

Section 10.3 Hyperbolas

Solutions to Even-Numbered Exercises

2. Center:
- $(0, 0)$

$$a = 5, b = 3$$

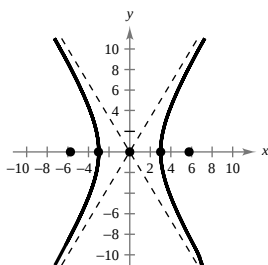
Vertical transverse axis

Matches graph (c)

6. $\frac{x^2}{9} - \frac{y^2}{25} = 1$

Center: $(0, 0)$

$$a = 3, b = 5, c = \sqrt{3^2 + 5^2} = \sqrt{34}$$

Vertices: $(\pm 3, 0)$ Foci: $(\pm \sqrt{34}, 0)$ Asymptotes: $y = \pm \frac{b}{a}x = \pm \frac{5}{3}x$ 

4. Center:
- $(-1, 2)$

$$a = 4, b = 3$$

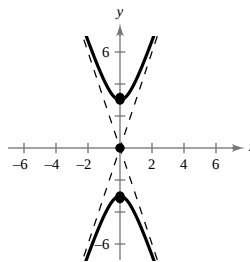
Horizontal transverse axis

Matches graph (d)

8. $\frac{y^2}{9} - \frac{x^2}{1} = 1$

$$a = 3, b = 1,$$

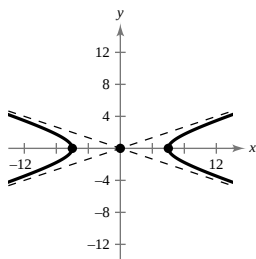
$$c = \sqrt{3^2 + 1^2} = \sqrt{10}$$

Center: $(0, 0)$ Vertices: $(0, \pm 3)$ Foci: $(0, \pm \sqrt{10})$ Asymptotes: $y = \pm 3x$ 

10. $\frac{x^2}{36} - \frac{y^2}{4} = 1$

$$a = 6, b = 2,$$

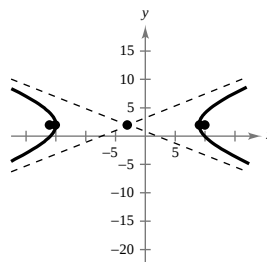
$$c = \sqrt{36 + 4} = 2\sqrt{10}$$

Center: $(0, 0)$ Vertices: $(\pm 6, 0)$ Foci: $(\pm 2\sqrt{10}, 0)$ Asymptotes: $y = \pm \frac{1}{3}x$ 

12. $\frac{(x+3)^2}{144} - \frac{(y-2)^2}{25} = 1$

Center: $(-3, 2)$

$$a = 12, b = 5, c = 13$$

Vertices: $(-15, 2), (9, 2)$ Foci: $(-16, 2), (10, 2)$ Asymptotes: $y = 2 \pm \frac{5}{12}(x + 3)$ 

$$14. \frac{(y-1)^2}{\frac{1}{4}} - \frac{(x+3)^2}{\frac{1}{16}} = 1$$

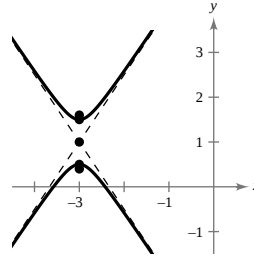
Center: $(-3, 1)$

$$a = \frac{1}{2}, b = \frac{1}{4}, c = \sqrt{\frac{1}{4} + \frac{1}{16}} = \frac{\sqrt{5}}{4}$$

Vertices: $\left(-3, \frac{1}{2}\right), \left(-3, \frac{3}{2}\right)$

Foci: $\left(-3, 1 \pm \frac{\sqrt{5}}{4}\right)$

Asymptotes: $y = 1 \pm \frac{\frac{1}{2}}{\frac{1}{4}}(x + 3) = 1 \pm 2(x + 3)$



$$16. x^2 - 9y^2 + 36y - 72 = 0$$

$$x^2 - 9(y^2 - 4y + 4) = 72 - 36$$

$$x^2 - 9(y - 2)^2 = 36$$

$$\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1$$

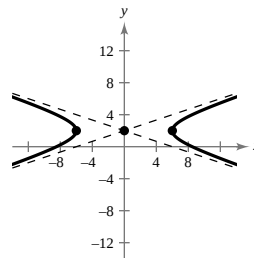
$$a = 6, b = 2, c = \sqrt{36 + 4} = 2\sqrt{10}$$

Center: $(0, 2)$

Vertices: $(\pm 6, 2)$

Foci: $(\pm 2\sqrt{10}, 2)$

Asymptotes: $y = 2 \pm \frac{1}{3}x$



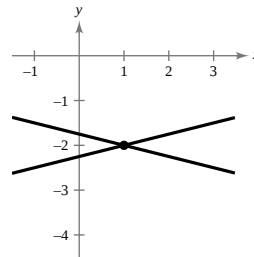
$$18. 16y^2 - x^2 + 2x + 64y + 63 = 0$$

$$16(y^2 + 4y + 4) - (x^2 - 2x + 1) = -63 + 64 - 1$$

$$16(y + 2)^2 - (x - 1) = 0$$

$$y + 2 = \pm \frac{1}{4}(x - 1)$$

Degenerate hyperbola is two intersecting lines.



$$20. 6y^2 - 3x^2 = 18$$

$$\frac{y^2}{3} - \frac{x^2}{6} = 1$$

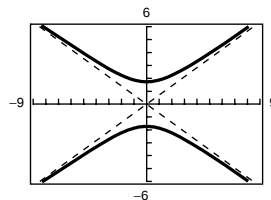
$$a = \sqrt{3}, b = \sqrt{6}, c = 3$$

Center: $(0, 0)$

Vertices: $(0, \pm\sqrt{3})$

Foci: $(0, \pm 3)$

Asymptotes: $y = \pm \frac{\sqrt{3}}{\sqrt{6}}x = \pm \frac{\sqrt{2}}{2}x$



$$\begin{aligned}
 22. \quad & 9x^2 - y^2 + 54x + 10y + 55 = 0 \\
 & 9(x^2 + 6x + 9) - (y^2 - 10y + 25) = -55 + 81 - 25 \\
 & \frac{(x+3)^2}{1/9} - \frac{(y-5)^2}{1} = 1
 \end{aligned}$$

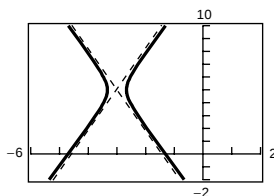
$$a = \frac{1}{3}, b = 1, c = \frac{\sqrt{10}}{3}$$

$$\text{Center: } (-3, 5)$$

$$\text{Vertices: } \left(-3 \pm \frac{1}{3}, 5\right)$$

$$\text{Foci: } \left(-3 \pm \frac{\sqrt{10}}{3}, 5\right)$$

$$\text{Asymptotes: } y = 5 \pm 3(x + 3)$$



$$24. \quad a = 2, c = 5$$

$$b^2 = c^2 - a^2 = 21 \Rightarrow b = \sqrt{21}$$

$$\frac{x^2}{4} - \frac{y^2}{21} = 1$$

$$26. \quad \text{Vertices: } (0, \pm 3) \Rightarrow a = 3$$

$$\text{Asymptotes: } y = \pm 3x \Rightarrow \frac{a}{b} = 3, b = 1$$

$$\text{Center: } (0, 0) = (h, k)$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{y^2}{9} - x^2 = 1$$

$$28. \quad \text{Foci: } (\pm 10, 0) \Rightarrow c = 10$$

$$\text{Asymptotes: } y = \pm \frac{3}{4}x \Rightarrow \frac{b}{a} = \frac{3m}{4m}$$

$$c^2 = a^2 + b^2 \Rightarrow 100 = (3m)^2 + (4m)^2$$

$$100 = 25m^2$$

$$2 = m$$

$$a = 4(2) = 8$$

$$b = 3(2) = 6$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{64} - \frac{y^2}{36} = 1$$

$$30. \quad \text{Vertices: } (2, 3), (2, -3) \Rightarrow a = 3$$

$$\text{Center: } (2, 0)$$

$$\text{Foci: } (2, 5), (2, -5) \Rightarrow c = 5$$

$$b^2 = c^2 - a^2 = 25 - 9 = 16$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{y^2}{9} - \frac{(x-2)^2}{16} = 1$$

$$32. \quad \text{Vertices: } (-2, 1), (2, 1) \Rightarrow a = 2$$

$$\text{Center: } (0, 1)$$

$$\text{Foci: } (-3, 1), (3, 1) \Rightarrow c = 3$$

$$b^2 = c^2 - a^2 = 9 - 4 = 5$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{4} - \frac{(y-1)^2}{5} = 1$$

34. $a = 2$, Center: $(0, 1)$

$$\frac{x^2}{4} - \frac{(y-1)^2}{b^2} = 1$$

Solution point: $(5, 4)$:

$$\frac{25}{4} - \frac{9}{b^2} = 1$$

$$\frac{9}{b^2} = \frac{21}{4}$$

$$b^2 = \frac{36}{21} = \frac{12}{7}$$

$$\frac{x^2}{4} - \frac{(y-1)^2}{\left(\frac{12}{7}\right)} = 1$$

38. Center: $(3, -3)$, $a = 3$

Asymptotes: $y = x - 6$, $y = -x$

$$1 = \frac{a}{b} = \frac{3}{b} \Rightarrow b = 3$$

$$\frac{(y+3)^2}{9} - \frac{(x-3)^2}{9} = 1$$

42. Foci: $(\pm 150, 0) \Rightarrow c = 150$

Center: $(0, 0) = (h, k)$

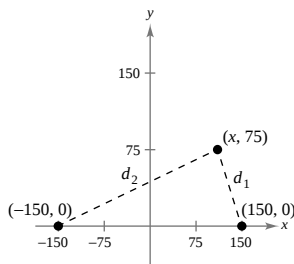
$$\frac{d_2}{186,000} - \frac{d_1}{186,000} = 0.001 \Rightarrow 2a = 186, a = 93$$

$$b^2 = c^2 - a^2 = 150^2 - 93^2 = 13,851$$

$$\frac{x^2}{93^2} - \frac{y^2}{13,851} = 1$$

$$x^2 = 93^2 \left(1 + \frac{75^2}{13,851} \right) \approx 12,161$$

$$x \approx 110.3 \text{ miles}$$



36. Center: $(1, 0)$, $a = 2$

$$\frac{y^2}{4} - \frac{(x-1)^2}{b^2} = 1$$

Solution point: $(0, \sqrt{5})$

$$\frac{5}{4} - \frac{1}{b^2} = 1$$

$$\frac{1}{b^2} = \frac{1}{4} \Rightarrow b = 2$$

$$\frac{y^2}{4} - \frac{(x-1)^2}{4} = 1$$

40. Vertices: $(3, 0)$, $(3, 4) \Rightarrow a = 2$

Asymptotes: $y = \frac{2}{3}x$, $y = 4 - \frac{2}{3}x$

$$\frac{a}{b} = \frac{2}{3} \Rightarrow b = 3$$

Center: $(3, 2) = (h, k)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-2)^2}{4} - \frac{(x-3)^2}{9} = 1$$

44. $x^2 + y^2 - 6x + 4y + 9 = 0$

$$A = 1, C = 1$$

$$A = C \Rightarrow \text{Circle}$$