128. True

130. Yes. The doubling time is given by

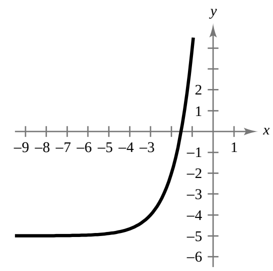
$$2P = Pe^{rt}$$
$$2 = e^{rt}$$
$$\ln 2 = rt$$
$$t = \frac{\ln 2}{r}$$

The time to quadruple is given by

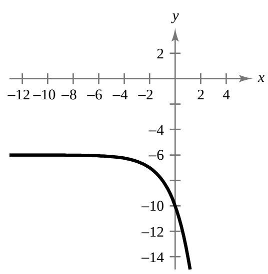
$$4P = Pe^{rt}$$
$$4 = e^{rt}$$
$$\ln 4 = rt$$
$$t = \frac{\ln 4}{r} = \frac{\ln 2^2}{\text{r}} = \frac{2 \ln 2}{r} = 2\left[\frac{\ln 2}{r}\right]$$

which is twice as long.

132. $f(x) = 3^{x+3} - 5$



134. $f(x) = -2^{x+2} - 6$



136. $\log_7 11 = \dfrac{\ln 11}{\ln 7} \approx 1.232$

138. $\log_9 6 = \dfrac{\ln 6}{\ln 9} = 0.815$

Section 3.5 Exponential and Logarithmic Models

Solutions to Even-Numbered Exercises

2. $y = 6e^{-x/4}$
This is an exponential decay model. Matches graph (e).

4. $y = 3e^{-(x+2)^2/5}$
Gaussian model
Matches (a)

6. $y = \dfrac{4}{1 + e^{-2x}}$
Logistics model
Matches (f)

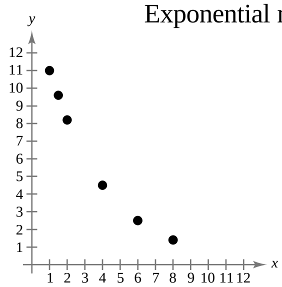
8. Linear model

10. Exponential model

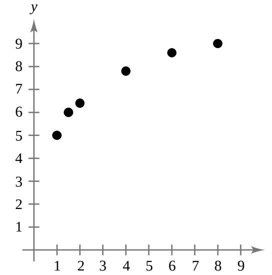
12. Logistics model

14. Linear model

16. Exponential model



18. Logarithmic model



20. Since $A = 20{,}000e^{0.105t}$, the time to double is given by $40{,}000 = 20{,}000e^{0.105t}$, and we have

$$t = \frac{\ln 2}{0.105} \approx 6.60 \text{ years.}$$

Amount after 10 years:

$$A = 20{,}000e^{0.105(10)} \approx \$57{,}153.02$$

22. Since $A = 10{,}000e^{rt}$ and $A = 20{,}000$ when $t = 12$, we have

$$20{,}000 = 10{,}000e^{12r}$$
$$2 = e^{12r}$$
$$r = \frac{\ln 2}{12} \approx 0.05776 \text{ or } 5.78\%$$

After 10 years, $A = 10{,}000e^{(0.05776)10} \approx \$17{,}817.57$.

24. Since $A = 600e^{rt}$ and $A = 1505.00$ when $t = 10$, we have

$$1505.00 = 600e^{r(10)} \implies r = \frac{1}{10}\ln\left(\frac{1505.00}{600}\right)$$
$$\approx 0.092 \text{ or } 9.2\%$$

Doubling time: $1200 = 600e^{rt}$

$$2 = e^{rt}$$
$$t = \frac{\ln 2}{r} = \frac{\ln 2}{0.092} \approx 7.5 \text{ years}$$

26. Since $A = Pe^{0.08t}$ and $A = 20{,}000$ when $t = 10$, we have

$$20{,}000 = Pe^{0.08(10)}$$
$$P = \frac{20{,}000}{e^{0.08(10)}} \approx \$8986.58.$$

The time to double is given by

$$t = \frac{\ln 2}{0.08} \approx 8.66 \text{ years.}$$

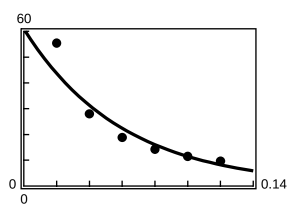
28.

$$3P = P(1 + r)^t$$
$$3 = (1 + r)^t$$
$$\ln 3 = \ln(1 + r)^t$$
$$\ln 3 = t\ln(1 + r)$$
$$\frac{\ln 3}{\ln(1 + r)} = t$$

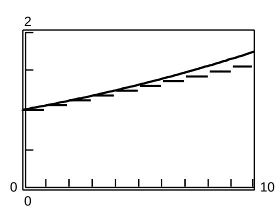
(a)

r	2%	4%	6%	8%	10%	12%
$t = \frac{\ln 3}{\ln(1 + r)}$	55.47	28.01	18.85	14.27	11.53	9.69

(b)



30.



From the graph, $5\frac{1}{2}\%$ compounded daily grows faster than 6% simple interest.

32.

$$\frac{1}{2}C = Ce^{k(1620)}$$
$$k = \frac{\ln 0.5}{1620}$$

Given $y = 1.5$ grams after 1000 years, we have

$$1.5 = Ce^{[(\ln 0.5)/1620](1000)}$$
$$C \approx 2.30 \text{ grams.}$$

34.

$$\frac{1}{2}C = Ce^{k(24{,}360)}$$
$$k = \frac{\ln 0.5}{24{,}360}$$

Given $y = 0.4$ grams after 1000 years, we have

$$0.4 = Ce^{[(\ln 0.5)/24{,}360](1000)}$$
$$C \approx 0.41 \text{ grams.}$$

36.

$$P = 240{,}360e^{0.012t}$$
$$275{,}000 = 240{,}360e^{0.012t}$$
$$\ln\frac{27{,}500}{24{,}036} = 0.012t$$
$$t = \frac{\ln(27{,}500/24{,}036)}{0.012} \approx 11$$

The population will reach 275,000 in 2011.

38. 1997: $t = 0$; 2020: $t = 23$. Model $y = ae^{bt}$

(a) *Croatia*: For $t = 0$, $5.0 = ae^{b(0)} = a$

$$\text{For } t = 23,\ 4.8 = 5.0e^{b(23)} \implies b = \frac{1}{23}\ln\left(\frac{4.8}{5.0}\right) \approx -0.00177$$

$$y = 5.0e^{-0.00177t}$$

In 2030, $t = 33$ and $y \approx 4.7$ million.

Mali: $a = 9.9$. For $t = 23$, $20.4 = 9.9e^{23(b)} \implies b = 0.0314$

$$y = 9.9e^{0.0314t}$$

In 2030, $t = 33$ and $y \approx 27.9$ million.

Singapore: $a = 3.5$. For $t = 23$, $4.3 = 3.5e^{23b} \implies b = 0.00895$

In 2030, $t = 33$ and $y \approx 4.7$ million.

Sweden: $a = 8.9$. For $t = 23$, $9.5 = 8.9e^{23b} \implies b = 0.00284$

In 2030, $t = 33$ and $y \approx 9.8$ million.

(b) b gives the growth rate.

(c) The negative value for b (Croatia) indicates that the population is decreasing.

40.
$$N = 250e^{kt}$$
$$280 = 250e^{k(10)}$$
$$k = \frac{\ln 1.12}{10}$$
$$N = 250e^{[(\ln 1.12)/10]t}$$
$$500 = 250e^{[(\ln 1.12)/10]t}$$
$$t = \frac{\ln 2}{(\ln 1.12)/10} \approx 61.16 \text{ hours}$$

42.
$$y = Ce^{kt}$$
$$\frac{1}{2}C = Ce^{5730k}$$
$$\ln\frac{1}{2} = 5730k$$
$$k = \frac{\ln(1/2)}{5730}$$

The ancient charcoal has only 15% as much radioactive carbon.

$$0.15C = Ce^{[(\ln 0.5)/5730]t}$$
$$\ln 0.15 = \frac{\ln 0.5}{5730}t$$
$$t = \frac{5730 \ln 0.15}{\ln 0.5} \approx 15{,}683 \text{ years}$$

44. (a) $V = mt + b$

$V(0) = 4600 \implies b = 4600$

$V(2) = 3000 \implies 3000 = m(2) + 4600 \implies m = -800$

$V = -800t + 4600$

(b) $V = ae^{kt}$

$V(0) = 4600 \implies a = 4600$

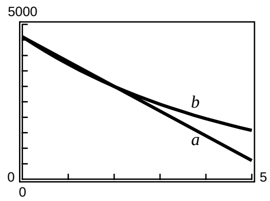
$V(2) = 3000 \implies 3000 = 4600e^{2k} \implies k = \frac{1}{2}\ln\left(\frac{30}{46}\right) \approx -0.2137$

$V = 4600e^{-0.2137t}$

—CONTINUED—

44. —CONTINUED—

(c)



The models depreciate the same in 2 years.

(d) Linear: $V(1) = 3800$

$V(3) = 2200$

Exponential: $V(1) = 3714.92$

$V(3) = 2422.88$

(e) The slope of the linear model, -800, is the annual decrease in book value.

46. $S = 10(1 - e^{kx})$

$x = 5$ (in hundreds), $S = 2.5$ (in thousands)

(a) $2.5 = 10(1 - e^{k(5)})$

$0.25 = 1 - e^{5k}$

$e^{5k} = 0.75$

$5k = \ln 0.75$

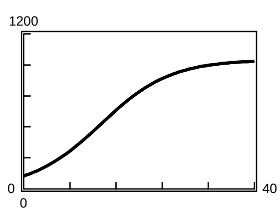
$k \approx -0.0575$

$S = 10(1 - e^{-0.0575x})$

(b) When $x = 7$, $S = 10(1 - e^{-0.0575(7)}) \approx 3.314$
which corresponds to 3314 units.

48. $p(t) = \dfrac{1000}{1 + 9e^{-0.1656t}}$

(a)



The horizontal asymptotes are $y = 0$ and $y = 1000$. The asymptote with the larger p-value, $y = 1000$, indicates that the population size will approach 1000 as time increases.

(b) $p(5) = \dfrac{1000}{1 + 9e^{-0.1656(5)}} \approx 203$ animals

(c) $500 = \dfrac{1000}{1 + 9e^{-0.1656t}}$

$1 + 9e^{-0.1656t} = 2$

$9e^{-0.1656t} = 1$

$e^{-0.1656t} = \dfrac{1}{9}$

$t = -\dfrac{\ln(1/9)}{0.1656} \approx 13$ months

50. $R = \log_{10} \dfrac{I}{I_0} = \log_{10} I$ since $I_0 = 1$.

(a) $8.6 = \log_{10} I$

$10^{8.6} = I \approx 398{,}107{,}171$

(b) $6.7 = \log_{10} I$

$10^{6.7} = I \approx 5{,}011{,}872$

52. $\beta(I) = 10 \log_{10}\left(\frac{I}{I_0}\right) = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$

(a) $\beta(10^{-3.5}) = 10 \log_{10}\left(\frac{10^{-3.5}}{10^{-12}}\right) = 10 \log_{10}(10^{8.5}) = 10(8.5) = 85$ decibels

(b) $\beta(10^{-3}) = 10 \log_{10}\left(\frac{10^{-3}}{10^{-12}}\right) = 10(9) = 90$ decibels

(c) $\beta(10^{-1.5}) = 10 \log_{10}\left(\frac{10^{-1.5}}{10^{-12}}\right) = 10(10.5) = 105$ decibels

54.
$$\beta = 10 \log_{10} \frac{I}{I_0}$$
$$10^{\beta/10} = \frac{I}{I_0}$$
$$I = I_0\, 10^{\beta/10}$$
$$\%\text{ decrease} = \frac{I_0\, 10^{8.8} - I_0\, 10^{7.2}}{I_0\, 10^{8.8}} \times 100 \approx 97\%$$

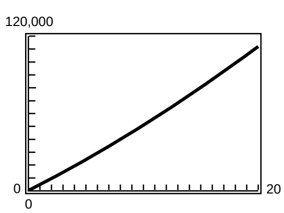
56.
$$5.8 = -\log_{10}[H^+]$$
$$10^{-5.8} = H^+$$
$$H^+ \approx 1.58 \times 10^{-6} \text{ moles per liter}$$

58.
$$pH - 1 = \log_{10}[H^+]$$
$$-(pH - 1) = \log_{10}[H^+]$$
$$10^{-(pH-1)} = [H^+]$$
$$10^{-pH+1} = [H^+]$$
$$10^{-pH} \cdot 10 = [H^+]$$

The hydrogen ion concentration is increased by a factor of 10.

60. $u = 120{,}000\left[\frac{0.075t}{1 - \left(\frac{1}{1 + 0.75/12}\right)^{12t}} - 1\right]$

(a)



(b) From the graph, when $u = 120{,}000$, $t \approx 21.2$ years. Yes, a mortgage of approximately 37.6 years will result in about \$240,000 of interest.

62. $(0, 4) \implies a = 4$

$(5, 1) \implies 1 = 4e^{b(5)} \implies b = \frac{1}{5}\ln\left(\frac{1}{4}\right)$

$= -\frac{1}{5}\ln 4 \approx -0.2773$

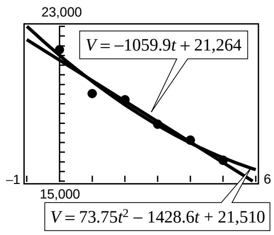
$y = 4e^{-0.2773x}$

64.
$$y = ae^{bx}$$
$$1 = ae^{b(0)} \implies 1 = a$$
$$\frac{1}{4} = e^{b(3)}$$
$$\ln\left(\frac{1}{4}\right) = 3b$$
$$\frac{\ln(1/4)}{3} = b \implies b \approx -0.4621$$

Thus,
$y = e^{-0.4621x}$.

66. (a) Linear Model: $V = 21{,}263.81 - 1059.86t$

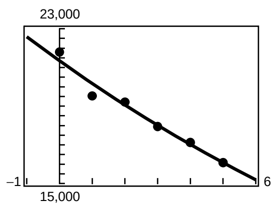
Quadratic Model: $V = 73.75t^2 - 1428.61t + 21509.64$



(b) The slope represents the average depreciation per year.

(c) No, it will ultimately rise to the right.

(d) Exponential Model: $V = 21{,}345.69(0.94487)^t$



The model fits well.

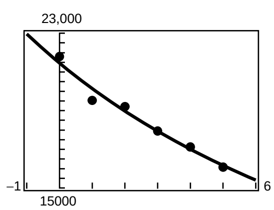
(e)

t	0	1	2	3	4	5
$\frac{1}{V}$	4.587	5.120	5.206	5.573	5.841	6.219

$\times 10^{-5}$

$$\frac{1}{V} = 4.661 \times 10^{-5} + 3.054 \times 10^{-6}t$$

$$V = \frac{1}{4.661 \times 10^{-5} + 3.054 \times 10^{-6}t} = \frac{10^9}{46610 + 3054t}$$



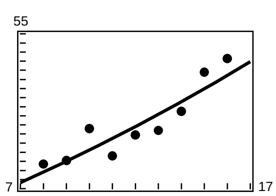
Fits well.

(f) $V = 0$. As time increases, the value approaches zero.

68. (a) Exponential Model: $y = 31.432(1.030)^t$

Linear Model: $y = 28.68 + 1.368t$

(b)

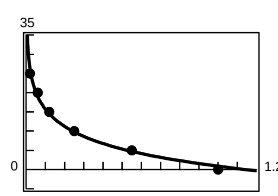


(c) For 2005, $t = 25$ and $y \approx 62.9$ million (linear model) and $y \approx 65.8$ million (exponential model).

70. (a) $h = 0$ is not in the domain of the logarithmic function.

(b) $h = 0.863 - 6.447 \ln p$

(c)



(d) If $p = 0.75$, then $h = 2.72$ kilometers.

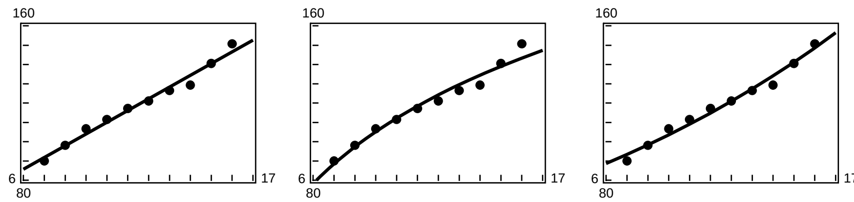
(e) If $h = 13$, then $p = 0.15$ atmosphere.

72. (a) $y_1 = 6.091x + 49.115$

$y_2 = -40.733 + 66.366 \ln x$

$y_3 = 65.046(1.053)^x$

(b)



(c)

	$y - y_1$	$(y - y_1)^2$	$(y - y_2)$	$(y - y_2)^2$	$(y - y_3)$	$(y - y_3)^2$
7	-1.752	3.0695	1.5907	2.5304	-3.373	11.374
8	0.257	.06605	0.8288	0.6869	-0.2213	0.0490
9	2.766	7.6508	1.612	2.5985	3.1677	10.034
10	1.475	2.1756	-0.5804	0.3368	2.4805	6.1527
11	1.084	1.1751	-1.206	1.4538	2.4024	5.7717
12	-1.107	1.2254	-3.08	9.4883	0.2182	0.0476
13	-1.798	3.2328	-2.992	8.9546	-0.7886	0.62185
14	-5.089	25.898	-5.111	26.119	-4.735	22.419
15	0.02	0.0004	1.511	2.2817	-0.6387	0.4080
16	4.129	17.049	7.427	55.166	2.0809	4.3303

(d)

Sum $(y - y_1)$	Sum $(y - y_1)^2$	Sum $(y - y_2)$	Sum $(y - y_2)^2$	Sum $(y - y_3)$	Sum $(y - y_3)^2$
-0.015	61.54	-8.2×10^{-5}	109.62	0.594	61.21

Models y_1 and y_3 seem best.

(e) The sums represent the sum of the errors and the sum of the squares of the errors, which is more useful.

74. False. The domain could be all real numbers.

76. True. For the Gaussian model, $y > 0$.

78. False. It is shifted vertically upwards 5 units.

80. Line with intercepts $(5, 0)$ and $(0, 2)$. Matches (c)

82. Line with intercepts $(2, 0)$ and $(0, 4)$. Matches (e).

84. Vertical line. Matches (a). Intercept $(-2, 0)$.

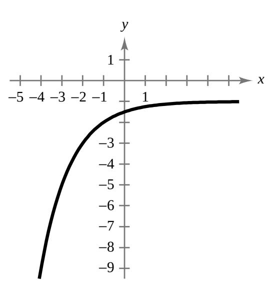
86.
$$\begin{array}{r|rrrr} \frac{3}{2} & 8 & -36 & 54 & -27 \\ & & 12 & -36 & 27 \\ \hline & 8 & -24 & 18 & 0 \end{array}$$

$$\frac{8x^3 - 36x^2 + 54x - 27}{x - \frac{3}{2}} = 8x^2 - 24x + 18, \ x \neq \frac{3}{2}$$

88.
$$\begin{array}{r|rrrrr} -5 & 1 & 0 & 0 & -3 & 1 \\ & & -5 & 25 & -125 & 640 \\ \hline & 1 & -5 & 25 & -128 & 641 \end{array}$$

$$\frac{x^4 - 3x + 1}{x + 5} = x^3 - 5x^2 + 25x - 128 + \frac{641}{x + 5}$$

90. $f(x) = -2^{x-1} - 1$



92. $f(x) = -3^x + 4$