CHAPTER 3
Exponential and Logarithmic Functions

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CHAPTER 3
Exponential and Logarithmic Functions

Section 3.1 Exponential Functions and Their Graphs

Solutions to Even-Numbered Exercises

2. $5000(2^{-1.5}) \approx 1767.767$

4. $5^{-\pi} \approx 0.006$

6. $100^{\sqrt{2}} \approx 673.639$

8. $e^{-3/4} \approx 0.472$

10. $e^{3.78} \approx 43.816$

12. $g(x) = 2^{2x+6}$
$= 2^{2x} \cdot 2^6$
$= 64(2^{2x})$
$= 64(2^2)^x$
$= 64(4^x)$
$= h(x)$
Thus, $g(x) = h(x)$ but $g(x) \neq f(x)$.

14. $f(x) = 5^{-x} + 3$
$g(x) = 5^{3-x} = 5^3 \cdot 5^{-x}$
$h(x) = -5^{x-3} = -(5^x \cdot 5^{-3})$
Thus, $f(x)$, $g(x)$ and $h(x)$ are all distinct.

16. $f(x) = -2^x$ is negative and decreasing. Matches graph (h).

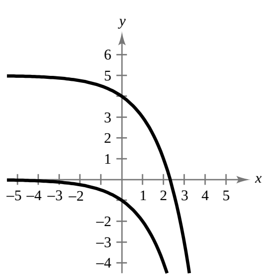
18. $f(x) = -2^{-x}$ is negative and increasing. Matches graph (b).

20. $f(x) = 2^x + 1$ is increasing and has $(0, 2)$ intercept. Matches graph (f).

22. $f(x) = 2^{x-2}$ is increasing and has $\left(0, \frac{1}{4}\right)$ intercept. Matches graph (d).

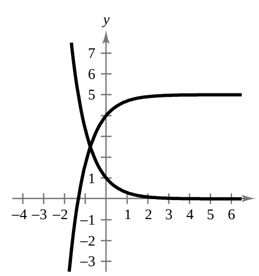
24. $f(x) = -2^x$
$g(x) = 5 - 2^x = 5 + f(x)$

Vertical shift five units upward



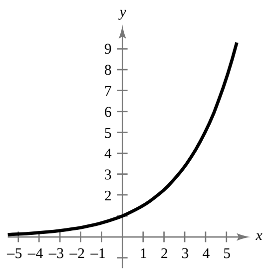
26. $f(x) = 0.3^x$
$g(x) = -0.3^x + 5 = -f(x) + 5$

Reflection in x-axis followed by vertical shift 5 units upward



28. $f(x) = \left(\frac{3}{2}\right)^x$

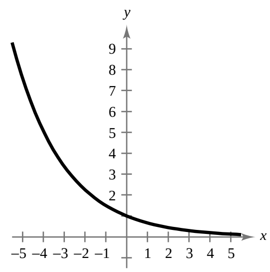
x	-2	-1	0	1	2
y	$\frac{4}{9}$	$\frac{2}{3}$	1	$\frac{3}{2}$	$\frac{9}{4}$



(a) Asymptote: $y = 0$
(b) Intercept: $(0, 1)$
(c) Increasing

30. $h(x) = \left(\frac{3}{2}\right)^{-x}$

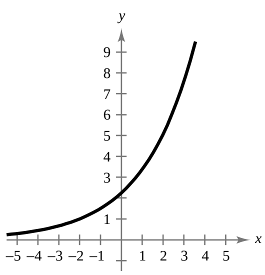
x	-2	-1	0	1	2
y	$\frac{9}{4}$	$\frac{3}{2}$	1	$\frac{2}{3}$	$\frac{4}{9}$



(a) Asymptote: $y = 0$
(b) Intercept: $(0, 1)$
(c) Decreasing

32. $g(x) = \left(\frac{3}{2}\right)^{x+2}$

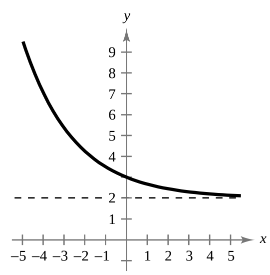
x	-4	-3	-2	-1	0
y	$\frac{4}{9}$	$\frac{2}{3}$	1	$\frac{3}{2}$	$\frac{9}{4}$



(a) Asymptote: $y = 0$
(b) Intercept: $\left(0, \frac{9}{4}\right)$
(c) Increasing

34. $f(x) = \left(\frac{3}{2}\right)^{-x} + 2$

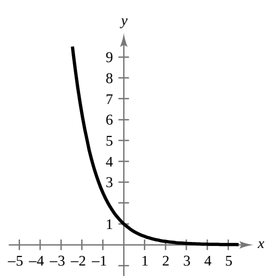
x	-2	-1	0	1	2
y	$\frac{17}{4}$	$\frac{7}{2}$	3	$\frac{8}{3}$	$\frac{22}{9}$



(a) Asymptote: $y = 2$
(b) Intercept: $(0, 3)$
(c) Decreasing

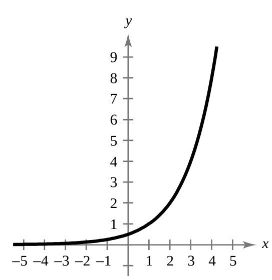
36. $f(x) = \left(\frac{5}{2}\right)^{-x}$

x	-2	-1	0	1	2
$f(x)$	6.25	2.5	1	0.4	0.16



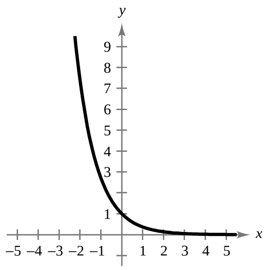
38. $f(x) = 2^{x-1}$

x	-1	0	1	2	3	4
$f(x)$	0.25	0.5	1	2	4	8



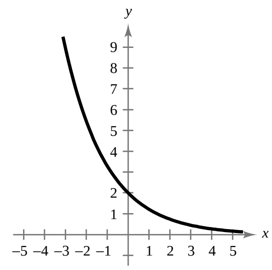
40. $f(x) = e^{-x}$

x	-3	-2	-1	0	1	2
$f(x)$	20.1	7.4	2.7	1	0.37	0.14



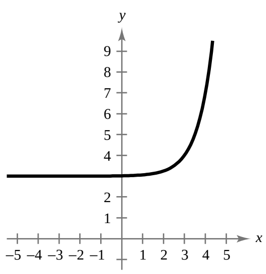
42. $f(x) = 2e^{-0.5x}$

x	-3	-2	-1	0	1	2
$f(x)$	9.0	5.4	3.3	2	1.2	0.7

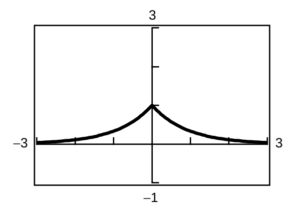


44. $f(x) = 4^{x-3} + 3$

x	-1	0	2	3	4	5
$f(x)$	3.004	3.02	3.25	4	7	19

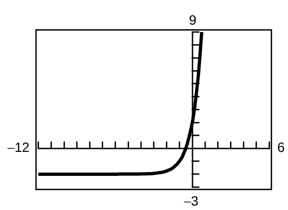


46.



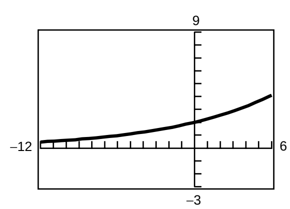
Asymptote: $y = 0$

48.



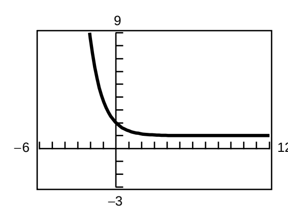
Asymptote: $y = -2$

50. $5(t) = 2e^{0.12t}$



Asymptote: $5 = 0$

52. $g(x) = 1 + e^{-x}$



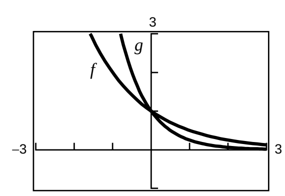
Asymptote: $y = 1$

54. (a)

x	-1	-0.5	0	0.5	1
$f(x)$	2	1.4142	1	0.7071	0.5
$g(x)$	4	2	1	0.5	0.25

$\left(\frac{1}{4}\right)^x < \left(\frac{1}{2}\right)^x$ for $x > 0$

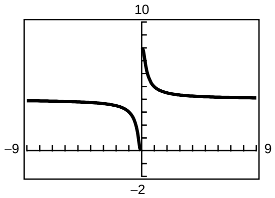
(b)



(i) $\left(\frac{1}{4}\right)^x < \left(\frac{1}{2}\right)^x$ for $x > 0$

(ii) $\left(\frac{1}{4}\right)^x > \left(\frac{1}{2}\right)^x$ for $x < 0$

56. (a)



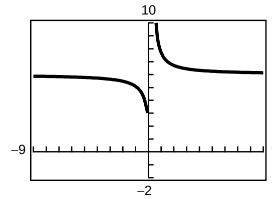
(b)

x	-15	-2	-1	-0.2	-0.1
$f(x)$	3.93	3.5	3.0	0.61	0.05

x	0	0.01	0.2	1	5
$f(x)$	undef.	8	7.4	4.9	4.2

Asymptote: $y = 4$

58. (a)



(b)

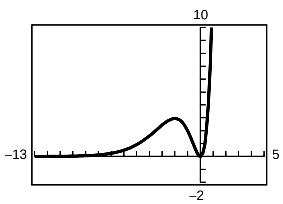
x	-15	-10	-1	-0.1	-0.01
$f(x)$	5.9	5.9	5.1	3.2	3

x	$\frac{0.2}{\ln 2}$	0.289	1	4	10
$f(x)$	undef.	2715	7.7	6.3	6.1

Asymptote: $y = 6$

$$x = \frac{0.2}{\ln 2} \approx 0.2885$$

60. (a)



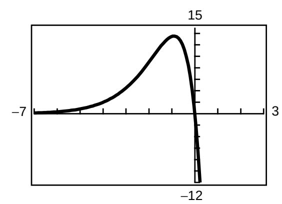
(b) Increasing on $(-\infty, -2)$ and $(0, \infty)$

Decreasing on $(-2, 0)$

(c) Relative maximum: $(-2, 2.943)$

Relative minimum: $(0, 0)$

62. (a)



(b) Increasing on $(-\infty, -0.91)$

Decreasing on $(-0.91, \infty)$

(c) Relative maximum: $(-0.910, 13.562)$

64. $P = 1000$, $r = 6\% = 0.06$, $t = 10$

n	1	2	4	12	365	Continuous
A	1790.85	1806.11	1814.02	1819.40	1822.03	1822.12

66. $P = 1000$, $r = 6\% = 0.06$, $t = 40$

n	1	2	4	12	365	Continuous
A	10,285.72	10,640.89	10,828.46	10,957.45	11,021.00	11,023.18

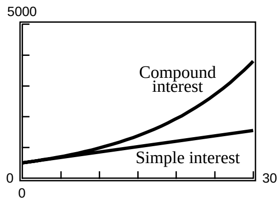
68. $P = 12{,}000$, $r = 6\% = 0.06$, compounded continuously: $A = Pe^{rt} = 12{,}000e^{(0.06)t}$

t	1	10	20	30	40	50
A	12,742.04	21,865.43	39,841.40	75,595.77	132,278.12	241,026.44

70. $P = 12{,}000$, $r = 7.5\% = 0.075$, $A = 12{,}000\left(1 + \frac{0.075}{365}\right)^{365t}$

t	1	10	20	30	40	50
A	12,934.51	25,402.04	53,771.98	113,826.52	240,952.18	510,056.46

72. (a)



(b) $A = 500(1.07)^t$
$A = 500(0.07)t + 500$

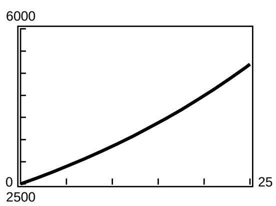
74. $P(t) = 2500e^{0.0293t}$

(a) 1992: $P(2) \approx 2651$

1995: $P(5) \approx 2894$

1998: $P(8) \approx 3160$

(b)



(c) 2005: $P(15) \approx 3880$

2010: $P(20) \approx 4492$

(d) $P(15) = 2500e^{0.0293(15)} \approx 3880$

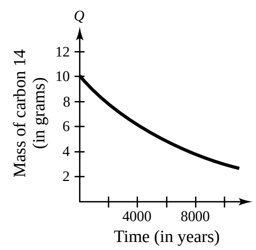
$P(20) = 2500e^{0.0293(20)} \approx 4492$

76. $Q = 10\left(\frac{1}{2}\right)^{t/5730}$

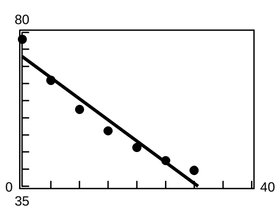
(a) $Q(0) = 10\left(\frac{1}{2}\right)^0 = 10$ grams

(b) $Q(2000) = 10\left(\frac{1}{2}\right)^{2000/5730} \approx 7.85$ grams

(c)

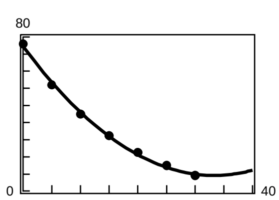


78. (a) $T = -1.239t + 73.021$

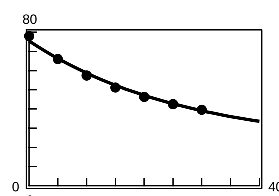


The temperature decreases at a slower rate as it approaches the room temperature.

(b) $T = 0.034t^2 - 2.264t + 77.295$
The parabola is increasing when $t = 60$.

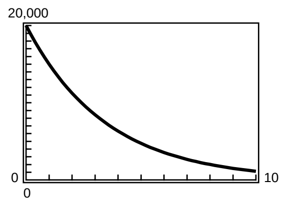


(c) $T = 54.438(0.964)^t + 21$



(d) The horizontal asymptote of the exponential is $T = 0$.

80. (a)



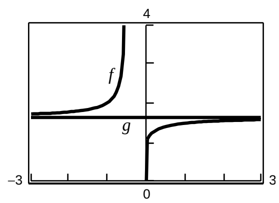
(b)

T	1	2	3	4	5	6	7	8	9	10
V	15,000	11,250	8437.50	6328.13	4746.09	3559.57	2669.68	2002.26	1501.69	1126.27

82. False. e is an irrational number.

84. $f(x) = \left(1 + \frac{0.5}{x}\right)^x$ and $g(x) = e^{0.5} \approx 1.6487$

(Horizontal line)



(a) As $x \to \infty$, $f(x) \to g(x)$.

(b) $\left(1 + \frac{r}{x}\right)^x \to e^r$ as $x \to \infty$

86. (c) and (d) are exponential functions.
(a) and (b) are polynomials.

88. f has an inverse because f is one-to-one.

$y = 5x - 7$

$x = 5y - 7$

$x + 7 = 5y$

$f^{-1}(x) = \frac{1}{5}(x + 7)$

90. f has an inverse because f is one-to-one.

$y = \sqrt[3]{x + 8}$

$x = \sqrt[3]{y + 8}$

$x^3 = y + 8$

$x^3 - 8 = y$

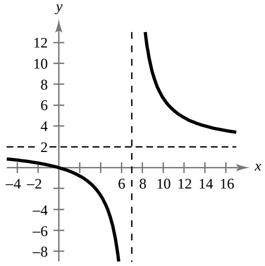
$f^{-1}(x) = x^3 - 8$

92. $f(x) = \frac{2x}{x - 7}$

Vertical asymptote: $x = 7$

Horizontal asymptote: $y = 2$

Intercept: $(0, 0)$



94. $f(x) = \frac{4x}{x^2 + 11x + 24} = \frac{4x}{(x + 3)(x + 8)}$

Vertical asymptote: $x = -3$, $x = -8$

Horizontal asymptotes: $y = 0$

Intercept: $(0, 0)$