Chapter P Prerequisites

Course/Section
Lesson Number
Date

Section P.3 Lines in the Plane

Section Objectives: Students will know how to find and use the slopes of lines to write and graph linear equations.

I. The Slope of a Line (pp. 25 - 26) Pace: 10 minutes

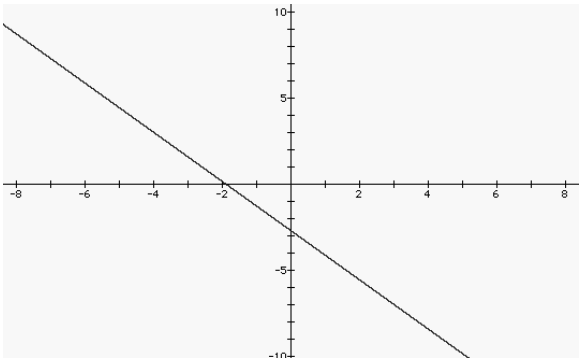
- Define the slope of a line to be the ratio of the change in y to the change in x. In addition, if we know two points on the line , (x_1, y_1) and (x_2, y_2), then the change in y is $y_2 - y_1$ and the change in x is $x_2 - x_1$. Therefore, the slope m of a nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Example 1. Find the slope of the line through each pair of points.

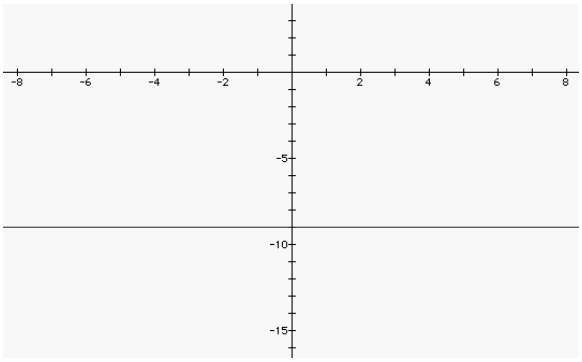
a) (3, -7) and (-4, 2)

$$m = \frac{3-(-7)}{-4-3} = -\frac{10}{7}$$



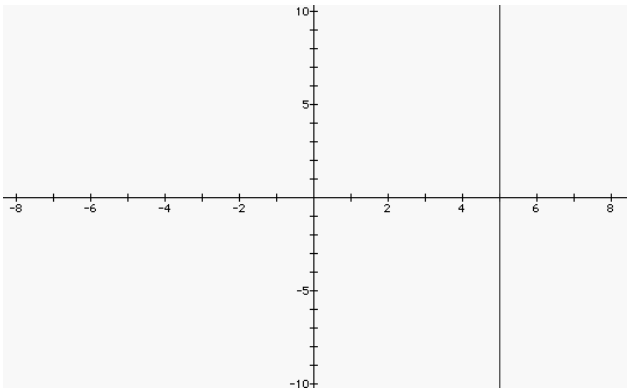
b) (2, -9) and (-6, -9)

$m = \frac{-9-(-9)}{-6-2} = \frac{0}{-8} = 0$. This line is horizontal.



c) (5, 4) and (5, -8)

$$m = \frac{-8-4}{5-5} = \frac{-12}{0}, \text{ no slope}$$



II. The Point-Slope Form of the Equation of a Line (pp. 27 - 28)
Pace: 15 minutes

- Discuss the following: if (x, y) is any other point on a line that contains (x_1, y_1) and with slope m, then

$$\frac{y-y_1}{x-x_1} = m, \text{ or } y - y_1 = m(x - x_1)$$

This is called point-slope form. State it as follows.
Point-Slope Form: An equation of the line through the point (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$.

Example 2. Find the slope-intercept form of the equation of the line with slope 4 that passes through the point (-6, 2).

$$y - 2 = 4(x - (-6))$$
$$y - 2 = 4x + 24$$
$$y = 4x + 26$$

Tip: Inform the students that any time they are instructed to find the equation of a line, they should think of point-slope form first.

Example 3. A company purchases a $20,000 machine. In 4 years the machine will be worth $10,000. Write a linear equation that relates the value V of the machine after t years.
First find the slope of the line through (0, 20,000) and (4, 10,000).

$$m = \frac{10,000 - 20,000}{4 - 0} = -\frac{10,000}{4} = -2500$$
$$V - 20,000 = -2500(t - 0)$$
$$V = -2500t + 20,000$$

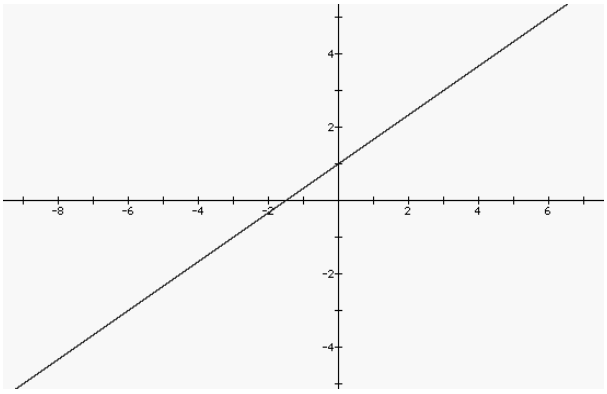
III. Sketching Graphs of Lines (pp. 29 - 30) Pace: 10 minutes

- Consider the line given by the linear equation $y = mx + b$. By replacing x with zero, we see that the y-intercept of the line is $(0, b)$. Note that $(1, m + b)$ is also a point on the line. From this we can see that the slope of the line is m, since a one unit change in x produces a m unit change in y. State the following definition.

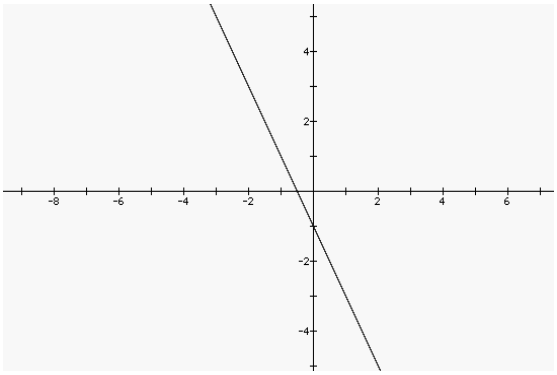
The graph of an equation of the form $y = mx + b$ is a line with slope m and y-intercept $(0, b)$. This form is called **slope-intercept form**.

Example 4. Sketch the graph of the following.

a) $y = \frac{2}{3}x + 1$. Plot the *y*-intercept (0, 1). From this point go up 2 and to the right 3. This produces another point on the line. Now draw the line through these two points.



b) $y = -2x - 1$. Plot the *y*-intercept (0, -1). From this point go down 2 and to the right 1. This produces another point on the line. Now draw the line through these two points.



- Discuss the *Exploration* on page 30 of the text.

IV. Parallel and Perpendicular Lines (pp. 31 - 32) Pace: 10 minutes

- State the following two facts.
  1. Two distinct nonvertical lines are parallel if and only if their slopes are equal. That is $m_1 = m_2$.
  2. Two nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of each other. That is $m_1 = 1/m_2$.

Example 5. Find the general form of the equation of the line that passes through the point (1, -3) and is (a) parallel to and (b) perpendicular to the line given by $2x - 3y = 1$.
First find the slope of the given line by writing it in slope-intercept form.

$$2x - 3y = 1$$
$$3y = -2x + 1$$
$$y = -\frac{2}{3}x + \frac{1}{3}$$

The slope of this line is –2/3.

$$y - (-3) = -\frac{2}{3}(x - 1)$$

a) $3y - 9 = -2x + 2$

$2x + 3y - 11 = 0$

$$y - (-3) = \frac{3}{2}(x - 1)$$

b) $2y + 6 = 3x - 3$

$3x - 2y - 9 = 0$