $y = 0$ is a horizontal asymptote.

(c) The domain is all real numbers except $x = 1$.

6. $f(x) = \dfrac{4x}{x^2-1}$

(a)

x	$f(x)$
0.5	$-2.\overline{66}$
0.9	-18.95
0.99	-199
0.999	-1999

x	$f(x)$
1.5	4.8
1.1	20.95
1.01	201
1.001	2001

x	$f(x)$
5	$0.83\overline{3}$
10	$0.\overline{40}$
100	0.04
1000	0.004

x	$f(x)$
-5	$-0.83\overline{3}$
-10	$0.\overline{40}$
-100	0.04
-1000	0.004

(b) The zeros of the denominator are $x = \pm 1$ so both $x = 1$ and $x = -1$ are vertical asymptotes. Because the degree of the numerator is less than the degree of the denominator, the x-axis or $y = 0$ is a horizontal asymptote.

(c) The domain is all real numbers except $x = \pm 1$.

8. $f(x) = \dfrac{1}{x-3}$

Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$
Matches graph (d).

10. $f(x) = \dfrac{1-x}{x}$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = -1$
Matches graph (e).

12. $f(x) = -\dfrac{x+2}{x+4}$

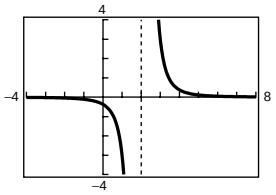
Vertical asymptote: $x = -4$
Horizontal asymptote: $y = -1$
Matches graph (f).

14. $f(x) = \dfrac{3}{(x-2)^3}$

(a) Domain: all real numbers except $x = 2$

(b) Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 0$
[Degree of $p(x)$ < degree of $q(x)$]

(c)



16. $f(x) = \dfrac{2-5x}{2+2x}$

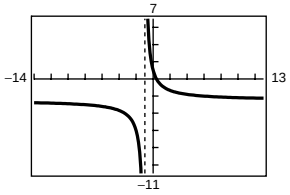
(a) Domain: all real numbers except $x = -1$

(b) Vertical asymptote: $x = -1$

Horizontal asymptote: $y = -\frac{5}{2}$

[Degree $p(x)$ = degree $q(x)$]

(c)



18. $f(x) = \dfrac{3x^2+1}{x^2+x+1}$

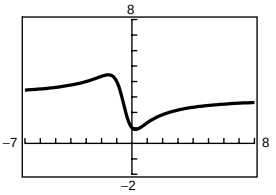
(a) Domain: All real numbers. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]

(b) Vertical asymptote: none

Horizontal asymptote: $y = 3$

[degree $p(x)$ = degree $q(x)$]

(c)



20. $f(x) = \dfrac{x^2(x-3)}{x^2-3x}$, $g(x) = x$

(a) Domain of f: all real numbers except 0 and 3
Domain of g: all real numbers

(b) Because $x^2 - 3x$ is a common factor of both the numerator and the denominator of $f(x)$, neither $x = 0$ nor $x = 3$ is a vertical asymptote of f. Thus, f has no vertical asymptotes.

(c)

x	-1	0	1	2	3	3.5	4
$f(x)$	-1	Undef.	1	2	Undef.	3.5	4
$g(x)$	-1	0	1	2	3	3.5	4

(d) f and g differ only where f is undefined.

22. $f(x) = \dfrac{2x - 8}{x^2 - 9x + 20}, g(x) = \dfrac{2}{x - 5}$

(a) Domain of f: all real numbers except 4 and 5
Domain of g: all real numbers except 5

(b) Because $x - 4$ is a common factor of both the numerator and the denominator of f, $x = 4$ is not a vertical asymptote of f. The only vertical asymptote is $x = 5$.

(c)

x	0	1	2	3	4	5	6
$f(x)$	$-\frac{2}{5}$	$-\frac{1}{2}$	$-\frac{2}{3}$	-1	Undef.	Undef.	2
$g(x)$	$-\frac{2}{5}$	$-\frac{1}{2}$	$-\frac{2}{3}$	-1	-2	Undef.	2

(d) f and g differ only at $x = 4$ where f is undefined and g is defined.

24. $f(x) = 2 + \dfrac{1}{x - 3}$

(a) As $x \to \pm\infty, f(x) \to 2$.

(b) As $x \to \infty, f(x) \to 2$ but is greater than 2.

(c) As $x \to -\infty, f(x) \to 2$ but is less than 2.

26. $f(x) = \dfrac{2x - 1}{x^2 + 1}$

(a) As $x \to \pm\infty, f(x) \to 0$.

(b) As $x \to \infty, f(x) \to 0$ but is greater than 0.

(c) As $x \to -\infty, f(x) \to 0$ but is less than 0.

28. $g(x) = \dfrac{x^3 - 8}{x^2 + 4}$

The zero of g corresponds to the zero of the numerator and is $x = 2$.

30. $h(x) = 6 + \dfrac{4}{x^2 + 2}$

There are no real zeros.

32. (a) $C = \dfrac{25{,}000(15)}{100 - 15} \approx 4411.76$

The cost would be \$4411.76.

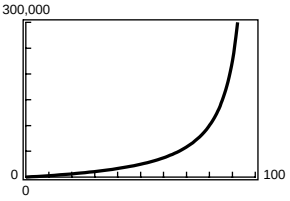
(b) $C = \dfrac{25{,}000(50)}{100 - 50} = 25{,}000$

The cost would be \$25,000.

(c) $C = \dfrac{25{,}000(90)}{100 - 90} = 225{,}000$

The cost would be \$225,000.

(d)



(e) No. The model is undefined for $p = 100$.

34. (a) Use data $\left(10, \frac{1}{7}\right), \left(20, \frac{1}{10}\right), \left(30, \frac{1}{14}\right), \left(40, \frac{1}{22}\right), \left(50, \frac{1}{40}\right)$. The least squares line for this data $(x, 1/y)$ is:

$$\frac{1}{y} = 0.164 - 0.0029x \Longrightarrow y = \frac{1}{0.164 - 0.0029x} = \frac{154{,}000}{25260 - 447x} = \frac{154{,}000}{3(8420 - 149x)}$$

(b)

x	10	20	30	40	50
y	7.4	9.4	13.0	20.9	52.9

(c) No, the function is negative for $x = 60$.