Section 3.2 Logarithmic Functions and Their Graphs

Solutions to Even-Numbered Exercises

2. $\log_3 81 = 4 \implies 3^4 = 81$

4. $\log_{10} \frac{1}{1000} = -3 \implies 10^{-3} = \frac{1}{1000}$

6. $\log_{16} 8 = \frac{3}{4} \implies 16^{3/4} = 8$

8. $\ln 4 = 1.386 \ldots \implies e^{1.386\ldots} = 4$

10. $8^2 = 64 \implies \log_8 64 = 2$

12. $9^{3/2} = 27 \implies \log_9 27 = \frac{3}{2}$

14. $10^{-3} = 0.001 \implies \log_{10} 0.001 = -3$

16. $e^x = 4 \implies x = \ln 4$

18. $e^\pi = 23.14\ldots \implies \pi = \ln 23.14\ldots$

20. $\log_{27} 9 = \log_{27} 27^{2/3} = \frac{2}{3}$

22. $\log_2 \frac{1}{8} = \log_2 2^{-3} = -3$

24. $\log_{10} 0.1 = \log_{10} 10^{-1} = -1$

26. $x = \log_5 5 = 1$

28. $\ln 1 = \ln x$
$1 = x$

30. $\log_2 2^{-1} = x$
$-1 = x$

32. $\log_{10} 145 \approx 2.161$

34. $\log_{10} \frac{25}{2} = \log_{10} (12.5)$
≈ 1.097

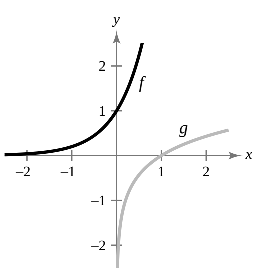
36. $\ln(\sqrt{5} - 2) \approx -1.444$

38. $\ln \sqrt{752} \approx 3.311$

40. $-3 \log_{10} 0.09 \approx 3.137$

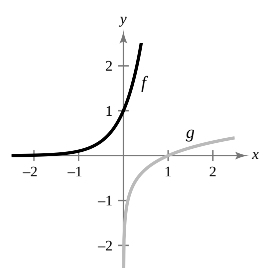
42. $-5.5 \ln 34 \approx -19.395$

44. $f(x) = 5^x, g(x) = \log_5 x$



f and g are inverses. Their graphs are reflected about the line $y = x$.

46. $f(x) = 10^x, g(x) = \log_{10} x$



f and g are inverses. Their graphs are reflected about the line $y = x$.

48. $f(x) = -\log_3 x$
Asymptote: $x = 0$
Point on graph: $(1, 0)$
Matches graph (f).

50. $f(x) = \log_3(x - 1)$
Asymptote: $x = 1$
Point on graph: $(2, 0)$
Matches graph (e).

52. $f(x) = -\log_3(-x)$
Asymptote: $x = 0$
Point on graph: $(-1, 0)$
Matches graph (a).

54. $g(x) = \log_6 x$

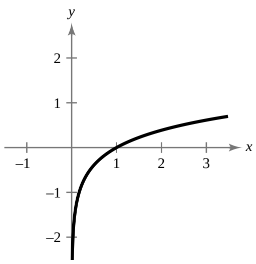
Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

x-intercept: $(1, 0)$

$y = \log_6 x \implies 6^y = x$

x	$\frac{1}{6}$	1	$\sqrt{6}$	36
y	-1	0	$\frac{1}{2}$	2

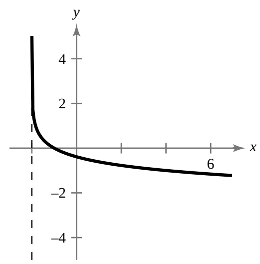


56. $f(x) = -\log_6(x + 2)$

Domain: $x + 2 > 0 \implies x > -2$

Vertical asymptote: $x + 2 = 0 \implies x = -2$

x-intercept: $(-1, 0)$



58. $y = \log_5(x - 1) + 4$

Domain: $x - 1 > 0 \implies x > 1$

The domain is $(1, \infty)$.

Vertical asymptote: $x - 1 = 0 \implies x = 1$

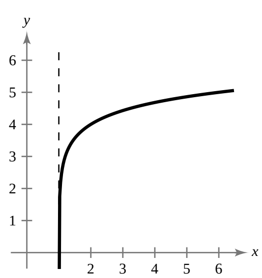
x-intercept: $\log_5(x - 1) + 4 = 0$

$$\log_5(x-1) = -4$$
$$5^{-4} = x - 1$$
$$\frac{1}{625} = x - 1$$
$$\frac{626}{625} = x$$

The x-intercept is $\left(\frac{626}{625}, 0\right)$.

$y = \log_5(x - 1) + 4 \implies 5^{y-4} + 1 = x$

x	1.00032	1.0016	1.008	1.04	1.2
y	-1	0	1	2	3



60. $f(x) = -\log_3(x + 2) - 4$

Domain: $(-2, \infty)$

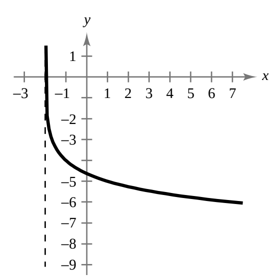
Vertical asymptote: $x = -2$

x-intercept: $\log_3(x + 2) = -4$

$$x + 2 = 3^{-4}$$
$$x = -2 + 3^{-4} = \frac{-161}{81}.$$

Intercept: $\left(\dfrac{-161}{81}, 0\right)$

x	-1	1	7
y	-4	-5	-6



62. $y = \log_{10}(-x)$

Domain: $-x > 0 \implies x < 0$

The domain is $(-\infty, 0)$.

Vertical asymptote: $x = 0$

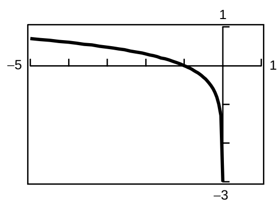
x-intercept: $\log_{10}(-x) = 0$

$$10^0 = -x$$
$$-1 = x$$

The x-intercept is $(-1, 0)$.

$y = \log_{10}(-x) \implies -10^y = x$

x	$-\frac{1}{100}$	$-\frac{1}{10}$	-1	-10
y	-2	-1	0	1



64. $h(x) = \ln(x + 1)$

Domain: $x + 1 > 0 \implies x > -1$

The domain is $(-1, \infty)$.

Vertical asymptote: $x + 1 = 0 \implies x = -1$

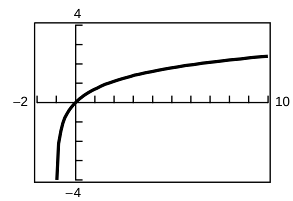
x-intercept: $\ln(x + 1) = 0$

$$e^0 = x + 1$$
$$1 = x + 1$$
$$0 = x$$

The x-intercept is $(0, 0)$.

$y = \ln(x + 1) \implies e^y - 1 = x$

x	-0.39	0	1.72	6.39	19.09
y	$-\frac{1}{2}$	0	1	2	3



66. $f(x) = \ln(3 - x)$

Domain: $3 - x > 0 \implies x < 3$

The domain is $(-\infty, 3)$.

Vertical asymptote: $3 - x = 0 \implies x = 3$

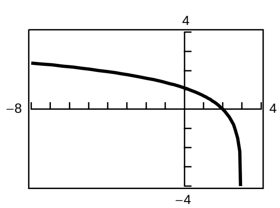
x-intercept: $\ln(3 - x) = 0$

$$e^0 = 3 - x$$
$$1 = 3 - x$$
$$2 = x$$

The x-intercept is $(2, 0)$.

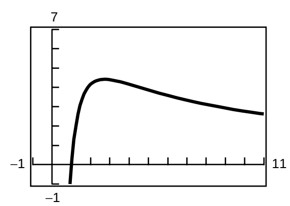
$y = \ln(3 - x) \implies x = 3 - e^y$

x	2.95	2.86	2.63	2	0.28
y	-3	-2	-1	0	1



68. $g(x) = \dfrac{12 \ln x}{x}$

(a)



(b) Domain: $(0, \infty)$

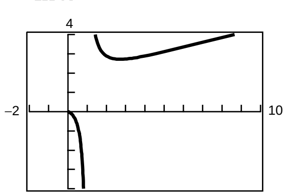
(c) Increasing on $(0, 2.72)$

Decreasing on $(2.72, \infty)$

(d) Relative maximum: $(2.72, 4.41)$

70. $f(x) = \dfrac{x}{\ln x}$

(a)



(b) Domain: $(0, 1)$ $(1, \infty)$

(c) Increasing on $(2.72, 0)$.

Decreasing on $(0, 1)$ $(1, 2.72)$

(d) Relative minimum: $(2.72, 2.72)$

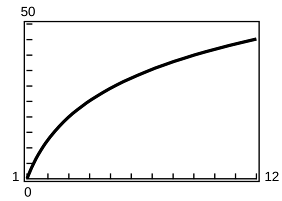
72. $W = 19{,}440(\ln 9 - \ln 3) \approx 21{,}357 \text{ ft} - 16$

74. $t = \dfrac{\ln k}{0.055}$

(a)

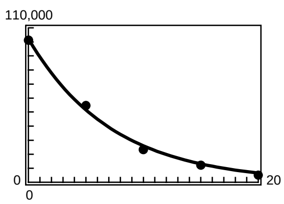
k	1	2	4	6	8	10	12
t	0	12.6	25.2	32.6	37.8	41.9	45.2

(b)



76. $P = 102.303e^{-0.137h}$

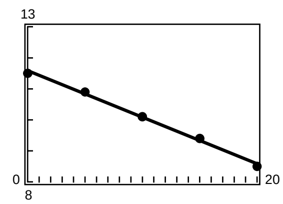
(a)



(b)

h	0	5	10	15	20
$\ln P$	11.526	10.910	10.056	9.406	8.531

$\ln P = -0.150h + 11.585$

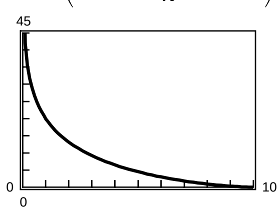


(c) $P = e^{-0.150h + 11.585} = 107{,}473.5e^{-0.15h}$

(d) Answers will vary.

78. $y = 10 \ln\left(\dfrac{10 + \sqrt{100 - x^2}}{x}\right) - \sqrt{100 - x^2}$

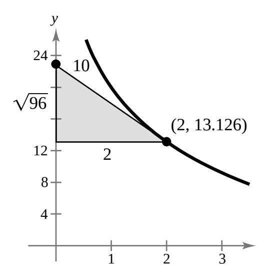
(a)



Domain: $0 < x \le 10$

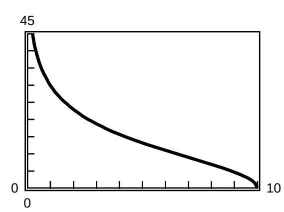
(b) Asymptote: $x = 0$

(c) When $x = 2$, $y \approx 13.126$. Since the rope is 10 feet long, the third side of the shaded right triangle is $\sqrt{100 - 2^2} = \sqrt{96}$. Thus, the person is at $13.126 + \sqrt{96} \approx 22.924$.



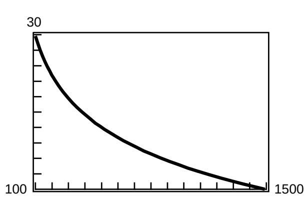
(d) $p = y + \sqrt{100 - x^2} = 10 \ln\left(\dfrac{10 + \sqrt{100 - x^2}}{x}\right)$

(e)



The position of the person changes most at the beginning.

80. $y = 80.4 - 11 \ln x$



$y(300) = 80.4 - 11 \ln 300 \approx 17.66 \text{ ft}^3/\text{min}$

82. $t = 16.625 \ln\left(\dfrac{897.72}{897.72 - 750}\right) \approx 30$ years

84. Total amount $= 897.72(30)(12) = \$323{,}179.20$

Interest $= 323{,}179.20 - 150{,}000 = \$173{,}179.20$

86. False. You would reflect $y = 6^x$ in the line $y = x$.

88. (a) False, y is not an exponential function of x. (y can never be 0.)

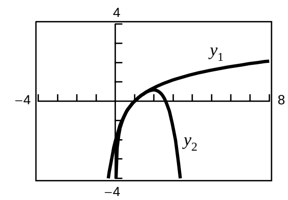
(b) True, y could be $\log_2 x$.

(c) True, x could be 2^y.

(d) False, y is not linear. (The points are not collinear.)

90. $y = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4$

The pattern implies that as we take more terms, the graph of y will more closely resemble that of $\ln x$ on the interval $(0, 2)$.



92. Vertical asymptote: $x = -8$

Horizontal asymptote: $y = 0$

94. $f(x) = \dfrac{x + 5}{(2x^2 + x - 15)} = \dfrac{x + 5}{(2x - 5)(x + 3)}$

Vertical asymptotes: $x = \dfrac{5}{2}, -3$

Horizontal asymptote: $y = 0$

96. $e^{12} \approx 162{,}754.791$

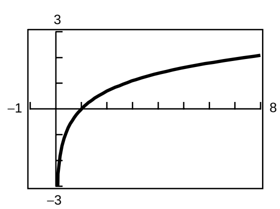
98. $e^{-5} \approx 0.007$

Section 3.3 Properties of Logarithms

Solutions to Even-Numbered Exercises

2. $f(x) = \ln x$

$g(x) = \dfrac{\log_{10} x}{\log_{10} e}$



$f(x) = g(x)$

4. $\log_7 4 = \dfrac{\ln 4}{\ln 7} \approx 0.712$

6. $\log_{1/8} 64 = \dfrac{\ln 64}{\ln\left(\frac{1}{8}\right)}$

$= \dfrac{\ln 8^2}{-\ln 8} = -2$

8. $\log_{1/3}(0.015) = \dfrac{\ln(0.015)}{\ln\left(\frac{1}{3}\right)}$

≈ 3.823

10. $\log_{20} 135 = \dfrac{\ln 135}{\ln 20} \approx 1.637$

12. (a) $\log_3 x = \dfrac{\log_{10} x}{\log_{10} 3}$

(b) $\log_3 x = \dfrac{\ln x}{\ln 3}$