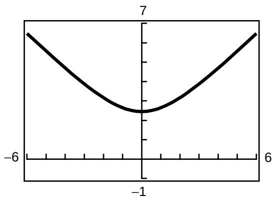
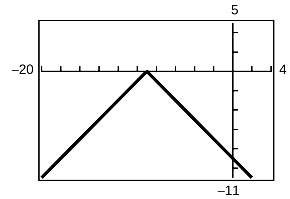
92.



Domain: all x

Range: $y \geq \sqrt{6}$

94.



Domain: all x

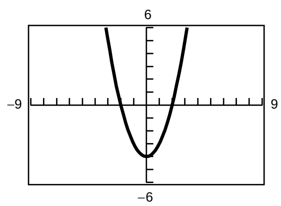
Range: $y \leq 0$

Review Exercises for Chapter 2

Solutions to Even-Numbered Exercises

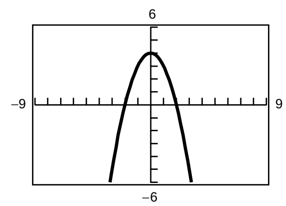
2. (a) $y = x^2 - 4$

Vertical shift 4 units downward



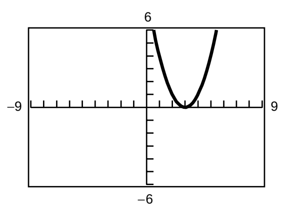
(b) $y = 4 - x^2$

Reflection in the x-axis and a vertical shift 4 units upward



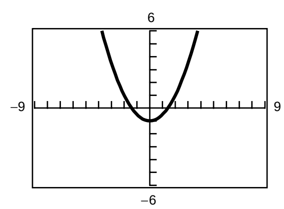
(c) $y = (x - 3)^2$

Horizontal shift 3 units to the right



(d) $y = \frac{1}{2}x^2 - 1$

Vertical shrink and a vertical shift 1 unit downward

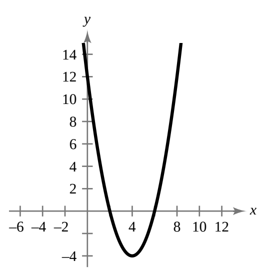


4. $f(x) = (x - 4)^2 - 4$

Vertex: $(4, -4)$

y-intercept: $(0, 12)$

x-intercepts: $(2, 0)$, $(6, 0)$



6. $f(x) = 3x^2 - 12x + 11$

$= 3\left(x^2 - 4x + 4 - 4 + \frac{11}{3}\right)$

$= 3\left[(x - 2)^2 - \frac{1}{3}\right]$

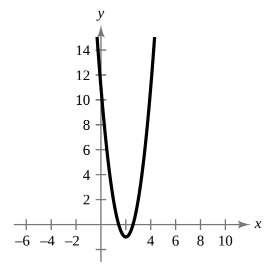
$= 3(x - 2)^2 - 1$

Vertex: $(2, -1)$

y-intercept: $(0, 11)$

x-intercepts: $x = \frac{12 \pm \sqrt{12}}{6} = 2 \pm \frac{1}{3}\sqrt{3}$

$\left(2 + \frac{1}{3}\sqrt{3}, 0\right), \left(2 - \frac{1}{3}\sqrt{3}, 0\right)$



8. Vertex: $(2, 3) \Longrightarrow f(x) = a(x - 2)^2 + 3$

Point: $(-1, 6) \Longrightarrow 6 = a(-1 - 2)^2 + 3$

$6 = 9a + 3$

$3 = 9a$

$\frac{1}{3} = a$

$f(x) = \frac{1}{3}(x - 2)^2 + 3$

10. $f(x) = x^2 + 8x + 10$

$= x^2 + 8x + 16 - 16 + 10$

$= (x + 4)^2 - 6$

The minimum occurs at the vertex $(-4, -6)$.

12. $h(x) = 3 + 4x - x^2$

$= -(x^2 - 4x - 3)$

$= -(x^2 - 4x + 4 - 4 - 3)$

$= -[(x - 2)^2 - 7]$

$= -(x - 2)^2 + 7$

The maximum occurs at the vertex $(2, 7)$.

14. $h(x) = 4x^2 + 4x + 13$

$= 4\left(x^2 + x + \frac{1}{4} - \frac{1}{4} + \frac{13}{4}\right)$

$= 4\left[\left(x + \frac{1}{2}\right)^2 + 3\right]$

$= 4\left(x + \frac{1}{2}\right)^2 + 12$

The minimum occurs at the vertex $\left(-\frac{1}{2}, 12\right)$.

16. $f(x) = 4x^2 + 4x + 5$

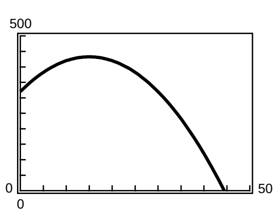
$= 4\left(x^2 + x + \frac{1}{4} - \frac{1}{4} + \frac{5}{4}\right)$

$= 4\left[\left(x + \frac{1}{2}\right)^2 + 1\right]$

$= 4\left(x + \frac{1}{2}\right)^2 + 4$

The minimum occurs at the vertex $\left(-\frac{1}{2}, 4\right)$.

18. (a)



(b) Vertex: $(15, 432.5)$

(c) Vertex at $x = \frac{-b}{2a} = \frac{-15}{2\left(-\frac{1}{2}\right)} = 15$

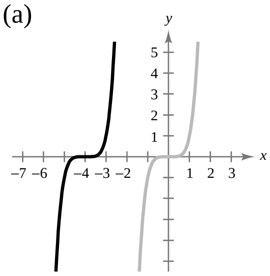
$P(15) = 432.5$

Vertex: $(15, 432.5)$

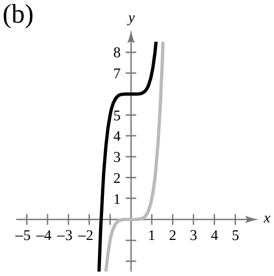
(d) The vertex represents the amount ($1500) of advertising that yields a maximum profit $43,250.

20. $y = x^5$

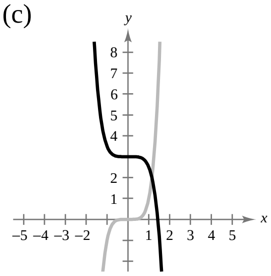
(a)



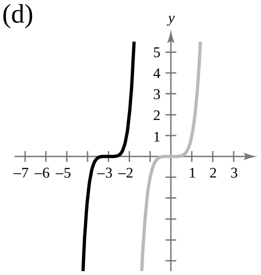
(b)



(c)

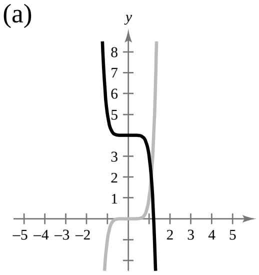


(d)

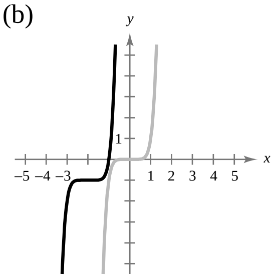


22. $y = x^7$

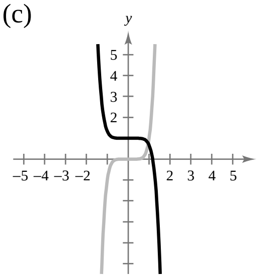
(a)



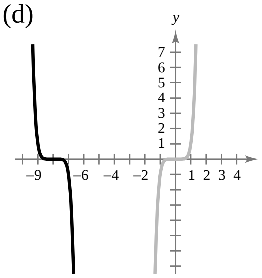
(b)



(c)



(d)



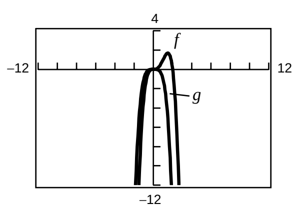
24. $f(x) = \frac{1}{2}x^3 + 2x$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

26. $h(x) = -x^5 - 7x^2 + 10x$

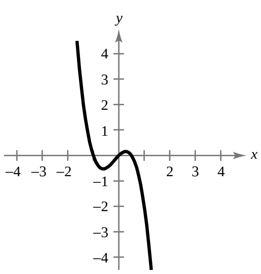
The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

28.



30. (a) Zeros of $h(x) = -2x^3 - x^2 + x = x(-2x^2 - x + 1) = x(x + 1)(2x - 1)$ are $0, -1, \frac{1}{2}$.

(b)



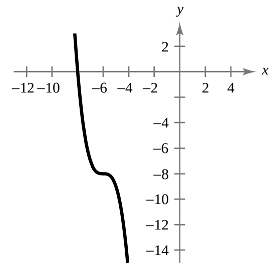
32. (a) $f(x) = -(x + 6)^3 - 8$

$(x + 6)^3 = -8$

$x + 6 = -2$

$x = -8$

(b)



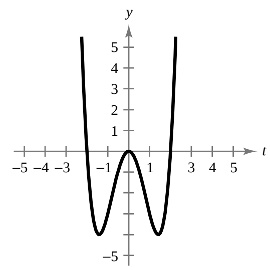
34. (a) $t^4 - 4t^2 = 0$

$t^2(t^2 - 4) = 0$

$t^2(t - 2)(t + 2) = 0$

$t = 0, 0, 2, -2$

(b)

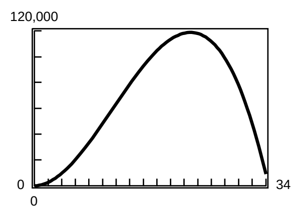


36. (a) Combined length and girth:

$2\pi r + h = 216 \Rightarrow h = 216 - 2\pi r$

Volume $= \pi r^2 h = \pi r^2(216 - 2\pi r)$

(b)



The volume is maximum when $r = \dfrac{72}{\pi} \approx 22.9$,
$h \approx 216 - 2\pi(22.9) = 72.1$

38. (a) $f(-3) < 0, f(-2) > 0 \Longrightarrow$ zero in $[-3, -2]$

$f(-1) > 0, f(0) < 0 \Longrightarrow$ zero in $[-1, 0]$

$f(3) < 0, f(4) > 0 \Longrightarrow$ zero in $[3, 4]$

(b) zeros: $-2.979, -0.554, 3.533$

40. (a) $f(-2) > 0, f(-1) < 0 \Longrightarrow$ zero in $[-2, -1]$

$f(0) < 0, f(1) > 0 \Longrightarrow$ zero in $[0, 1]$

(b) zeros: $-1.897, 0.738$

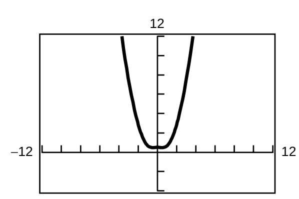
42. $y_1 = \dfrac{x^4 + 1}{x^2 + 2}$

$$y_2 = x^2 - 2 + \frac{5}{x^2 + 2}$$

$$= \frac{x^2(x^2 + 2)}{x^2 + 2} - \frac{2(x^2 + 2)}{x^2 + 2} + \frac{5}{x^2 + 2}$$

$$= \frac{x^4 + 2x^2 - 2x^2 - 4 + 5}{x^2 + 2}$$

$$= \frac{x^4 + 1}{x^2 + 2} = y_1$$



44.
$$\begin{array}{r} \frac{4}{3} \\ 3x-2\overline{\big)\,4x+7} \\ \underline{4x-\frac{8}{3}} \\ \frac{29}{3} \end{array}$$

$$\frac{4x+7}{3x-2} = \frac{4}{3} + \frac{29}{3(3x-2)}$$

46.
$$\begin{array}{r} 3x^2 \qquad\quad + 3 \\ x^2-1\overline{\big)\,3x^4+0x^3+0x^2+0x+0} \\ \underline{3x^4 \qquad -3x^2 \qquad\qquad} \\ 3x^2 \qquad + 0 \\ \underline{3x^2 \qquad - 3} \\ 3 \end{array}$$

$$\frac{3x^4}{x^2-1} = 3x^2 + 3 + \frac{3}{x^2-1}$$

48.
$$\begin{array}{r} 3x^2+5x+8 \\ 2x^2+0x-1\overline{\big)\,6x^4+10x^3+13x^2-5x+2} \\ \underline{6x^4+\ \ 0x^3-\ \ 3x^2 \qquad\qquad\quad} \\ 10x^3+16x^2-5x \qquad \\ \underline{10x^3+\ \ 0x^2-5x} \qquad \\ 16x^2-\ \ 0+2 \\ \underline{16x^2+\ \ 0-8} \\ 10 \end{array}$$

$$\frac{6x^4+10x^3+13x^2-5x+2}{2x^2-1} = 3x^2+5x+8+\frac{10}{2x^2-1}$$

50.
$$\begin{array}{c|cccc} 5 & 0.1 & 0.3 & 0 & -0.5 \\ & & 0.5 & 4 & 20 \\ \hline & 0.1 & 0.8 & 4 & 19.5 \end{array}$$

$$\frac{0.1x^3+0.3x^2-0.5}{x-5} = 0.1x^2+0.8x+4+\frac{19.5}{x-5}$$

52.
$$\begin{array}{c|cccc} \frac{1}{2} & 2 & 2 & -1 & 2 \\ & & 1 & \frac{3}{2} & \frac{1}{4} \\ \hline & 2 & 3 & \frac{1}{2} & \frac{9}{4} \end{array}$$

$$\frac{2x^3+2x^2-x+2}{x-(1/2)} = 2x^2+3x+\frac{1}{2}+\frac{9/4}{x-(1/2)}$$

54. (a)
$$\begin{array}{c|ccccc} -1 & 20 & 9 & -14 & -3 & 0 \\ & & -20 & 11 & 3 & 0 \\ \hline & 20 & -11 & -3 & 0 & 0 \end{array} \quad f(-1)=0$$

(b)
$$\begin{array}{c|ccccc} \frac{3}{4} & 20 & 9 & -14 & -3 & 0 \\ & & 15 & 18 & 3 & 0 \\ \hline & 20 & 24 & 4 & 0 & 0 \end{array} \quad f\left(\tfrac{3}{4}\right)=0$$

(c)
$$\begin{array}{c|ccccc} 0 & 20 & 9 & -14 & -3 & 0 \\ & & 0 & 0 & 0 & 0 \\ \hline & 20 & 9 & -14 & -3 & 0 \end{array} \quad f(0)=0$$

(d)
$$\begin{array}{c|ccccc} 1 & 20 & 9 & -14 & -3 & 0 \\ & & 20 & 29 & 15 & 12 \\ \hline & 20 & 29 & 15 & 12 & 12 \end{array} \quad f(1)=12$$

56.
$$\begin{array}{c|cccc} -\frac{2}{3} & 3 & 2 & -15 & -10 \\ & & -2 & 0 & 10 \\ \hline & 3 & 0 & -15 & 0 \end{array} \quad f\left(-\tfrac{2}{3}\right)=0$$

$$\begin{aligned} 3x^3+2x^2-15x-10 &= \left(x+\tfrac{2}{3}\right)(3x^2-15) \\ &= (3x+2)(x^2-5) \\ &= (3x+2)\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right) \end{aligned}$$

Zeros: $-\frac{2}{3}, \pm\sqrt{5}$

58. $f(x) = 10x^3+21x^2-x-6$
$$\begin{array}{c|cccc} -2 & 10 & 21 & -1 & -6 \\ & & -20 & -2 & 6 \\ \hline & 10 & 1 & -3 & 0 \end{array}$$

Zeros: $-2, -\frac{3}{5}, \frac{1}{2}$

60. $10x^3-13x^2-17x+6 = (x-2)(x+1)(10x-3) \implies$ Zeros: $2, -1, \frac{3}{10}$

62. $f(x) = 5x^4 + 126x^2 + 25$

$f(x) = (5x^2 + 1)(x^2 + 25)$

$5x^2 + 1 = 0$

$x^2 = -\frac{1}{5}$

$x = \pm\frac{\sqrt{5}}{5}i$

$x^2 + 25 = 0$

$x^2 = -25$

$x = \pm 5i$

64.
$$\begin{array}{r|rrrr} 8 & 2 & -5 & -14 & 8 \\ & & 16 & 88 & \\ \hline & 2 & 11 & 74 & 592 \end{array}$$

All positive $\implies x = 8$ is upper bound

$$\begin{array}{r|rrrr} -4 & 2 & -5 & -14 & 8 \\ & & -8 & 52 & -152 \\ \hline & 2 & -13 & 38 & -144 \end{array}$$

Alternating signs $\implies x = -4$ is lower bound.

66. $-\sqrt{-12} + 3 = -2\sqrt{3}\,i + 3 = 3 - 2\sqrt{3}\,i$

68. $-i^2 - 4i = 1 - 4i$

70. $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = -\sqrt{2}\,i$

72. $(1 + 6i)(5 - 2i) = 5 - 2i + 30i + 12 = 17 + 28i$

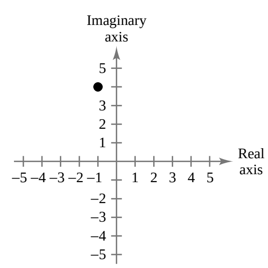
74. $i(6 + i)(3 - 2i) = i(18 + 3i - 12i + 2) = i(20 - 9i) = 9 + 20i$

76. $(4 - i)^2 - (4 + i)^2 = (16 - 8i - 1) - (16 + 8i - 1) = -16i$

78. $\frac{3 + 2i}{5 + i} \cdot \frac{5 - i}{5 - i} = \frac{15 + 10i - 3i + 2}{25 + 1} = \frac{17}{26} + \frac{7}{26}i$

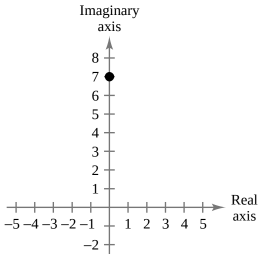
80.
$$\begin{aligned} \frac{1}{(2 + i)^4} &= \frac{1}{(4 + 4i - 1)(4 + 4i - 1)} \\ &= \frac{1}{(3 + 4i)(3 + 4i)} \\ &= \frac{1}{9 + 24i - 16} \\ &= \frac{1}{-7 + 24i} \cdot \frac{-7 - 24i}{-7 - 24i} \\ &= \frac{-7 - 24i}{49 + 576} \\ &= \frac{-7}{625} - \frac{24}{625}i \end{aligned}$$

82.



$-1 + 4i$

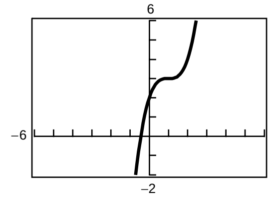
84.



$7i$

86. $g(x) = x^3 - 3x^2 + 3x + 2$

(a)

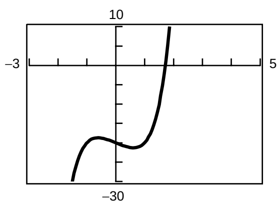


(b) One real zero because the graph has only one x-intercept.

(c) The zero is $x \approx -0.44$.

88. $f(x) = x^5 + 2x^3 - 3x - 20$

(a)



(b) One real zero because the graph has only one x-intercept.

(c) The zero is $x \approx 1.72$.

90. $f(x) = x^3 - 5x^2 - 7x + 51$

$$= (x + 3)(x^2 - 8x + 17)$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(17)}}{2} = \frac{8 \pm \sqrt{-4}}{2} = 4 \pm i$$

zeros: $-3, 4 + i, 4 - i$

$f(x) = (x + 3)(x - 4 - i)(x - 4 + i)$

92. $f(x) = 2x^3 - 9x^2 + 22x - 30$

$$= (2x - 5)(x^2 - 2x + 6)$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(6)}}{2} = 1 \pm \sqrt{5}i$$

zeros: $\frac{5}{2}, 1 + \sqrt{5}i, 1 - \sqrt{5}i$

$f(x) = (2x - 5)\left(x - 1 - \sqrt{5}i\right)\left(x - 1 + \sqrt{5}i\right)$

94. $f(x) = x^4 + 10x^3 + 26x^2 + 10x + 25$

$$= (x^2 + 1)(x^2 + 10x + 25)$$

$$= (x^2 + 1)(x + 5)^2 = (x + i)(x - i)(x + 5)^2$$

zeros: $\pm i, -5, -5$

96. $f(x) = (x - 1)(x + 4)(x + 3 - 5i)(x + 3 + 5i)$

$$= (x^2 + 3x - 4)((x + 3)^2 + 25)$$

$$= (x^2 + 3x - 4)(x^2 + 6x + 34)$$

$$= x^4 + 9x^3 + 48x^2 + 78x - 136$$

98. $f(x) = (x + 4)(x + 4)\left(x - 1 - \sqrt{3}i\right)\left(x - 1 + \sqrt{3}i\right)$

$$= (x^2 + 8x + 16)((x - 1)^2 + 3)$$

$$= (x^2 + 8x + 16)(x^2 - 2x + 4)$$

$$= x^4 + 6x^3 + 4x^2 + 64$$

100. $f(x) = x^4 - 4x^3 + 3x^2 + 8x - 16$

(a) $f(x) = (x^2 - x - 4)(x^2 - 3x + 4)$

(b) $x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(-4)}}{2} = \dfrac{1}{2} \pm \dfrac{\sqrt{17}}{2}$

$$f(x) = \left(x - \frac{1}{2} - \frac{\sqrt{17}}{2}\right)\left(x - \frac{1}{2} + \frac{\sqrt{17}}{2}\right)(x^2 - 3x + 4)$$

(c) $x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(4)}}{2} = \dfrac{3}{2} \pm \dfrac{\sqrt{7}}{2}i$

$$f(x) = \left(x - \frac{1}{2} - \frac{\sqrt{17}}{2}\right)\left(x - \frac{1}{2} + \frac{\sqrt{17}}{2}\right)\left(x - \frac{3}{2} + \frac{\sqrt{7}}{2}i\right)\left(x - \frac{3}{2} - \frac{\sqrt{7}}{2}i\right)$$

102. Domain: all $x \neq -12$

Horizontal asymptote: $y = 5$

Vertical asymptote: $x = -12$

104. The denominator $x^2 + x + 3$ has no zeros.

Domain: all x

Horizontal asymptote: $y = 2$

Vertical asymptotes: none

106. $y = 0$

108. No horizontal asymptote
(degree $p(x) >$ degree $q(x)$)

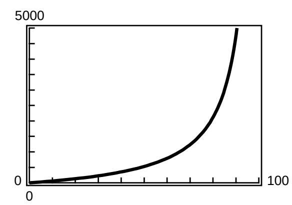
110. $y = \pm 1$

112. (a) When $p = 25$, $C = \dfrac{528(25)}{100 - 25} = \$176$ million.

(b) When $p = 50$, $C = \dfrac{528(50)}{100 - 50} = \$528$ million.

(c) When $p = 75$, $C = \dfrac{528(75)}{100 - 75} = \$1584$ million.

(d)



(e) As $p \Longrightarrow 100$, C tends to infinity.
No, it is not possible.

114. $f(x) = \dfrac{x - 3}{x - 2}$

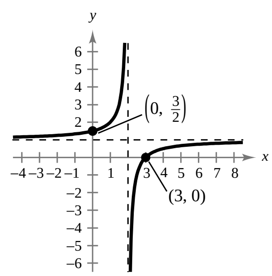
x-intercept: $(3, 0)$

y-intercept: $\left(0, \dfrac{3}{2}\right)$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

x	-1	0	1	3	4	5
y	$\frac{4}{3}$	$\frac{3}{2}$	2	0	$\frac{1}{2}$	$\frac{2}{3}$



116. $y = \dfrac{2x^2}{x^2 - 4}$

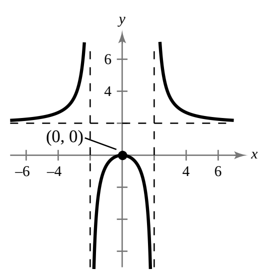
Intercept: $(0, 0)$

y-axis symmetry

Vertical asymptotes: $x = 2, x = -2$

Horizontal asymptote: $y = 2$

x	± 5	± 4	± 3	± 1	0
y	$\frac{50}{21}$	$\frac{8}{3}$	$\frac{18}{5}$	$-\frac{2}{3}$	0



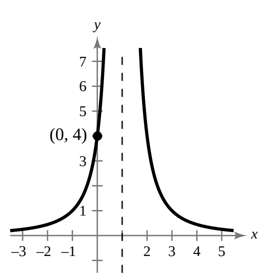
118. $h(x) = \dfrac{4}{(x - 1)^2}$

y-intercept: $(0, 4)$

Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 0$

x	-2	-1	0	2	3	4
y	$\frac{4}{9}$	1	4	4	1	$\frac{4}{9}$

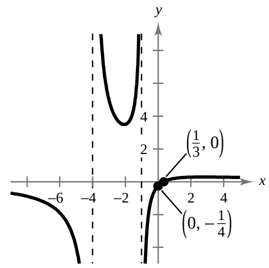


120. $f(x) = \dfrac{3x - 1}{x^2 + 5x + 4} = \dfrac{3x - 1}{(x + 4)(x + 1)}$

Intercepts: $\left(0, -\frac{1}{4}\right), \left(\frac{1}{3}, 0\right)$

Vertical asymptotes: $x = -4, x = -1$

Horizontal asymptotes: $y = 0$

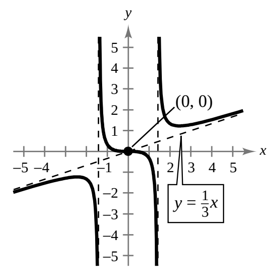


122. $f(x) = \dfrac{x^3}{3x^2 - 6} = \dfrac{1}{3}x + \dfrac{2x}{3x^2 - 6} = \dfrac{1}{3}\left[x + \dfrac{2x}{x^2 - 2}\right]$

Intercepts: $(0, 0)$

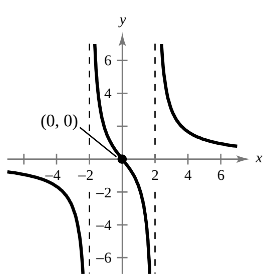
Vertical asymptotes: $x = \pm\sqrt{2}$

Slant asymptote: $y = \dfrac{1}{3}x$



124. $y = \dfrac{5x}{x^2 - 4}$

Intercept: $(0, 0)$
Vertical asymptotes: $x = 2, x = -2$
Horizontal asymptote: $y = 0$

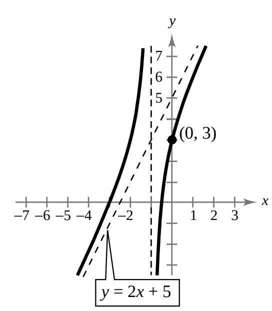


126. $f(x) = \dfrac{2x^2 + 7x + 3}{x + 1} = 2x + 5 - \dfrac{2}{x + 1}$

Intercepts: $(0, 3), (-3, 0), \left(-\frac{1}{2}, 0\right)$

Vertical asymptote: $x = -1$

Slant asymptote: $y = 2x + 5$



128. False. The degree of the numerator is two more than the degree of the denominator.