Chapter P Prerequisites

Course/Section
Lesson Number
Date

Section P.1 Graphical Representation of Data

Section Objectives: Students will know how to plot points in the coordinate plane and use the Distance and Midpoint Formulas.

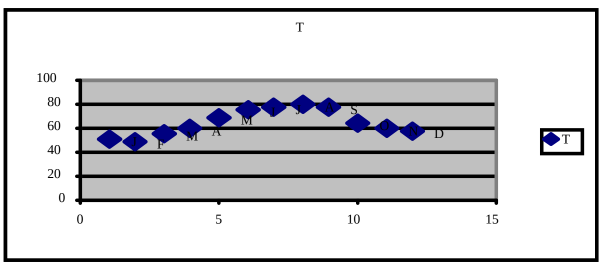
I. The Cartesian Plane (pp. 2 - 3) Pace: 5 minutes

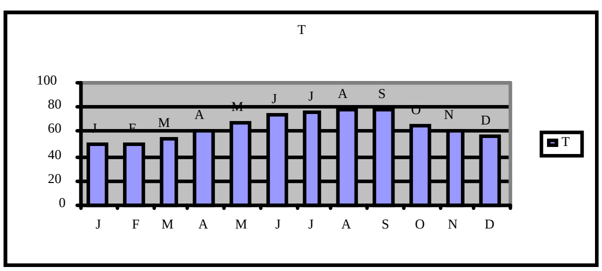
- Draw two real number lines, one horizontal and the other vertical, intersecting at their origins. This forms the rectangular coordinate system. The point of intersection of the two lines is the origin. The horizontal line is called the x-axis, while the vertical is the y-axis. The lines divide the plane into four sections called quadrants. Points in the rectangular coordinate system are represented by ordered pairs of real numbers, (x, y). The x-coordinate represents the directed distance from the y-axis to the point, and y-coordinate represents the directed distance from the x-axis to the point.

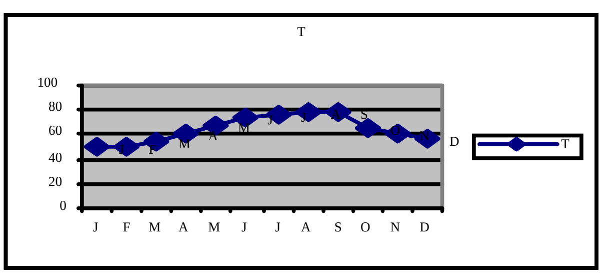
II. Representing Data Graphically (pp. 4 - 5) Pace: 5 minutes

Example 1. In the table below, there are the average monthly temperatures for a small southern California town. Sketch a scatter plot, a bar graph, and a line graph of the data.

m	J	F	M	A	M	J	J	A	S	O	N	D
T	51	50	55	60	68	75	77	79	78	65	60	57







III. The Distance Formula (pp. 5 – 6) Pace: 10 minutes

- Use the Pythagorean Theorem to develop the distance formula as follows. Plot two arbitrary points and label them (x_1, y_1) and (x_2, y_2). Use the line segment between the two points as the hypotenuse of the right triangle, formed by a horizontal and a vertical leg, with these points at the acute vertices. Then

$$c^2 = a^2 + b^2$$
$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2. Find the distance between (2, -5) and (8, 3).

$$\begin{aligned} d &= \sqrt{(8-2)^2 + (3-(-5))^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

Example 3. Show that the points (1, -3), (3, 2), and (-2, 4) from an isosceles triangle.

$$d_1 = \sqrt{(3-1)^2 + (2-(-3))^2} = \sqrt{29}$$
$$d_2 = \sqrt{(-2-3)^2 + (4-2)^2} = \sqrt{29}$$

Since two of the sides have equal length, the triangle is isosceles.

IV. **The Midpoint Formula** (p. 7) Pace: 5 minutes

- The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

We can show this by showing that the distance from each endpoint to the midpoint is the same, and that the distance from one endpoint to the midpoint is half the distance between the two endpoints.

$$d_1 = \sqrt{\left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_1+y_2}{2} - y_1\right)^2}$$

$$= \sqrt{\left(\frac{x_1+x_2}{2} - \frac{2x_1}{2}\right)^2 + \left(\frac{y_1+y_2}{2} - \frac{2y_1}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$

$$d_2 = \sqrt{\left(x_2 - \frac{x_1+x_2}{2}\right)^2 + \left(y_2 - \frac{y_1+y_2}{2}\right)^2}$$

$$= \sqrt{\left(\frac{2x_2}{2} - \frac{x_1+x_2}{2}\right)^2 + \left(\frac{2y_2}{2} - \frac{y_1+y_2}{2}\right)^2}$$

$$= \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$

and

$$d_1 = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4}(x_2-x_1)^2 + \frac{1}{4}(y_2-y_1)^2}$$

$$= \frac{1}{2}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Example 4. Find the midpoint of the line segment joining the points (-9, 5) and (4, 2).

$$\left(\frac{-9+4}{2}, \frac{5+2}{2}\right) = \left(\frac{5}{2}, \frac{7}{2}\right)$$

V. The Equation of a Circle (p. 8) Pace: 5 minutes

A circle with center at (h, k) and a radius r, consists of all points (x, y), whose distance from (h, k) is r. From the distance formula, we have $\sqrt{(x-h)^2+(y-k)^2} = r \Rightarrow (x-h)^2+(y-k)^2 = r^2$ as the standard form equation of a circle.

Example 5. Find the standard form of the equation of the circle with center at (2, -5) and radius 4.

$(x-2)^2+(y-(-5))^2 = 4^2$

or

$(x-2)^2+(y+5)^2 = 16$