Chapter P Prerequisites

Course/Section
Lesson Number
Date

Section P.5 Solving Inequalities Algebraically and Graphically

Section Objectives: Students will know how to solve linear inequalities, inequalities involving absolute value, polynomial inequalities and rational inequalities.

I. Properties of Inequalities (pp. 54 - 55) Pace: 5 minutes

- We will start our discussion of inequalities with some terminology. A **solution** of an inequality is a number *r*, such that when *x* is replaced by *r* the resulting inequality is a true statement. The **solution set** of an inequality is the set of all solutions of the inequality. Two inequalities are said to be **equivalent** if they have the same solution set. The set of all points on the real number line that correspond to a solution of the inequality is the **graph of the inequality**.

- In order to solve inequalities we look at some properties of inequalities that are very similar to the properties of equations that we saw earlier in the chapter. Let *a, b, c,* and *d* be real numbers
  1. Transitive Property: If $a < b$ and $b < c$, then $a < c$
  2. Addition of Inequalities: If $a < b$ and $c < d$, then $a + c < b + d$
  3. Addition of a Constant: If $a < b$, then $a + c < b + c$
  4. Multiplication by a Constant: If $a < b$ and $c > 0$, then $ac < bc$
     If $a < b$ and $c < 0$, then $ac > bc$
- Note that these properties are all valid for the other three inequality symbols also. In addition, note that the main difference between solving an equation and solving an inequality, is with an inequality, when you multiply both sides by a negative number you must reverse the inequality symbol.

II. Solving a Linear Inequality (pp. 55 - 56) Pace: 10 minutes

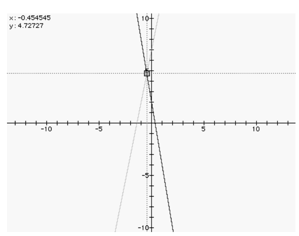
Example 1. Solve the following inequalities.

a)
$$2x+1 \le 5$$
$$2x \le 4$$
$$x \le 2$$
The solution set is (-∞, 2]

b) ***Algebraic Solution***

$$2-6x > 5x+7$$
$$2-11x > 7$$
$$-11x > 5$$
$$x < -\frac{5}{11}$$
The solution set is (-∞, -5/11)

Graphical Solution



c) ***Algebraic Solution***

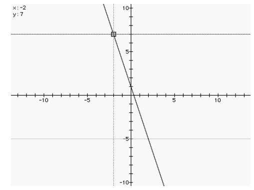
$-5 < 1 - 3x \le 7$

$-6 < -3x \le 6$

$2 > x \ge -2$

The solution set is [-2, 2)

Graphical Solution



III. **Inequalities Involving Absolute Value** (p. 57) Pace: 10 minutes

- Let x be a variable or a variable expression, and let $a \ge 0$.
  1. $|x| < a$ if and only if $x > -a$ and $x < a$ (i.e., $-a < x < a$)
  2. $|x| > a$ if and only if $x < -a$ or $x > a$

Example 2. Solve each inequality.

a) ***Algebraic Solution***

$|2 - x| < 5$

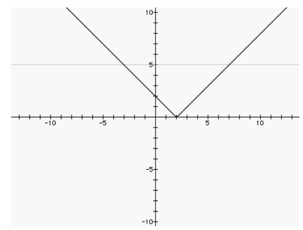
$-5 < 2 - x < 5$

$-7 < -x < 3$

$7 > x > -3$

The solution set is (-3, 7)

Graphical Solution



b) ***Algebraic Solution***

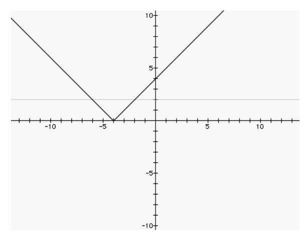
$|x + 4| \ge 2$

$x + 4 \le -2$ or $x + 4 \ge 2$

$x \le -6$ $\quad$ $x \ge -2$

The solution set is $(-\infty, -6] \cup [-2, \infty)$

Graphical Solution



IV. **Polynomial Inequalities** (pp. 58 – 60) Pace: 15 minutes

- Graph a polynomial such as $y = x^2 + x - 6$,using a graphing utility. Explain that are no sign changes between consecutive zeros of the polynomial. Hence, to solve a polynomial inequality we need to find the zeros of the polynomial, called **critical numbers** of the inequality, use them to create **test intervals**, and test a number from each test interval in the original inequality.

Example 3. Solve the following inequalities.

a) $x^2 + x - 6 > 0$

$(x + 3)(x - 2) > 0$ The critical numbers are –3 and 2.

Interval	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
x-value	-4	0	3
Result	6 > 0	-6 > 0	6 > 0

The solution set is $(-\infty, -3) \cup (2, \infty)$

b) ***Algebraic Solution***

$x^3 - 4x^2 - x \geq -4$

$x^3 - 4x^2 - x + 4 \geq 0$

$(x + 1)(x - 1)(x - 4) \geq 0$

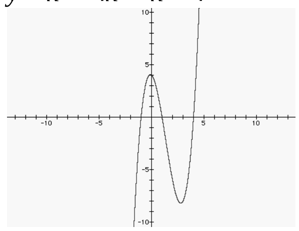
Critical Numbers: -1, 1, 4

Test Intervals: (-∞, -1), (-1, 1), (4, ∞)

The solution set is (-∞, -1] ∪ [1, 4]

Graphical Solution

$y = x^3 - 4x^2 - x + 4$



Tip: You check these solutions by graphing the polynomial with a graphing utility. While using the graphing utility, you could figure out the unusual solution sets obtained from:

a) $x^2 - 4x + 1 \geq 0$
b) $x^2 + 4x + 1 < 0$
c) $x^2 + 2x + 1 \leq 0$
d) $x^2 + 2x + 1 > 0$

V. Rational Inequalities (p. 61) Pace: 10 minutes

- Explain that the concept of critical numbers and test intervals can be extended to rational inequalities with one exception, rational expressions can also change signs at its undefined values. Therefore there are two types of critical numbers for rational inequalities.

Example 4. Solve

Algebraic Solution

$$\frac{2}{x-1} \geq -1$$

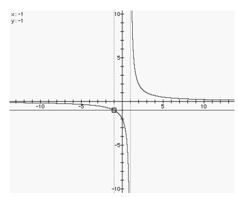
$$\frac{2}{x-1} + \frac{x-1}{x-1} \geq 0$$

$$\frac{x+1}{x-1} \geq 0$$

Interval	(-∞, -1)	(-1, 1)	(1, ∞)
x-value	-2	0	2
Result	1/3 ≥ 0	-1 ≤ 0	3 ≥ 0

The solution set is (-∞, -1] ∪ (1, ∞).

Graphical Solution



VI. Applications (p. 62) Pace: 5 minutes

Example 5. A projectile, fired upward with an initial velocity of 48 feet per second from ground level, can be modeled by the equation

$$h = -16t^2 + 48t$$

where *h* is the height of the projectile in feet and *t* is time in seconds. During which interval of time is the projectile higher than 32 feet?

$$-16t^2 + 48t > 32$$

$$t^2 - 3t + 2 < 0$$

$$(t-1)(t-2) < 0$$

Interval	(-∞, 1)	(1, 2)	(2, ∞)
x-value	0	1.5	3
Result	2 < 0	-0.25 < 0	2 < 0

The projectile will be higher than 32 feet between one and two seconds.