Chapter P Prerequisites

Course/Section
Lesson Number
Date

Section P.4 Solving Equations Algebraically and Graphically

Section Objectives: Students will know how to solve linear equations, quadratic equations, polynomial equations, equations involving radicals, equations involving fractions, and equations involving absolute value.

I. Equations and Solutions of Equations (pp. 38 - 39) Pace: 5 minutes

- An **equation** in *x* is a statement that variable expressions are equal. A **solution** of an equation is a number *r*, such that when *x* is replaced by *r* the resulting equation is a true statement. The **solution set** of an equation is the set of all solutions of the equation. Two equations are said to be **equivalent** if they have the same solution set. To **solve** an equation means to find its solution set. We will encounter two types of equations. One is the **identity**, in which every real number in the domain of the variable is a solution. The other is the **conditional equation**, in which only some of the numbers in the domain of the variable are solutions. This second type is the type that we solve.

Example 1. Solve

a)

$$\frac{x}{3} - \frac{2x}{5} = \frac{1}{6}$$

$$30\left(\frac{x}{3} - \frac{2x}{5}\right) = 30 \cdot \frac{1}{6}$$

$$10x - 12x = 5$$

$$-2x = 5$$

$$x = -\frac{5}{2}$$

b)

$$\frac{x}{x-1} = \frac{1}{x-1} - \frac{1}{x-3}$$

$$(x-1)(x-3) \cdot \frac{x}{x-1} = (x-1)(x-3) \cdot \left(\frac{1}{x-1} - \frac{1}{x-3}\right)$$

$$x(x-3) = (x-3) - (x-1)$$

$$x^2 - 3x = -2$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x = 2$$

Note that $x = 1$ is an extraneous solution since it causes division by zero.

II. Intercepts and Solutions (pp. 39 - 41) Pace: 10 minutes

- A point at which the graph of an equation meets the *x*-axis is called an *x*-intercept. Since all points on the *x*-axis have a *y*-coordinate of zero, we find *x*-intercepts by replacing *y* with zero and solving for *x*. A point at which the graph of an equation meets the *y*-axis is called a *y*-intercept. Since all points on the *y*-axis have a *x*-coordinate of zero, we find *y*-intercepts by replacing *x* with zero and solving for *y*.

Example 2. Find the *x*- and *y*-intercepts of the graph of each equation.

a) $2x + 3y = 6$

To find the *y*-intercept, let $x = 0$ and solve for *y*.

$$2(0) + 3y = 6$$
$$3y = 6$$
$$y = 2$$

Hence (0, 2) is the *y*-intercept.

To find the *x*-intercept, let $y = 0$ and solve for *x*.

$$2x + 3(0) = 6$$
$$2x = 6$$
$$x = 3$$

Hence (3, 0) is the *x*-intercept.

b) $y = x^2 + x - 6$

To find the *y*-intercept, let $x = 0$ and solve for *y*.

$$y = 0^2 + 0 - 6 = -6$$

Hence (0, -6) is the *y*-intercept.

To find the *x*-intercept, let $y = 0$ and solve for *x*.

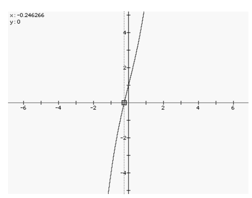
$$0 = x^2 + x - 6$$
$$0 = (x + 3)(x - 2)$$
$$x = -3 \text{ or } x = 2$$

Hence (-3, 0) and (2, 0) are the *x*-intercepts.

III. Finding Solutions Graphically (pp. 41 - 42) Pace: 5 minutes

- State the following steps for **graphical approximations of solutions of an equation**.
- Write the equation in *general form*, $f(x) = 0$.
- Use a graphing utility to graph the function $y = f(x)$. Be sure the viewing window shows all the relevant features of the graph.
- Use the *zero* or *root* feature or the *zoom* and *trace* features of the graphing utility to approximate the *x*-intercept of the graph of *f*.

Example 3. Use a graphing utility to approximate the solutions of $x^3 + 4x + 1 = 0$.



IV. Points of Intersection of Two Graphs (pp. 43 - 45) Pace: 10 minutes

- State that the points at which two graphs of equations meet are called **points of intersection.** Their corresponding ordered pairs are solutions to both the equations.

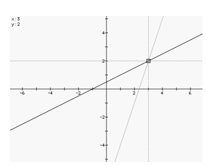
Example 4. Find the points of intersection of the graphs of $x - 2y = -1$ and $3x - y = 7$.

Algebraic Solution

$y = 0.5x + 0.5$
$y = 3x - 7$
$0.5x + 0.5 = 3x - 7$
$x = 3$
$y = 2$

Point of intersection is (3, 2).

Graphical Solution



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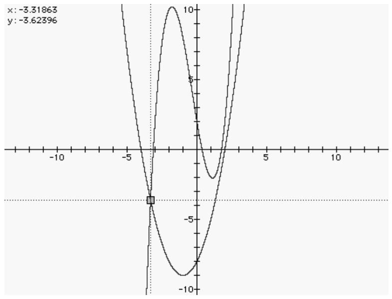
Example 5. Approximate the points of intersection of the graphs of the following equations.

$y = x^2 + 2x - 8$
$y = x^3 + x^2 - 6x + 2$



Point of intersection is (-3.31863, -3.62396).

V. Solving Polynomial Equations Algebraically (pp. 45 - 46)

Pace: 20 minutes

- Define a quadratic equation to be any equation that can be written equivalently in the form
$$ax^2 + bx + c = 0$$
where a, b, and c are real numbers with $a \neq 0$.

- We will look at three different ways of solving these equations in this section. The first is by factoring. This method uses the Zero-Factor Property, which states if $ab = 0$, then $a = 0$ or $b = 0$.

Example 6. Solve by factoring.

$$x^2 + 7x + 12 = 0$$
$$(x+3)(x+4) = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$
$$x + 4 = 0 \Rightarrow x = -4$$

Tip: Use a graphing utility to show that these solutions are x-intercepts.

- State the fact that if $u^2 = d$, where $d > 0$ and u is an algebraic expression, then $u = \pm\sqrt{d}$. Solving equations using this fact is called **extracting square roots**.

Tip: Students have a tendency to forget the "±." You may have to go to great lengths to overcome this.

Example 7. Solve by extracting square roots.

$$16x^2 = 25$$
$$x^2 = \frac{25}{16}$$
$$x = \pm\sqrt{\frac{25}{16}} = \pm\frac{5}{4}$$

- State the following fact: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$. Hence, if we have $x^2 + bx = c$, add $(b/2)^2$ to both sides, rewrite the left side, and solve by extracting square roots.

Example 8. Solve by completing the square.

$$x^2 + 4x = 5$$
$$x^2 + 4x + 4 = 5 + 4$$
$$(x+2)^2 = 9$$
$$x + 2 = \pm 3$$
$$x = -2 \pm 3$$

The solutions are –5 and 1.

- State the **Quadratic Formula.**
  The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are
  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 9. Solve by using the Quadratic Formula.

$$3x^2 - x - 5 = 0$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-5)}}{2(3)}$$
$$x = \frac{1 \pm \sqrt{61}}{6}$$

- Now, we look at polynomial equations with higher degree. Our main tool will be the Zero-Product Property.

Example 10. Solve

a)

$$x^3 = 9x$$
$$x^3 - 9x = 0$$
$$x\left(x^2 - 9\right) = 0$$
$$x(x+3)(x-3) = 0$$
$$x = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$
$$x - 3 = 0 \Rightarrow x = 3$$

Tip**:** Remind your students not to divide by *x;* they will lose a solution that way.

b)

$$x^3 - x^2 - 4x + 4 = 0$$
$$x^2(x-1) - 4(x-1) = 0$$
$$\left(x^2 - 4\right)(x-1) = 0$$
$$(x+2)(x-2)(x-1) = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x - 1 = 0 \Rightarrow x = 1$$

VI. Other Types of Equations (p. 47 - 49) Pace: 10 minutes

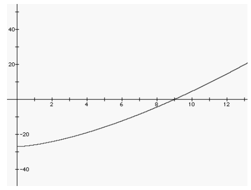
Tip**:** Students should be warned that the following methods can produce extraneous roots. Therefore all alleged solutions should be checked in the original equation.

Example 11. Solve

a) ***Algebraic Solution***

$$x^{3/2} - 27 = 0$$
$$x^{3/2} = 27$$
$$x = 27^{2/3} = 9$$

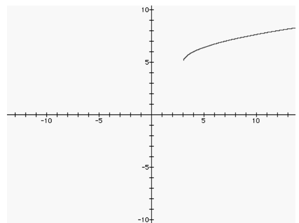
Graphical Solution



b) ***Algebraic Solution***

$$\sqrt{x-3} + 5 = 0$$
$$\sqrt{x-3} = -5$$
$$\sqrt{x-3}^{\,2} = (-5)^2$$
$$x - 3 = 25$$
$$x = 28$$

Graphical Solution



By checking 28 in the original equation, we can see that there are no solutions.

c)

$$\sqrt{x-5} + \sqrt{x+7} = 6$$
$$\sqrt{x-5} = 6 - \sqrt{x+7}$$
$$\sqrt{x-5}^{\,2} = \left(6 - \sqrt{x+7}\right)^2$$
$$x - 5 = 36 - 12\sqrt{x+7} + (x+7)$$
$$-48 = -12\sqrt{x+7}$$
$$4^2 = \sqrt{x+7}^{\,2}$$
$$16 = x + 7$$
$$9 = x$$

This value checks in the original equation.

- Remind the students that the first step in solving equations that contain fractions is to multiply both sides by the LCD.

Example 12. Solve

$$\frac{x}{x-1} - \frac{6}{x} = 2$$
$$x(x-1)\left(\frac{x}{x-1} - \frac{6}{x}\right) = 2x(x-1)$$
$$x^2 - 6(x-1) = 2x^2 - 2x$$
$$x^2 + 4x = 6$$
$$x^2 + 4x + 4 = 6 + 4$$
$$(x+2)^2 = 10$$
$$x + 2 = \pm\sqrt{10}$$
$$x = -2 \pm \sqrt{10}$$

- To solve equations involving absolute values, we rewrite it equivalently as two separate equations. For instance:

$$|x + 4| = 5 \Leftrightarrow x + 4 = 5 \text{ and } x + 4 = -5.$$

Example 13. Solve

$$|x^2 - 6| = x$$

$$x^2 - 6 = x$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x - 3 = 0 \Rightarrow x = 3$$

$$x + 2 = 0 \Rightarrow x = -2$$

and

$$x^2 - 6 = -x$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x + 3 = 0 \Rightarrow x = -3$$

$$x - 2 = 0 \Rightarrow x = 2$$

The solutions are ±2 and ±3.