14. (a) $\log_{1/3} x = \dfrac{\log_{10} x}{\log_{10}\left(\frac{1}{3}\right)} = \dfrac{-\log_{10} x}{\log_{10} 3}$

(b) $\log_{1/3} x = \dfrac{\ln x}{\ln\left(\frac{1}{3}\right)} = \dfrac{-\ln x}{\ln 3}$

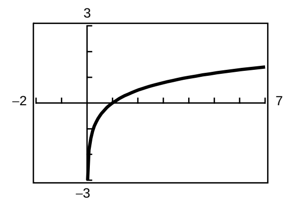
16. (a) $\log_x\left(\frac{3}{4}\right) = \dfrac{\log_{10}\left(\frac{3}{4}\right)}{\log_{10} x}$

(b) $\log_x\left(\frac{3}{4}\right) = \dfrac{\ln\left(\frac{3}{4}\right)}{\ln x}$

18. (a) $\log_{7.1} x = \dfrac{\log_{10} x}{\log_{10} 7.1}$

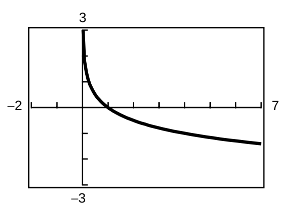
(b) $\log_{7.1} x = \dfrac{\ln x}{\ln 7.1}$

20. $f(x) = \log_4 x = \dfrac{\ln x}{\ln 4}$

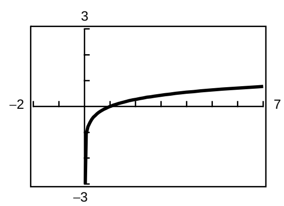


22. $f(x) = \log_{1/4} x = \dfrac{\ln x}{\ln\left(\frac{1}{4}\right)}$

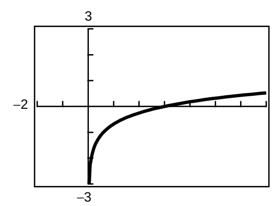
$= \dfrac{-\ln x}{\ln 4}$



24. $f(x) = \log_{12.4} x = \dfrac{\ln x}{\ln 12.4}$



26. $f(x) = \log_5\left(\frac{x}{3}\right) = \dfrac{\ln\left(\frac{x}{3}\right)}{\ln(5)}$



28. $\log_{10} 10z = \log_{10} 10 + \log_{10} z = 1 + \log_{10} z$

30. $\log_{10} \dfrac{y}{2} = \log_{10} y - \log_{10} 2$

32. $\log_6 z^{-3} = -3 \log_6 z$

34. $\ln \sqrt[3]{t} = \ln t^{1/3} = \dfrac{1}{3} \ln t$

36. $\ln \dfrac{xy}{z} = \ln x + \ln y - \ln z$

38. $\ln\left(\dfrac{x^2 - 1}{x^3}\right) = \ln(x^2 - 1) - \ln x^3$

$= \ln[(x + 1)(x - 1)] - \ln x^3$

$= \ln(x + 1) + \ln(x - 1) - 3 \ln x$

40. $\ln\sqrt{\dfrac{x^2}{y^3}} = \ln\left(\dfrac{x^2}{y^3}\right)^{1/2} = \dfrac{1}{2}\ln\left(\dfrac{x^2}{y^3}\right)$

$= \dfrac{1}{2}(\ln x^2 - \ln y^3)$

$= \dfrac{1}{2}(2 \ln x - 3 \ln y)$

42. $\ln\left(\frac{x}{\sqrt{x^2+1}}\right) = \ln x - \ln\sqrt{x^2+1}$
$= \ln x - \ln(x^2+1)^{1/2}$
$= \ln x - \frac{1}{2}\ln(x^2+1)$

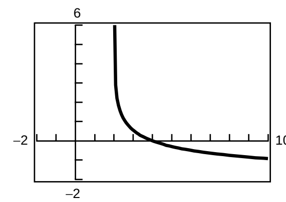
44. $\ln\sqrt{x^2(x+2)} = \ln[x^2(x+2)]^{1/2}$
$= \ln[x(x+2)^{1/2}]$
$= \ln x + \ln(x+2)^{1/2}$
$= \ln x + \frac{1}{2}\ln(x+2)$

46. $\log_b \frac{\sqrt{x}\,y^4}{z^4} = \log_b \sqrt{x}\,y^4 - \log_b z^4$
$= \log_b x^{1/2} + \log_b y^4 - \log_b z^4$
$= \frac{1}{2}\log_b x + 4\log_b y - 4\log_b z$

48. $y_1 = \ln\left(\frac{\sqrt{x}}{x-2}\right)$

$y_2 = \frac{1}{2}\ln x - \ln(x-2)$

$y_1 = y_2$



50. $\ln y + \ln z = \ln yz$

52. $\log_5 8 - \log_5 t = \log_5 \frac{8}{t}$

54. $-6\log_6 2x = \log_6(2x)^{-6} = \log_6\left(\frac{1}{64x^6}\right)$

56. $\frac{5}{2}\log_7(z-4) = \log_7(z-4)^{5/2}$

58. $2\ln 8 + 5\ln z = \ln 8^2 + \ln z^5$
$= \ln 64z^5$

60. $3\ln x + 2\ln y - 4\ln z = \ln x^3 + \ln y^2 - \ln z^4$
$= \ln x^3y^2 - \ln z^4$
$= \ln\frac{x^3y^2}{z^4}$

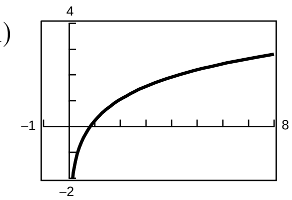
62. $4[\ln z + \ln(z+5)] - 2\ln(z-5) = 4[\ln z(x+5)] - \ln(z-5)^2$
$= \ln[z(z+5)]^4 - \ln(z-5)^2$
$= \ln\frac{z^4(z+5)^4}{(z-5)^2}$

64. $2[\ln x - \ln(x+1) - \ln(x-1)] = 2\left[\ln\frac{x}{x+1} - \ln(x-1)\right]$
$= 2\left[\ln\frac{x}{(x+1)(x-1)}\right]$
$= 2\left[\ln\frac{x}{x^2-1}\right]$
$= \ln\left(\frac{x}{x^2-1}\right)^2$

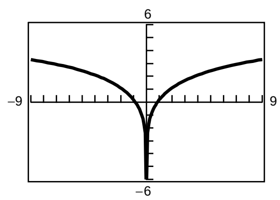
66. $\frac{1}{2}[\ln(x+1) + 2\ln(x-1)] + 3\ln x = \frac{1}{2}[\ln(x+1) + \ln(x-1)^2] + \ln x^3$
$= \frac{1}{2}[\ln(x+1)(x-1)^2] + \ln x^3$
$= \ln[(x+1)(x-1)^2]^{1/2} + \ln x^3$
$= \ln[(x+1)^{1/2}(x-1)] + \ln x^3$
$= \ln\left[x^3(x-1)\sqrt{x+1}\right]$

68. $\frac{3}{2}\ln 5t^6 - \frac{3}{4}\ln t^4 = \ln(5t^6)^{3/2} - \ln(t^4)^{3/4}$
$= \ln 5^{3/2}\, t^9 - \ln t^3$
$= \ln\frac{5\sqrt{5}t^9}{t^3}$
$= \ln 5\sqrt{5}\,t^6$

70. $y_1 = \ln x + \frac{1}{3}\ln(x+1)$
$y_2 = \ln\left(x\sqrt[3]{x+1}\right)$
$y_1 = y_2$



72. $y_1 = \frac{1}{4}\ln[x^4(x^2+1)]$
$y_2 = \ln x + \frac{1}{4}\ln(x^2+1)$



They are not equivalent. The domain of y_1 is all real numbers except 0. The domain of y_2 is $x > 0$.

74. $\log_6 \sqrt[3]{6} = \log_6 6^{1/3} = \frac{1}{3}\log_6 6 = \frac{1}{3}(1) = \frac{1}{3}$

76. $\log_5 \frac{1}{125} = \log_5 5^{-3} = -3\log_5 5 = -3(1) = -3$

78. $\log_4(-16)$ is undefined because -16 is not in the domain of $\log_4 x$.

80. $\log_4 2 + \log_4 32 = \log_4 4^{1/2} + \log_4 4^{5/2}$
$= \frac{1}{2}\log_4 4 + \frac{5}{2}\log_4 4$
$= \frac{1}{2}(1) + \frac{5}{2}(1)$
$= 3$

82. $3\ln e^4 = (3)(4)\ln e$
$= 12(1) = 12$

84. $\ln 1 = 0$

86. $\ln\sqrt[5]{e^3} = \ln e^{3/5} = \frac{3}{5}$

88. $\log_5\left(\frac{1}{15}\right) = \log_5 1 - \log_5 15 = 0 - (\log_5 3 + \log_5 5)$
$= -1 - \log_5 3$

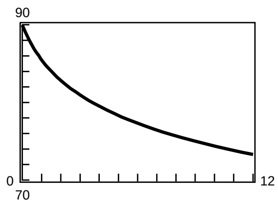
90. $\log_2(4^2 \cdot 3^4) = \log_2 4^2 + \log_2 3^4$
$= 2\log_2 4 + 4\log_2 3$
$= 2\log_2 2^2 + 4\log_2 3$
$= 4\log_2 2 + 4\log_2 3$
$= 4 + 4\log_2 3$

92. $\log_{10}\frac{9}{300} = \log_{10}\frac{3}{100}$
$= \log_{10} 3 - \log_{10} 100$
$= \log_{10} 3 - \log_{10} 10^2$
$= \log_{10} 3 - 2\log_{10} 10$
$= \log_{10} 3 - 2$

94. $\ln \frac{6}{e^2} = \ln 6 - \ln e^2$
$= \ln 6 - 2 \ln e$
$= \ln 6 - 2$

96. $f(t) = 90 - 15 \log_{10}(t + 1), 0 \le t \le 12$

(a)



(b) When $t = 0$, $f(0) = 90$.

(c) $f(6) \approx 77$

(d) $f(12) \approx 73$

(e) $f(t) = 75$ when $t \approx 9$ months.

98. If $y = ab^x$, then $\ln y = \ln(ab^x) = \ln a + x \ln b$, which is linear. If $y = \frac{1}{cx + d}$, then $\frac{1}{y} = cx + d$.

100. $f(ax) = f(a) + f(x), a > 0, x > 0$

True, because

$f(ax) = \ln ax = \ln a + \ln x$
$= f(a) + f(x)$.

102. $\sqrt{f(x)} = \frac{1}{2} f(x)$; False.

$\sqrt{f(x)} = \sqrt{\ln x}$ can't be simplified further.

$f(\sqrt{x}) = \ln \sqrt{x} = \ln x^{1/2} = \frac{1}{2} \ln x = \frac{1}{2} f(x)$

104. If $f(x) < 0$, then $0 < x < 1$.

True.

106. $\ln 2 \approx 0.6931$, $\ln 3 \approx 1.0986$, $\ln 5 \approx 1.6094$

$\ln 2 \approx 0.6931$

$\ln 3 \approx 1.0986$

$\ln 4 = \ln 2 + \ln 2 \approx 0.6931 + 0.6931 = 1.3862$

$\ln 5 \approx 1.6094$

$\ln 6 = \ln 2 + \ln 3 \approx 0.6931 + 1.0986 = 1.7917$

$\ln 8 = \ln 2^3 = 3 \ln 2 \approx 3(0.6931) = 2.0793$

$\ln 9 = \ln 3^2 = 2 \ln 3 \approx 2(1.0986) = 2.1972$

$\ln 10 = \ln 5 + \ln 2 \approx 1.6094 + 0.6931 = 2.3025$

$\ln 12 = \ln 2^2 + \ln 3 = 2 \ln 2 + \ln 3 \approx 2(0.6931) + 1.0986 = 2.4848$

$\ln 15 = \ln 5 + \ln 3 \approx 1.6094 + 1.0986 = 2.7080$

$\ln 16 = \ln 2^4 = 4 \ln 2 \approx 4(0.6931) = 2.7724$

$\ln 18 = \ln 3^2 + \ln 2 = 2 \ln 3 + \ln 2 \approx 2(1.0986) + 0.6931 = 2.8903$

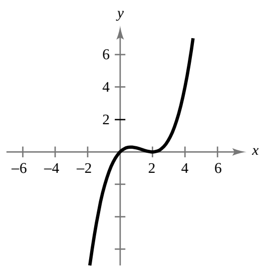
$\ln 20 = \ln 5 + \ln 2^2 = \ln 5 + 2 \ln 2 \approx 1.6094 + 2(0.6931) = 2.9956$

108. Let $x = \log_b u$, then $u = b^x$ and $u^n = b^{nx}$.

$\log_b u^n = \log_b b^{nx} = nx = n \log_b u$

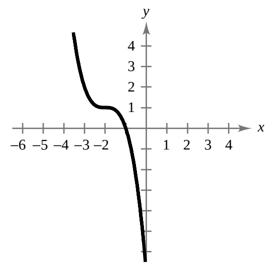
110. $f(x) = -\frac{1}{2}(x^2 + 4x)$

Intercepts: $(0, 0), (2, 0)$



112. $f(x) = -(x + 2)^3 + 1$

Translation and reflection of $y = x^3$.



114.
$$2x^3 + 20x^2 + 50x = 0$$
$$2x(x^2 + 10x + 25) = 0$$
$$2x(x + 5)^2 = 0$$
$$x = 0, -5, -5$$

116.
$$9x^4 - 37x^2 + 4 = 0$$
$$(x^2 - 4)(9x^2 - 1) = 0$$
$$(x - 2)(x + 2)(3x - 1)(3x + 1) = 0$$
$$x = \pm 2, \pm\frac{1}{3}$$

118.
$$9x^4 - 226x^2 + 25 = 0$$
$$(x^2 - 25)(9x^2 - 1) = 0$$
$$(x - 5)(x + 5)(3x + 1)(3x - 1) = 0$$
$$x = \pm 5, \pm\frac{1}{3}$$

120. $\sqrt[5]{8251} \approx 6.072$

122. $170(4^{-1.1}) \approx 36.998$

124. $\log_{10}\left(\frac{7}{5}\right) \approx 0.146$

126. $\ln\left(5 - \sqrt{7}\right) \approx 0.856$

Section 3.4 Solving Exponential and Logarithmic Equations

Solutions to Even-Numbered Exercises

2. $2^{3x+1} = 32$

(a) $x = -1$

$2^{3(-1)+1} = 2^{-2} = \frac{1}{4}$

No, $x = -1$ is not a solution.

(b) $x = 2$

$2^{3(2)+1} = 2^7 = 128$

No, $x = 2$ is not a solution.

4. $-4e^{x-1} = -60$

(a) $x = 1 + \ln 15$: $-4e^{(1+\ln 15)-1} = -4e^{\ln 15} = -4(15) = -60$. Yes.

(b) $x \approx 3.7081$: $-4e^{3.7081-1} = -4e^{2.7081} \approx -60$. Yes.

(c) $x = \ln 16$: $-4e^{\ln 16 - 1} \approx -23.5 \neq -60$. No.