36. The moth will become satiated at the horizontal asymptote:

$y = \dfrac{1.568}{6.360} \approx 0.247$ mg

38. False. A rational function can have at most n vertical asymptotes, where n is the degree of the denominator.

40. $f(x) = \dfrac{1}{(x + 2)(x - 1)} = \dfrac{1}{x^2 + x - 2}$

is one possible answer.

42. $f(x) = \dfrac{2x^2}{x^2 + 1}$ is one posible answer.

44. $225x - 50x^3 = 0$

$25x(9 - 2x^2) = 0$

$x = 0, \pm 3/\sqrt{2}$

46. $2z^2 - 3z - 35 = 0$

$(2z + 7)(-5) = 0$

$z = -\frac{7}{2}, 5$

48. $27x^3 - 147x = 0$

$3x(9x^2 - 49) = 0$

$x = 0, \pm\frac{7}{3}$

50.

$$\begin{array}{r|rrr} 4 & 1 & 5 & 6 \\ & & 4 & 36 \\ \hline & 1 & 9 & 42 \end{array}$$

$$\frac{x^2 + 5x + 6}{x - 4} = x + 9 + \frac{42}{x - 4}$$

52.

$$\begin{array}{r|rrr} -5 & 2 & 1 & -11 \\ & & -10 & 45 \\ \hline & 2 & -9 & 34 \end{array}$$

$$\frac{2x^2 + x - 11}{x + 5} = 2x - 9 + \frac{34}{x + 5}$$

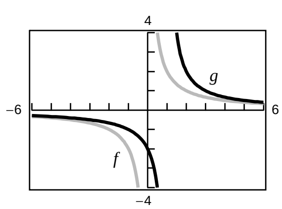
54. $(x - 8)(x - 5i)(x + 5i) = (x - 8)(x^2 + 25) = x^3 - 8x^2 + 25x - 200$

56.
$$\begin{aligned} (x - 6)(x - (3 + i))(x - (3 - i)) &= (x - 6)((x - 3)^2 + 1) \\ &= (x - 6)(x^2 - 6x + 10) \\ &= x^3 - 12x^2 + 46x - 60 \end{aligned}$$

Section 2.7 Graphs of Rational Functions

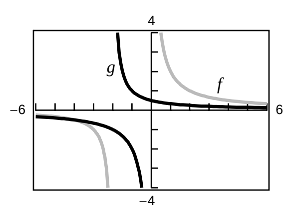
Solutions to Even-Numbered Exercises

2.



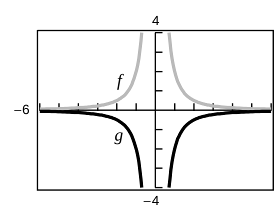
Horizontal shift one unit to the right

4.



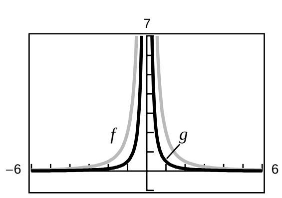
Horizontal shift two units to the left, and vertical shrink

6.



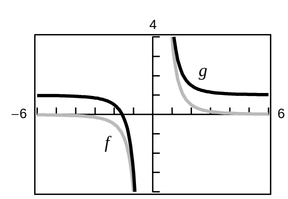
Reflection in the x-axis

8.



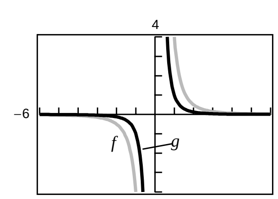
Each y-value is multiplied by $\frac{1}{4}$. Vertical shrink

10.



Vertical shift one unit upward

12.



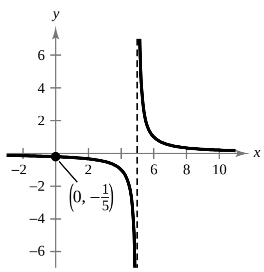
Each y-value is multiplied by $\frac{1}{4}$. Vertical shrink

14. $f(x) = \dfrac{1}{x - 5}$

y-intercept: $\left(0, -\frac{1}{5}\right)$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = 0$



16. $P(x) = \dfrac{1 - 3x}{1 - x} = \dfrac{3x - 1}{x - 1}$

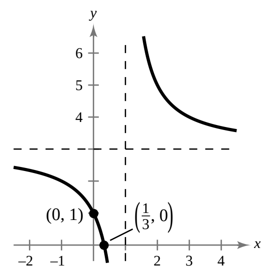
x-intercept: $\left(\dfrac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 3$

x	-1	0	2	3
y	2	1	5	4



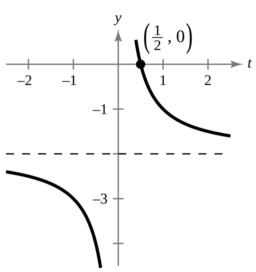
18. $f(t) = \dfrac{1 - 2t}{t} = -\dfrac{2t - 1}{t}$

t-intercept: $\left(\dfrac{1}{2}, 0\right)$

Vertical asymptote: $t = 0$

Horizontal asymptote: $y = -2$

x	-2	-1	$\frac{1}{2}$	1	2
y	$-\frac{5}{2}$	-3	0	-1	$-\frac{3}{2}$



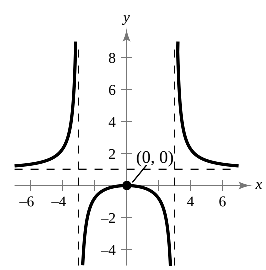
20. $h(x) = \dfrac{x^2}{x^2 - 9}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = \pm 3$

Horizontal asymptote: $y = 1$

y-axis symmetry



22. $g(x) = \dfrac{x}{x^2 - 9}$

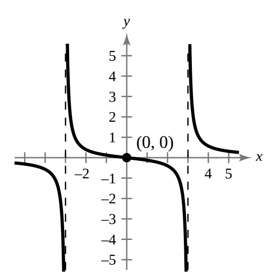
Intercepts: $(0, 0)$

Vertical asymptote: $x = \pm 3$

Horizontal asymptote: $y = 0$

Origin symmetry

x	-5	-4	-2	0	2	4	5
y	$-\frac{5}{16}$	$-\frac{4}{7}$	$\frac{2}{5}$	0	$-\frac{2}{5}$	$\frac{4}{7}$	$\frac{5}{16}$



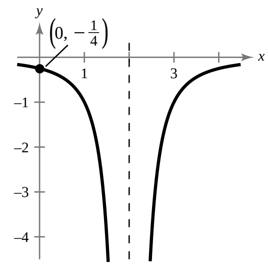
24. $f(x) = -\dfrac{1}{(x-2)^2}$

y-intercept: $\left(0, -\dfrac{1}{4}\right)$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}$	3	$\frac{7}{2}$	4
y	$-\frac{1}{4}$	$-\frac{4}{9}$	-1	-4	-4	-1	$-\frac{4}{9}$	$-\frac{1}{4}$

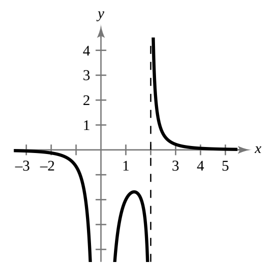


26. $h(x) = \dfrac{2}{x^2(x-2)}$

Vertical asymptotes: $x = 0, x = 2$

Horizontal asymptote: $y = 0$

x	-2	-1	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}$	3
y	$-\frac{1}{8}$	$-\frac{2}{3}$	$-\frac{16}{5}$	-2	$-\frac{16}{9}$	$\frac{16}{25}$	$\frac{2}{9}$



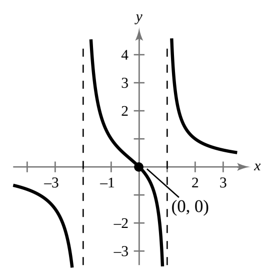
28. $f(x) = \dfrac{2x}{x^2 + x - 2} = \dfrac{2x}{(x+2)(x-1)}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = -2, 1$

Horizontal asymptote: $y = 0$

x	-4	-3	-1	0	$\frac{1}{2}$	2	3
y	$-\frac{4}{5}$	$-\frac{3}{2}$	1	0	$-\frac{4}{5}$	1	$\frac{3}{5}$



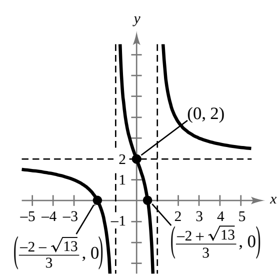
30. $f(x) = 2 + \dfrac{8}{3x - \dfrac{3}{x}} = 2 + \dfrac{8x}{3x^2 - 3} = \dfrac{6x^2 + 8x - 6}{3x^2 - 3}, x \neq 0$

Vertical asymptotes: $x = \pm 1$

Intercepts: $(-1.869, 0), (0.535, 0)$ [(0, 2) is not on graph.]

Horizontal asymptote: $y = 2$

x	-3	-2	-0.5	0.5	2	3
y	1	$\frac{2}{9}$	3.78	$\frac{2}{9}$	3.78	3



32. $f(x) = \dfrac{3-x}{2-x} = \dfrac{x-3}{x-2}$

x-intercept: $(3, 0)$

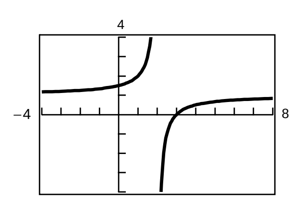
y-intercept: $\left(0, \dfrac{3}{2}\right)$

Vertical asymptote: $x = 2$

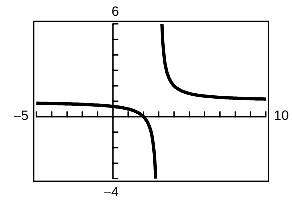
Horizontal asymptote: $y = 1$

Domain: all $x \neq 2$

x	0	1	3	4
y	$\frac{3}{2}$	2	0	$\frac{1}{2}$



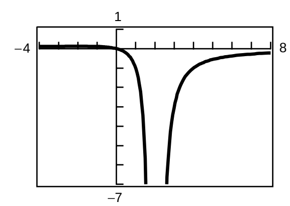
34. $h(x) = \dfrac{x-2}{x-3}$



x-intercept: $(2, 0)$
y-intercept: $\left(0, \dfrac{2}{3}\right)$
Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 1$
Domain: all $x \neq 3$

x	0	1	2	4	5	6
y	$\frac{2}{3}$	$\frac{1}{2}$	0	2	$\frac{3}{2}$	$\frac{4}{3}$

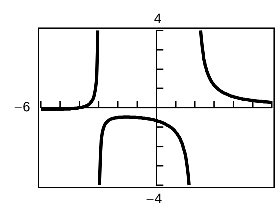
36. $g(x) = -\dfrac{x}{(x-2)^2}$



Domain: all real numbers except 2 OR $(-\infty, 2) \cup (2, \infty)$
Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 0$

x	-1	0	1	$\frac{3}{2}$	$\frac{5}{2}$	3	4
y	$\frac{1}{9}$	0	-1	-6	-10	-3	-1

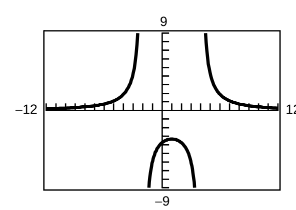
38. $f(x) = \dfrac{x+4}{x^2+x-6}$



Domain: all real numbers except -3 and 2 OR $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$
Vertical asymptotes: $x = -3, x = 2$
Horizontal asymptote: $y = 0$

x	-6	-4	-2	-1	0	1	3	4
y	$-\frac{1}{12}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{5}{4}$	$\frac{7}{6}$	$\frac{4}{7}$

40. $f(x) = 5\left(\dfrac{1}{x-4} - \dfrac{1}{x+2}\right) = \dfrac{30}{(x-4)(x+2)}$

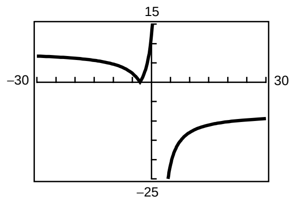


Domain: all real numbers except -2 and 4
Vertical asymptotes: $x = -2, x = 4$
Horizontal asymptote: $y = 0$

x	-4	-3	-1	0	1	2	3	5	6	7
y	$\frac{15}{8}$	$\frac{30}{7}$	-6	$-\frac{15}{4}$	$-\frac{10}{3}$	$-\frac{15}{4}$	-6	$\frac{30}{7}$	$\frac{15}{8}$	$\frac{10}{9}$

42. $f(x) = \dfrac{-x}{\sqrt{9+x^2}}$

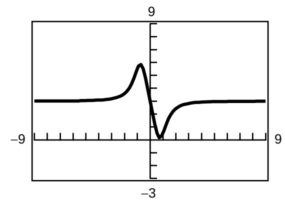
44. $f(x) = \dfrac{-8|3 + x|}{x - 2} = \dfrac{8|3 + x|}{2 - x}$



There are two horizontal asymptotes, $y = -8$ and $y = 8$.

Vertical asymptote: $x = 2$

46. $g(x) = \dfrac{3x^4 - 5x + 3}{x^4 + 1}$



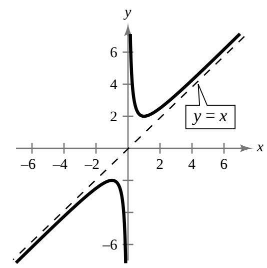
The graph crosses its horizontal asymptote, $y = 3$.

48. $g(x) = \dfrac{x^2 + 1}{x} = x + \dfrac{1}{x}$

Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

Origin symmetry



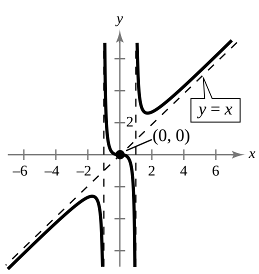
50. $f(x) = \dfrac{x^3}{x^2 - 1} = x + \dfrac{x}{x^2 - 1}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = \pm 1$

Slant asymptote: $y = x$

Origin symmetry

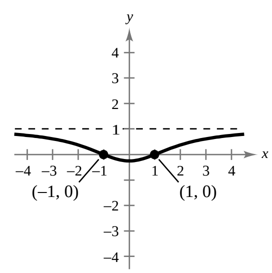


52. $f(x) = \dfrac{x^2 - 1}{x^2 + 4}$

No vertical asymptotes

Horizontal asymptote: $y = 1$

Intercepts: $(\pm 1, 0)$, $\left(0, -\frac{1}{4}\right)$

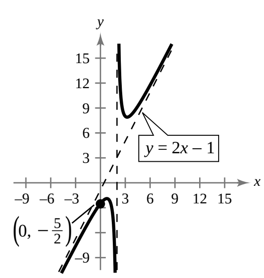


54. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2} = 2x - 1 + \dfrac{3}{x - 2}$

y-intercept: $\left(0, -\dfrac{5}{2}\right)$

Vertical asymptote: $x = 2$

Slant asymptote: $y = 2x - 1$



56. (a) x-intercept: $(0, 0)$

(b) $0 = \dfrac{2x}{x - 3}$

$0 = 2x$

$0 = x$

58. (a) x-intercepts: $(1, 0)$, $(2, 0)$

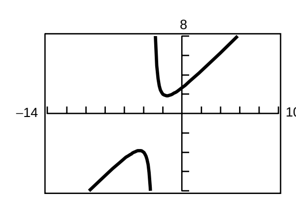
(b) $0 = x - 3 + \dfrac{2}{x}$

$0 = x^2 - 3x + 2$

$0 = (x - 1)(x - 2)$

$x = 1, 2$

60.



Domain: all $x \neq -3$

Vertical asymptote: $x = -3$

Slant asymptote: $y = x + 2$

62. $h(x) = \dfrac{12 - 2x - x^2}{2(4 + x)} = -\dfrac{1}{2}x + 1 + \dfrac{2}{4 + x}$

Domain: all real numbers except -4

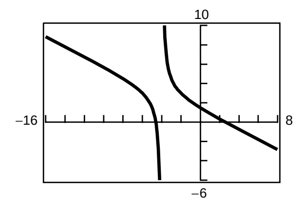
OR $(-\infty, -4) \cup (-4, \infty)$

x-intercepts: $(-4.61, 0)$, $(2.61, 0)$

y-intercept: $\left(0, \dfrac{3}{2}\right)$

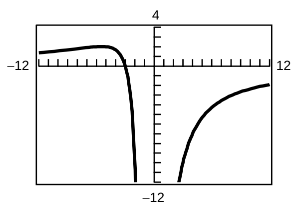
Vertical asymptote: $x = -4$

Slant asymptote: $y = -\dfrac{1}{2}x + 1$



64. $y = 20\left(\dfrac{2}{x + 1} - \dfrac{3}{x}\right)$

(a)



x-intercept: $(-3, 0)$

(b) $0 = 20\left(\dfrac{2}{x + 1} - \dfrac{3}{x}\right)$

$\dfrac{3}{x} = \dfrac{2}{x + 1}$

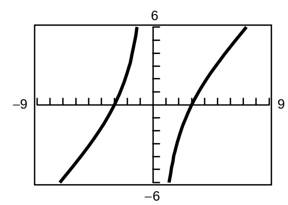
$2x = 3(x + 1)$

$2x = 3x + 3$

$-3 = x$

66. $y = x - \dfrac{9}{x}$

(a)



x-intercepts: $(-3, 0)$, $(3, 0)$

(b) $0 = x - \dfrac{9}{x}$

$\dfrac{9}{x} = x$

$9 = x^2$

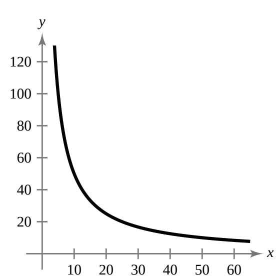
$\pm 3 = x$

68. (a) Area $= xy = 500$

$y = \dfrac{500}{x}$

(b) Domain: $x > 0$

(c)



For $x = 30$, $y = \dfrac{500}{30} = 16\frac{2}{3}$ meters.

70. (a) The line passes through the points $(a, 0)$ and $(3, 2)$ and has a slope of

$$m = \frac{2 - 0}{3 - a} = \frac{2}{3 - a}.$$

$$y - 0 = \frac{2}{3 - a}(x - a) \text{ by the point-slope form}$$

$$y = \frac{2(x - a)}{3 - a} = \frac{-2(a - x)}{-1(a - 3)}$$

$$= \frac{2(a - x)}{a - 3}, \; 0 \le x \le a$$

(b) The area of a triangle is $A = \frac{1}{2}bh$.

$b = a$

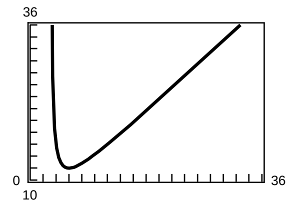
$h = y$ when $x = 0$, so $h = \dfrac{2(a - 0)}{a - 3} = \dfrac{2a}{a - 3}$.

$$A = \frac{1}{2}a\left(\frac{2a}{a - 3}\right) = \frac{a^2}{a - 3}$$

(c) $A = \dfrac{a^2}{a - 3} = a + 3 + \dfrac{9}{a - 3}$

Vertical asymptote: $a = 3$

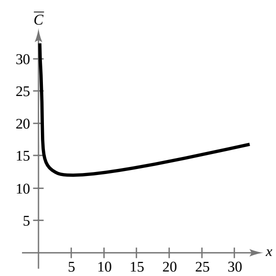
Slant asymptote: $A = a + 3$



A is a minimum when $a = 6$ and $A = 12$.

72. $\overline{C} = \dfrac{C}{x} = \dfrac{0.2x^2 + 10x + 5}{x}$

x	0.5	1	2	3	4	5	6	7
$\overline{C}$	20.1	15.2	12.9	≈ 12.3	12.05	12	≈ 12.0	12.1



The minimum average cost occurs when $x = 5$.

74. (a) Rate $\times$ Time $=$ Distance or $\dfrac{\text{Distance}}{\text{Rate}} = \text{Time}$

$$\frac{100}{x} + \frac{100}{y} = \frac{200}{50} = 4$$

$$\frac{25}{x} + \frac{25}{y} = 1$$

$$25y + 25x = xy$$

$$25x = xy - 25y$$

$$25x = y(x - 25)$$

$$y = \frac{25x}{x - 25}$$

(b) Vertical asymptote: $x = 25$

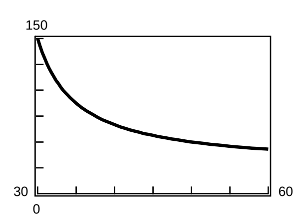
Horizontal asymptote: $y = 25$

(c)

x	30	35	40	45	50	55	60
y	150	87.5	66.7	56.3	50	45.8	42.9

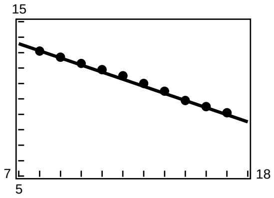
The results in the table are unexpected. You would expect the average speed for the round trip to be the average of the average speeds for the two parts of the trip.

(d)



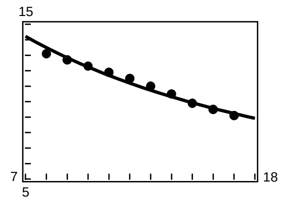
(e) No, it is not possible to average 20 miles per hour in one direction and still average 50 miles per hour on the round trip. At 20 miles per hour you would use more time in one direction than is required for the round trip at an average speed of 50 miles per hour.

76. (a) $N_1 = -0.4552t + 16.8394$



(b) $\dfrac{1}{N_2} = 0.003780t + 0.04371$

$$N_2 = \frac{1}{0.003780t + 0.04371} = \frac{1000}{3.78t + 43.71}$$



(c)

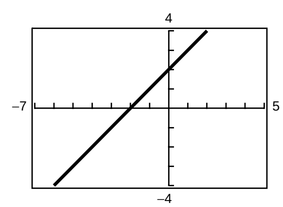
t	8	9	10	11	12	13	14	15	16	17
N_1	13.2	12.7	12.3	11.8	11.4	10.9	10.5	10.0	9.6	9.1
N_2	13.5	12.9	12.3	11.7	11.2	10.8	10.3	10.0	9.6	9.3

Both models fit the data well. Either model will do.

78. False. A rational function can cross its horizontal asymptote. See Exercises 45, 46.

80. $g(x) = \dfrac{x^2 + x - 2}{x - 1} = \dfrac{(x + 2)(x - 1)}{x - 1}$

Since $g(x)$ is not reduced $(x - 1)$ is a factor of both the numerator and the denominator, $x = 1$ is not a horizontal asymptote.



82. $y = x + 1 + \dfrac{a}{x - 2}$ has slant asymptote $y = x + 1$ and vertical asymptote $x = 2$.

$$0 = -2 + 1 + \frac{a}{-2 - 2}$$

$$1 = \frac{a}{-4}$$

$$a = -4$$

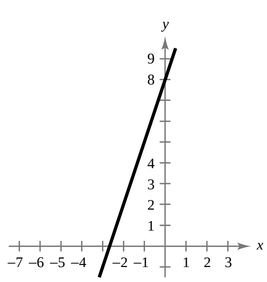
Hence, $y = x + 1 + \dfrac{-4}{x - 2} = \dfrac{x^2 - x - 6}{x - 2}$

84. $y = \dfrac{2(x - 3)}{(x + 1)}$

This has a vertical asymptote at $x = -1$, the zero of the denominator, a horizontal asymptote at $y = 2$ because the degree of the denominator equals the degree of the numerator, and has $x = 3$ as a zero of the function because 3 is the zero of the numerator.

86. $-y + 3x + 8 = 0$

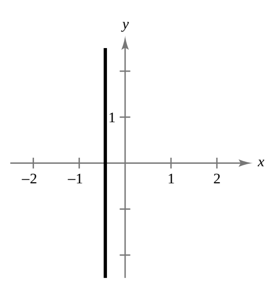
$y = 3x + 8$



88. $7x + 3 = 0$

$x = -\frac{3}{7}$

vertical line



90. $x - y - 1 = 0$

$y = x - 1$