

C H A P T E R 1 0

Topics in Analytic Geometry

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CHAPTER 10

Topics in Analytic Geometry

Section 10.1 Introduction to Conics: Parabolas

Solutions to Even-Numbered Exercises

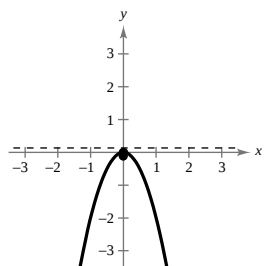
2. $x^2 = 2y$

Vertex: $(0, 0)$

$p = \frac{1}{2} > 0$

Opens upward.

Matches graph (b).



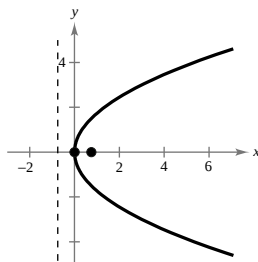
4. $y^2 = 12x$

Vertex: $(0, 0)$

$p = 3 > 0$

Opens to the right.

Matches graph (f).



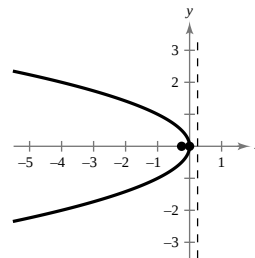
6. $(x + 3)^2 = -2(y - 1)$

Vertex: $(-3, 1)$

$p = -\frac{1}{2} < 0$

Opens downward.

Matches graph (c).

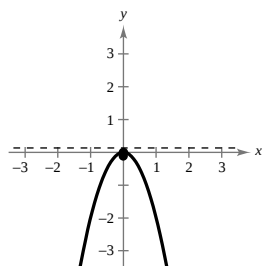


8. $y = 2x^2 \Rightarrow x^2 = -\frac{1}{2}y = 4(-\frac{1}{8})y$

Vertex: $(0, 0)$

Focus: $(0, -\frac{1}{8})$

Directrix: $y = \frac{1}{8}$

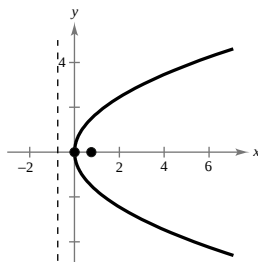


10. $y^2 = 3x \Rightarrow 4(\frac{3}{4})x$

Vertex: $(0, 0)$

Focus: $(\frac{3}{4}, 0)$

Directrix: $x = -\frac{3}{4}$



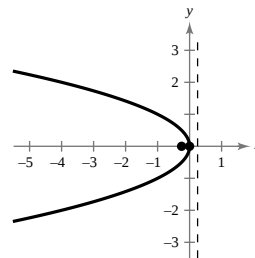
12. $x + y^2 = 0$

$y^2 = -x = 4(-\frac{1}{4})x$

Vertex: $(0, 0)$

Focus: $(-\frac{1}{4}, 0)$

Directrix: $x = \frac{1}{4}$



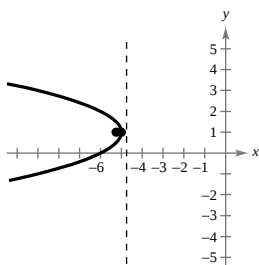
14. $(x + 5) + (y - 1)^2 = 0$

$(y - 1)^2 = -(x + 5) = 4(-\frac{1}{4})(x + 5)$

Vertex: $(-5, 1)$

Focus: $(-5 - \frac{1}{4}, 1) = (-\frac{21}{4}, 1)$

Directrix: $x = -5 + \frac{1}{4} = -\frac{19}{4}$

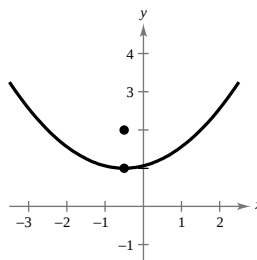


16. $(x + \frac{1}{2})^2 = 4(y - 1) \Rightarrow p = 1$

Vertex: $(-\frac{1}{2}, 1)$

Focus: $(-\frac{1}{2}, 1 + 1) = (-\frac{1}{2}, 2)$

Directrix: $y = 0$



18. $4x - y^2 - 2y - 33 = 0$

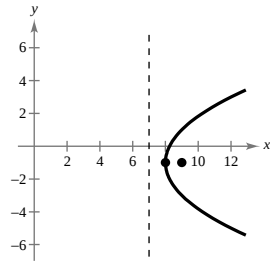
$$y^2 + 2y + 1 = 4x - 33 + 1$$

$$(y + 1)^2 = 4(1)(x - 8)$$

Vertex: $(8, -1)$

Focus: $(9, -1)$

Directrix: $x = 7$



20. $y^2 - 4y - 4x = 0$

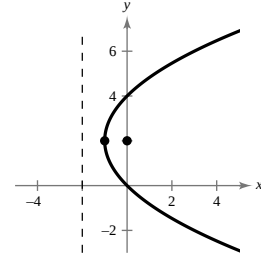
$$y^2 - 4y + 4 = 4x + 4$$

$$(y - 2)^2 = 4(1)(x + 1)$$

Vertex: $(-1, 2)$

Focus: $(0, 2)$

Directrix: $x = -2$



22. $x^2 - 2x + 8y + 9 = 0$

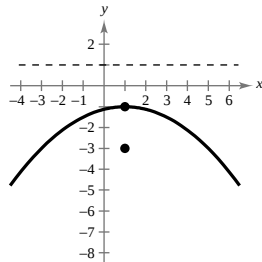
$$x^2 - 2x + 1 = -8y - 9 + 1$$

$$(x - 1)^2 = -8(y + 1) = 4(-2)(y + 1)$$

Vertex: $(1, -1)$

Focus: $(1, -3)$

Directrix: $y = 1$



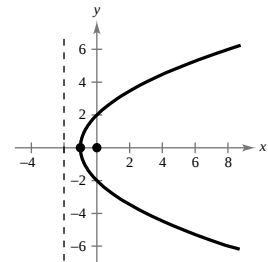
24. $y^2 - 4x - 4 = 0$

$$y^2 = 4x + 4 = 4(1)(x + 1)$$

Vertex: $(-1, 0)$

Focus: $(0, 0)$

Directrix: $x = -2$



26. $y^2 = 9x, y \geq 0$

$$y = \sqrt{9x} = 3\sqrt{x}$$

28. Point: $(-2, 6)$

$$x = ay^2$$

$$-2 = a(6)^2$$

$$-\frac{1}{18} = a$$

$$x = -\frac{1}{18}y^2$$

30. Focus: $(2, 0) \Rightarrow p = 2$

$$y^2 = 4px$$

$$y^2 = 8x$$

32. Focus: $(0, -2) \Rightarrow p = -2$

$$x^2 = 4py$$

$$x^2 = -8y$$

34. Directrix: $y = 3 \Rightarrow p = -3$

$$x^2 = 4py$$

$$x^2 = -12y$$

36. Directrix: $x = -3 \Rightarrow p = 3$

$$y^2 = 4px$$

$$y^2 = 12x$$

38. Vertical axis, passes through $(-3, -3)$

$$x^2 = 4py$$

$$(-3)^2 = 4p(-3)$$

$$9 = -12p \Rightarrow p = -\frac{3}{4}$$

$$x^2 = 4\left(-\frac{3}{4}\right)y$$

$$x^2 = -3y$$

40. Vertex: $(5, 3) \Rightarrow h = 5,$
 $k = 3$

Passes through: $(4.5, 4)$

$$(y - k)^2 = 4p(x - h)$$

$$(y - 3)^2 = 4p(x - 5)$$

$$1 = 4p(4.5 - 5)$$

$$p = -\frac{1}{2}$$

$$(y - 3)^2 = -2(x - 5)$$

44. Vertex: $(-1, 2) \Rightarrow h = -1, k = 2$

Focus: $(-1, 0) \Rightarrow p = -2$

$$(x - h)^2 = 4p(y - k)$$

$$(x + 1)^2 = 4(-2)(y - 2)$$

$$(x + 1)^2 = -8(y - 2)$$

48. Focus: $(0, 0)$; Directrix: $y = 8 \Rightarrow p = -4$

$$h = 0, k = 4$$

$$(x - h)^2 = 4p(y - k)$$

$$x^2 = 4(-4)(y - 4)$$

$$x^2 = -16(y - 4)$$

52. $2y = x^2$

$$4\left(\frac{1}{2}\right)y = x^2$$

$$p = \frac{1}{2}$$

Focus: $\left(0, \frac{1}{2}\right)$

$$d_1 = \frac{1}{2} - b$$

$$d_2 = \sqrt{(-3 - 0)^2 + \left(\frac{9}{2} - \frac{1}{2}\right)^2} = 5$$

$$\frac{1}{2} - b = 5$$

$$b = -\frac{9}{2}$$

$$m = \frac{-(9/2) - (9/2)}{0 + 3} = -3$$

Tangent line: $y = -3x - \frac{9}{2} \Rightarrow 6x + 2y + 9 = 0$

x-intercept: $\left(-\frac{3}{2}, 0\right)$

42. Vertex: $(3, -3) \Rightarrow h = 3,$
 $k = -3$

Passes through: $(0, 0)$

$$(x - h)^2 = 4p(y - k)$$

$$(x - 3)^2 = 4p(y + 3)$$

$$9 = 12p$$

$$p = \frac{3}{4}$$

$$(x - 3)^2 = 3(y + 3)$$

46. Vertex: $(-2, 1) \Rightarrow h = -2, k = 1$

Directrix: $x = 1 \Rightarrow p = -3$

$$(y - k)^2 = 4p(x - h)$$

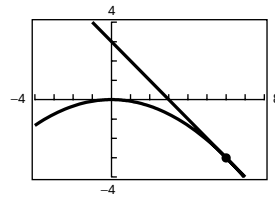
$$(y - 1)^2 = 4(-3)(x - (-2))$$

$$(y - 1)^2 = -12(x + 2)$$

50. $x^2 + 12y = 0 \Rightarrow y_1 = -\frac{1}{12}x^2$

$$x + y - 3 = 0 \Rightarrow y_2 = 3 - x$$

Using the trace or intersect feature,
the point of tangency is $(6, -3)$.



54. $y = -2x^2, (2, -8)$

$$x^2 = -\frac{1}{2}y = 4\left(-\frac{1}{8}\right)y \Rightarrow p = -\frac{1}{8}. \text{ Focus: } \left(0, -\frac{1}{8}\right)$$

$$d_1 = \frac{1}{8} + b$$

$$d_2 = \sqrt{(2 - 0)^2 + \left(-8 + \frac{1}{8}\right)^2} = \frac{65}{8}$$

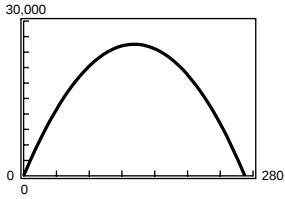
$$d_1 = d_2 \Rightarrow \frac{1}{8} + b = \frac{65}{8} \Rightarrow b = 8$$

$$m = \frac{-8 - 8}{2 - 0} = -8$$

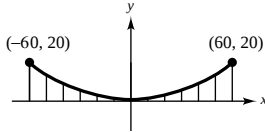
$$y = -8x + 8$$

Intercept: $(1, 0)$

56.


 $R = 378x - \frac{7}{5}x^2$ is a maximum (25, 515) at $x = 135$ units.

58. (a)



(b) $(x - 0)^2 = 4p(y - 0)$

$x^2 = 4py$

At (60, 20): $60^2 = 4p(20) \Rightarrow p = 45$

$x^2 = 4(45)y$

$y = \frac{x^2}{180}$

(c)

x	0	20	40	60
y	0	$2\frac{2}{9}$	$8\frac{8}{9}$	20

60. Vertex: (0, 0)

$y^2 = 4px$

Point: (1000, 800)

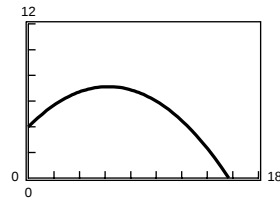
$800^2 = 4p(1000) \Rightarrow p = 160$

$y^2 = 4(160)x$

$y^2 = 640x$

62. $y = -0.08x^2 + x + 4$

(a)



(b) The highest point is approximately (6.25, 7.125). The range is approximately 15.69 feet.

64. $y = -\frac{16}{v^2}x^2 + s$

550 miles per hour = 806.67 feet per second.

$y = -\frac{16}{806.67^2}x^2 + 42,000$

$y = 0 \Rightarrow \frac{16}{806.67^2}x^2 = 42,000$

$x^2 \approx 1,708,115,666.67 \Rightarrow x \approx 41,329.37$ feet

66. True

68. $\pm 10, \pm 5, \pm 2, \pm 1, \pm \frac{5}{2}, \pm \frac{1}{2}$

70. $\pm 22, \pm 11, \pm 2, \pm 1, \pm \frac{22}{3}, \pm \frac{11}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}$