6. $\log_6\left(\frac{5}{3}x\right) = 2 \iff \dfrac{\ln\left(\frac{5}{3}x\right)}{\ln 6} = 2$

(a) $x \approx 20.2882$: $\dfrac{\ln\left(\frac{5}{3} \cdot 20.2882\right)}{\ln 6} = 1.965 \neq 2$. No.

(b) $x = \frac{108}{5}$: $\log_6\left(\frac{5}{3} \cdot \frac{108}{5}\right) = \log_6(36) = 2$. Yes.

(c) $x = 7.2$: $\dfrac{\ln\left(\frac{5}{3}(7.2)\right)}{\ln 6} \approx 1.3869 \neq 2$. No.

8. $\frac{1}{5}\ln(2 + x) = \frac{1}{2} \iff \ln(2 + x) = \frac{5}{2}$

(a) $x = e^{2.5} - 2$: $\ln(2 + e^{2.5} - 2) = \ln e^{2.5} = 2.5$. Yes.

(b) $x \approx \frac{4073}{400}$: $\ln\left(2 + \frac{4073}{400}\right) \approx 2.5$. Yes.

(c) $x = \frac{1}{2}$: $\ln\left(2 + \frac{1}{2}\right) \approx 0.9163 \neq 2.5$. No.

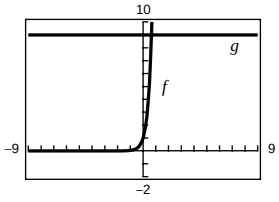
10. $f(x) = g(x)$

$27^x = 9$

$27^x = 27^{2/3}$

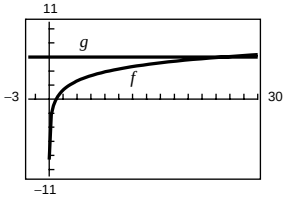
$x = \frac{2}{3}$

Point of intersection: $\left(\frac{2}{3}, 9\right)$



12. $f(x) = 3\log_5 x = 3 \cdot \dfrac{\ln x}{\ln 5}$

$g(x) = 6$



Intersection point: $(25, 6)$

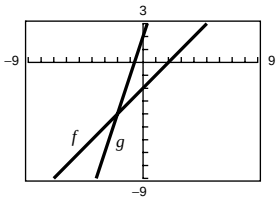
$3\log_5 x = 6$

$\log_5 x = 2$

$x = 5^2 = 25 \implies (25, 6)$

14. $f(x) = \ln e^{x-2} = x - 2$

$g(x) = 3x + 2$



Intersection point: $(-2, -4)$

$x - 2 = 3x + 2$

$-4 = 2x$

$x = -2 \implies (-2, -4)$

16. $3^x = 243$
$3^x = 3^5$
$x = 5$

18. $7^x = \frac{1}{49}$
$7^x = 7^{-2}$
$x = -2$

20. $\left(\frac{1}{2}\right)^x = 32$
$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-5}$
$x = -5$

22. $\left(\frac{3}{4}\right)^x = \frac{27}{64}$
$\left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^3$
$x = 3$

24. $\ln x - \ln 2 = 0$
$\ln x = \ln 2$
$x = 2$

26. $e^x = 2$
$x = \ln 2 \approx 0.693$

28. $\ln x = -1$
$e^{-1} = x$
$x = \dfrac{1}{e} \approx 0.368$

30. $\log_4 x = 3$
$4^3 = x$
$x = 64$

32. $\log_{10} x - 2 = 0$
$\log_{10} x = 2$
$10^2 = x$
$x = 100$

34. $\log_{10} x = -\dfrac{1}{2}$
$x = 10^{-1/2} = \dfrac{1}{\sqrt{10}}$
≈ 0.316

36. $\ln(3x + 5) = 8$
$e^8 = 3x + 5$
$x = \dfrac{1}{3}(e^8 - 5)$
≈ 991.986

38. $\ln e^{2x-1} = 2x - 1$

40. $-1 + \ln e^{2x} = -1 + 2x = 2x - 1$

42. $-8 + e^{\ln x^3} = -8 + x^3 = x^3 - 8$

44. $6^{5x} = 3000$
$\ln 6^{5x} = \ln 3000$
$(5x) \ln 6 = \ln 3000$
$5x = \dfrac{\ln 3000}{\ln 6}$
$x = \dfrac{\ln 3000}{5 \ln 6} \approx 0.894$

46. $4e^{2x} = 40$
$e^{2x} = 10$
$2x = \ln 10$
$x = \frac{1}{2}\ln 10 \approx 1.151$

48. $4^{-3t} = 0.10$
$\ln 4^{-3t} = \ln 0.10$
$(-3t) \ln 4 = \ln 0.10$
$-3t = \dfrac{\ln 0.10}{\ln 4}$
$t = -\dfrac{\ln 0.10}{3 \ln 4} \approx 0.554$

50. $8^{-2-x} = 431$
$\log_8 431 = -2 - x$
$x = -2 - \log_8 431 = -2 - \dfrac{\ln 431}{\ln 8} \approx -4.917$

52. $1000e^{-4x} = 75$

$e^{-4x} = \frac{3}{40}$

$\ln e^{-4x} = \ln \frac{3}{40}$

$-4x = \ln \frac{3}{40}$

$x = -\frac{1}{4} \ln \frac{3}{40} \approx 0.648$

54. $-14 + 3e^x = 11$

$3e^x = 25$

$e^x = \frac{25}{3}$

$\ln e^x = \ln \frac{25}{3}$

$x = \ln \frac{25}{3} \approx 2.120$

56. $e^{2x} - 5e^x + 6 = 0$

$(e^x - 2)(e^x - 3) = 0$

$e^x = 2$ or $e^x = 3$

$x = \ln 2 \approx 0.693$ or

$x = \ln 3 \approx 1.099$

58. $\dfrac{525}{1 + e^{-x}} = 275$

$1 + e^{-x} = \dfrac{525}{275}$

$e^{-x} = \dfrac{525}{275} - 1 = \dfrac{250}{275} = \dfrac{10}{11}$

$-x = \ln \dfrac{10}{11}$

$x = -\ln \dfrac{10}{11} = \ln \dfrac{11}{10} \approx 0.095$

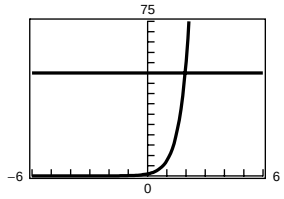
60. $\left(16 + \dfrac{0.878}{26}\right)^{3t} = 30$

$3t \ln\left(16 + \dfrac{0.878}{26}\right) = \ln 30$

$t = \dfrac{\ln 30}{3 \ln\left(16 + \dfrac{0.878}{26}\right)} \approx \dfrac{3.4012}{8.3241} \approx 0.409$

62.

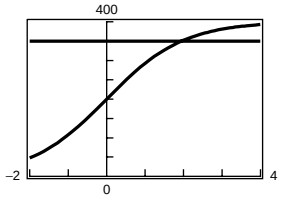
x	1.6	1.7	1.8	1.9	2.0
$f(x)$	24.53	29.96	36.60	44.70	54.60



$x \approx 1.956$

64.

x	0	1	2	3	4
$f(x)$	200	292	352	381	393



$x \approx 1.946$

66. $4^{-x/2} = 0.10$

The zero of $y = 4^{-x/2} - 0.1$ is $x = 3.322$.

68. $8(10^{3x}) = 12$

The zero of $y = 8(10^{3x}) - 12$ is $x = 0.059$.

70. $8(3^{6-x}) = 40$

The zero of $y = 8(3^{6-x}) - 40$ is $x = 4.535$.

72. $\left(4 - \dfrac{2.471}{40}\right)^{9t} = 21$

$3.938225^{9t} = 21$

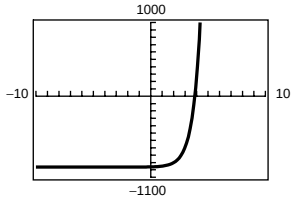
The zero of $y = 3.938225^{9t} - 21$ is $t = 0.247$.

74. $\dfrac{119}{e^{6x} - 14} = 7$

The zero of $y = \dfrac{119}{e^{6x} - 14} - 7$ is $x = 0.572$.

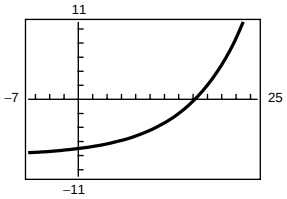
76. $f(x) = 3e^{3x/2} - 962$

The zero is $x \approx 3.847$.



78. $h(t) = e^{0.125t} - 8$

The zero is $t \approx 16.636$.



80. $\ln x = 2$

$e^{\ln x} = e^2$

$x = e^2 \approx 7.389$

82. $\ln 4x = 1$

$e^{\ln 4x} = e^1$

$4x = e$

$x = \dfrac{e}{4} \approx 0.680$

84. $2 \ln x = 7$

$\ln x = \frac{7}{2}$

$x = e^{7/2} \approx 33.115$

86. $\log_{10} x^2 = 6$

$10^{\log_{10} x^2} = 10^6$

$x^2 = 10^6$

$x = \pm\sqrt{10^6} = \pm 1000$

88. $4 \log_{10}(x - 6) = 11$

$\log_{10}(x - 6) = \frac{11}{4}$

$10^{11/4} = x - 6$

$x = 6 + 10^{11/4} \approx 568.341$

90. $\ln\sqrt{x - 8} = 5$

$\frac{1}{2}\ln(x - 8) = 5$

$\ln(x - 8) = 10$

$e^{10} = x - 8$

$x = 8 + e^{10} \approx 22{,}034.466$

92. $\ln(x^2 + 1) = 8$

$e^8 = x^2 + 1$

$x = \pm\sqrt{e^8 - 1} = \pm 54.589$

94. $\log_3 x + \log_3(x^2 - 8) = \log_3 8x$

$\log_3[x(x^2 - 8)] = \log_3 8x$

$x(x^2 - 8) = 8x$

$x^3 - 16x = 0$

$x(x + 4)(x - 4) = 0 \implies x = 0, \pm 4$

The solution is $x = 4$ [$x = 0, -4$ extraneous].

96. $\ln(x + 1) - \ln(x - 2) = \ln x^2$

$\ln\left(\dfrac{x + 1}{x - 2}\right) = \ln x^2$

$\dfrac{x + 1}{x - 2} = x^2$

$x + 1 = x^3 - 2x^2$

$0 = x^3 - 2x^2 - x - 1$

From the graph, we have $x \approx 2.547$.

98. $\log_{10} 4x - \log_{10}\left(12 + \sqrt{x}\right) = 2$

$\log_{10} \dfrac{4x}{12 + \sqrt{x}} = 2$

$\dfrac{4x}{12 + \sqrt{x}} = 10^2 = 100$

$4x = 1200 + 100\sqrt{x}$

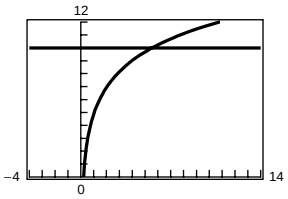
$x - 25\sqrt{x} - 300 = 0$ Quadratic in $\sqrt{x}$

$\sqrt{x} = \dfrac{25 \pm \sqrt{(-25)^2 - 4(-300)}}{2} = \dfrac{25 \pm \sqrt{1825}}{2}$

Taking the positive root, and squaring, $x = 1146.5$.

100.

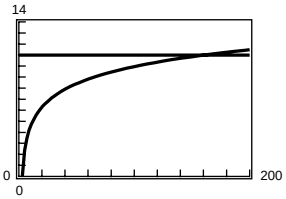
x	4	5	6	7	8
$f(x)$	8.99	9.66	10.20	10.67	11.07



$x \approx 5.606$

102.

x	150	155	160	165	170
$f(x)$	10.85	10.92	10.99	11.06	11.13



$x \approx 160.489$

104. Solving $y = \log_{10} x^2 - 4 = 0$, $x = \pm 100$

106. Solving $y = \ln 4x - 3 = 0$, $x = 5.021$

108.
$$\log_2 x + \log_2(x+5) = \log_2(x+4)$$
$$\log_2(x(x+5)) = \log_2(x+4)$$
$$x(x+5) = x+4$$
$$x^2 + 5x = x + 4$$
$$x^2 + 4x - 4 = 0$$
$$x = 0.828 \quad (-4.828 \text{ is extraneous})$$

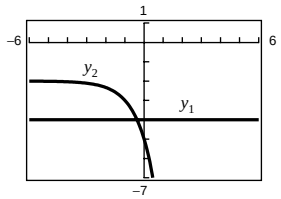
110. $\ln x + \ln(x^2+4) = 10$

Solving $y = \ln x + \ln(x^2+4) - 10 = 0$, $x = 27.984$

112. $y_1 = -4$

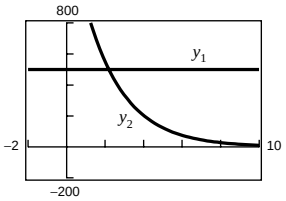
$y_2 = -3^{x+1} - 2$

The graphs intersect at $(x, y) = (-0.369, -4)$.



114. $y_1 = 500$

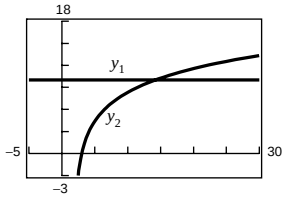
$y_2 = 1500e^{-x/2}$



From the graph, we have $(x, y) \approx (2.197, 500)$

116. $y_1 = 10$

$y_2 = 4\ln(x-2)$



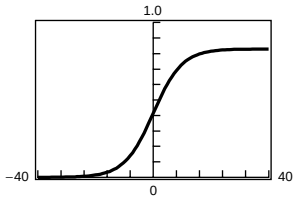
From the graph, we have $(x, y) \approx (14.182, 10)$.

118. (a)
$$r = 0.12$$
$$A = Pe^{rt}$$
$$2000 = 1000e^{0.12t}$$
$$2 = e^{0.12t}$$
$$\ln 2 = \ln e^{0.12t}$$
$$\ln 2 = 0.12t$$
$$\frac{\ln 2}{0.12} = t$$
$$t \approx 5.8 \text{ years}$$

(b)
$$3000 = 1000e^{0.12t}$$
$$3 = e^{0.12t}$$
$$\ln 3 = 0.12t$$
$$t = \frac{\ln 3}{0.12}$$
$$t \approx 9.2 \text{ years}$$

120. $P = \dfrac{0.83}{1 + e^{-0.2n}}$

(a)



(b) Horizontal asymptotes: $y = 0, y = 0.83$
The upper asymptote, $y = 0.83$, indicates that the proportion of correct responses will approach 0.83 as the number of trials increases.

(c) When $P = 60\%$ or $P = 0.60$:

$$0.60 = \frac{0.83}{1 + e^{-0.2n}}$$

$$1 + e^{-0.2n} = \frac{0.83}{0.60}$$

$$e^{-0.2n} = \frac{0.83}{0.60} - 1$$

$$\ln e^{-0.2n} = \ln\left(\frac{0.83}{0.60} - 1\right)$$

$$-0.2n = \ln\left(\frac{0.83}{0.60} - 1\right)$$

$$n = -\frac{\ln\left(\frac{0.83}{0.60} - 1\right)}{0.2} \approx 5 \text{ trials}$$

122. $p = 5000\left(1 - \dfrac{4}{4 + e^{-0.002x}}\right)$

(a) When $p = \$600$:

$$600 = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

$$0.12 = 1 - \frac{4}{4 + e^{-0.002x}}$$

$$\frac{4}{4 + e^{-0.002x}} = 0.88$$

$$4 = 3.52 + 0.88e^{-0.002x}$$

$$0.48 = 0.88e^{-0.002x}$$

$$\frac{6}{11} = e^{-0.002x}$$

$$\ln\frac{6}{11} = \ln e^{-0.002x}$$

$$\ln\frac{6}{11} = -0.002x$$

$$x = \frac{\ln(6/11)}{0.002} \approx 303 \text{ units}$$

(b) When $p = \$400$:

$$400 = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

$$0.08 = 1 - \frac{4}{4 + e^{-0.002x}}$$

$$\frac{4}{4 + e^{-0.002x}} = 0.92$$

$$4 = 3.68 + 0.92e^{-0.002x}$$

$$0.32 = 0.92e^{-0.002x}$$

$$\frac{8}{23} = e^{-0.002x}$$

$$\ln\frac{8}{23} = \ln e^{-0.002x}$$

$$x = \frac{\ln(8/23)}{0.002} \approx 528 \text{ units}$$

124. $N = 68(10^{-0.04x})$

When $N = 21$:

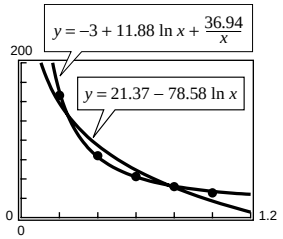
$$21 = 68(10^{-0.04x})$$

$$\frac{21}{68} = 10^{-0.04x}$$

$$\log_{10}\frac{21}{68} = -0.04x$$

$$x = -\frac{\log(21/68)}{0.04} \approx 12.76 \text{ inches}$$

126. (a) $y = 21.37 - 78.58 \ln x$



Answers will vary.

(b) Graph the models together with $y = 30$. they intersect at $y = 1.2$ and 0.9, respectively.
1.2 meters, 0.9 meters.
Answers will vary.