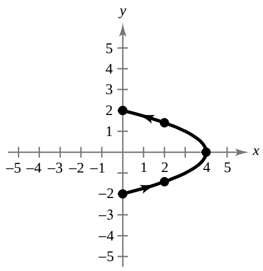
10. $x = 4\cos^2\theta, y = 2\sin\theta$

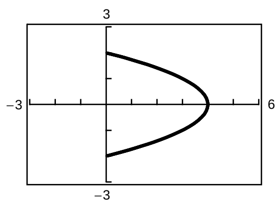
(a)

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
x	0	2	4	2	0
y	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2

(b)



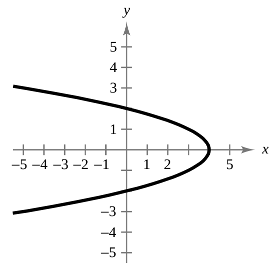
(c)



(d) $\frac{x}{4} = \cos^2\theta, \frac{y}{2} = \sin^2\theta$

$$\frac{x}{4} + \frac{y^2}{4} = 1$$

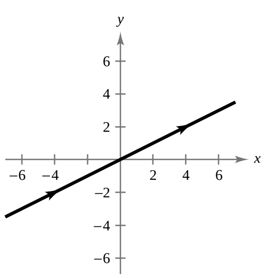
Parabola



The rectangular version continues the graph into the second and third quadrants.

12. $x = t,\ y = \frac{1}{2}t$

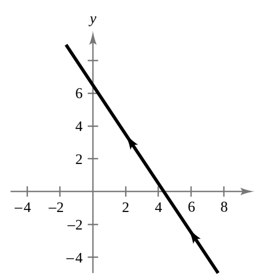
$$y = \frac{1}{2}x \quad \text{or} \quad x - 2y = 0$$



14. $x = 3 - 2t, y = 2 + 3t$

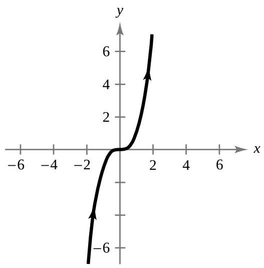
$$y = 2 + 3\left(\frac{3 - x}{2}\right)$$

$$3x + 2y - 13 = 0$$



16. $x = t, \ y = t^3$

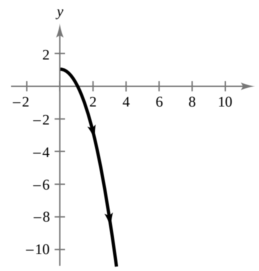
$y = x^3$



18. $x = \sqrt{t}$

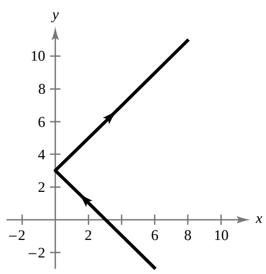
$y = 1 - t$

$y = 1 - x^2, \ x \geq 0$



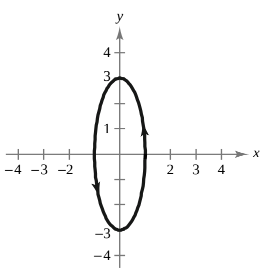
20. $x = |t - 1|$

$y = t + 2$



Eliminating the parameter t, $t = y - 2$ and $x = |t - 1| = |(y - 2) - 1| = |y - 3|$.

22.



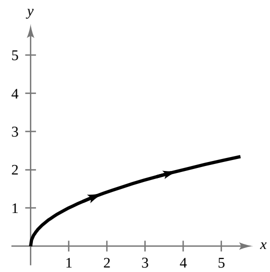
$$\left.\begin{aligned} x = \cos\theta &\Longrightarrow x^2 = \cos^2\theta \\ y = 3\sin\theta &\Longrightarrow \frac{y^2}{9} = \sin^2\theta \end{aligned}\right\} \sin^2\theta + \cos^2\theta = 1 = x^2 + \frac{y^2}{9}$$

Ellipse

24. $x = e^{2t}$

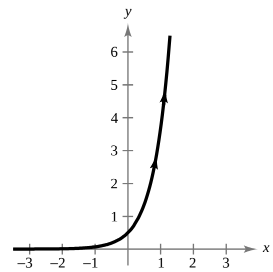
$y = e^t \Longrightarrow y^2 = e^{2t}$

$y^2 = x,\ y > 0;\ y = \sqrt{x},\ x > 0$

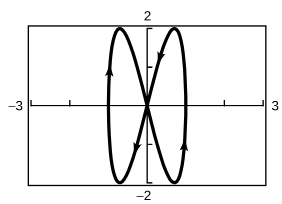


26. $x = \ln 2t \quad \Longrightarrow \quad e^x = 2t \quad \Longrightarrow \quad t = \frac{1}{2}e^x$

$y = 2t^2 = 2\left(\frac{1}{2}e^x\right)^2 = \frac{1}{2}e^{2x}$



28.



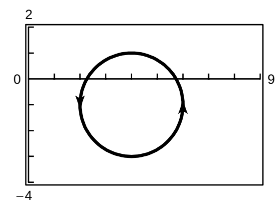
$x = \cos\theta$

$y = 2\sin 2\theta = 4\sin\theta\cos\theta$

$y^2 = 16\sin^2\theta\cos^2\theta = 16(1 - \cos^2\theta)\cos^2\theta$

$y^2 = 16(1 - x^2)x^2 = 16x^2(1 - x^2)$

30.



$x = 4 + 2\cos\theta \Longrightarrow \cos\theta = \dfrac{x - 4}{2}$

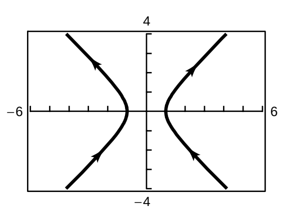
$y = -1 + 2\sin\theta \Longrightarrow \sin\theta = \dfrac{y + 1}{2}$

$$\left(\frac{x - 4}{2}\right)^2 + \left(\frac{y + 1}{2}\right)^2 = \cos^2\theta + \sin^2\theta = 1$$

$(x - 4)^2 + (y + 1)^2 = 4$

Circle

32.



$x = \sec\theta$

$y = \tan\theta$

$\tan^2\theta + 1 = \sec^2\theta$

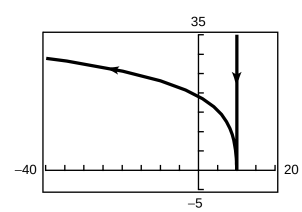
$y^2 + 1 = x^2$

$x^2 - y^2 = 1$

Hyperbola

34. $x = 10 - 0.01e^t$

$y = 0.4t^2$



36. Each curve represents a portion of the line $y = x$.

(a) Domain: $-\infty < x < \infty$
Orientation: Left to right

(b) Domain: $x \geq 0$
Orientation: Depends on t

(c) Domain: $-\infty < x < \infty$
Orientation: Right to left

(d) Domain: $-\infty < x < \infty$
Orientation: Left to right

38. $x = h + r\cos\theta$

$y = k + r\sin\theta$

$\dfrac{(x-h)}{r} = \cos\theta, \qquad \dfrac{y-k}{r} = \sin\theta$

$$\cos^2\theta + \sin^2\theta = \frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$$

$$(x-h)^2 + (y-k)^2 = r^2$$

40. $x = h + a\sec\theta$

$y = k + b\tan\theta$

$\dfrac{x-h}{a} = \sec\theta, \qquad \dfrac{y-k}{b} = \tan\theta$

$$\sec^2\theta - \tan^2\theta = \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

42. $x = 1 + t(5-1)$

$y = 4 + t(-2-4)$

$x = 1 + 4t$

$y = 4 - 6t$

(Solution not unique.)

44. From Exercise 38:

$x = -3 + 3\cos\theta, \quad h = -3, r = 3$

$y = 1 + 3\sin\theta, \quad k = 1, r = 3$

46. $a = 1, c = 2, b = \sqrt{c^2 - a^2} = \sqrt{3}$

From Exercise 40, $x = \sqrt{3}\tan\theta, y = \sec\theta$.

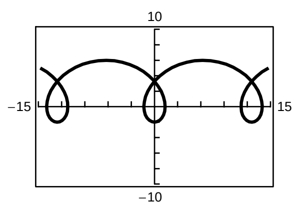
48. $y = x^2$

Examples:

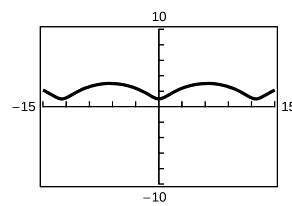
$x = t,\ y = t^2$

$x = \frac{1}{2}t,\ y = \frac{1}{4}t^2$

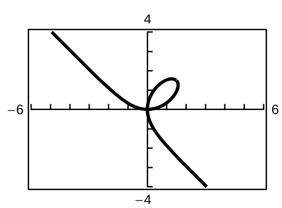
50.



52.



54.



56. Matches (c).

58. Matches (a).

60. (a) $x = (\cos 35°)v_0 t$

$y = 7 + (\sin 35°)v_0 t - 16t^2$

(b) If the ball is caught at time t_1, then:

$90 = (\cos 35°)v_0 t_1$

$4 = 7 + (\sin 35°)v_0 t_1 - 16t_1^2$.

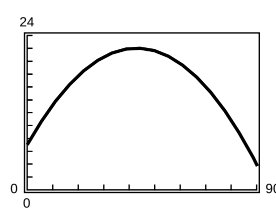
$$v_0 t = \frac{90}{\cos 35°} \Longrightarrow -3 = (\sin 35°)\frac{90}{\cos 35°} - 16t_1^2$$

$$\Longrightarrow 16t_1^2 = 90\tan 35° + 3$$

$$\Longrightarrow t_1 \approx 2.03 \text{ seconds}$$

$$\Longrightarrow v_0 = \frac{90}{t_1\cos 35°} \approx 54.09 \text{ ft/sec}$$

(c)



Maximum height $\approx$ 22 feet

(d) From part (b), $t_1 \approx 2.03$ seconds.

62. False. It is the line $y = x$ for $x \geq 0$.

64. $5x^2 + 8 = 0$

$$x^2 = -\frac{8}{5}$$

$$x = \pm\sqrt{\frac{8}{5}}i = \pm\frac{2}{5}\sqrt{10}\,i$$

66. $4x^2 + 4x - 11 = 0$

$$x = \frac{-4 \pm \sqrt{16 + 4(4)(11)}}{8} = \frac{-1 \pm \sqrt{12}}{2} = -\frac{1}{2} \pm \sqrt{3}$$

68. $\displaystyle\sum_{n=1}^{50} 8n = 8\,\frac{50(51)}{2} = 10{,}200$

70. $\displaystyle\sum_{n=1}^{40} \left(300 - \frac{1}{2}n\right) = 300(40) - \frac{1}{2}\,\frac{40(41)}{2} = 11{,}590$

72. $\displaystyle\sum_{n=0}^{18} 8\left(\frac{1}{2}\right)^n = 8\,\frac{1 - \left(\frac{1}{2}\right)^{19}}{1 - \frac{1}{2}} = 16\left[1 - \left(\tfrac{1}{2}\right)^{19}\right] \approx 15.99997$

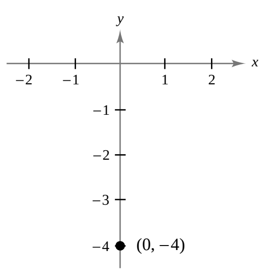
Section 10.6 Polar Coordinates

Solutions to Even-Numbered Exercises

2. Polar coordinates: $\left(4, \dfrac{3\pi}{2}\right)$

$$x = 4\cos\left(\frac{3\pi}{2}\right) = 0,\ y = 4\sin\left(\frac{3\pi}{2}\right) = -4$$

Rectangular coordinates: $(0, -4)$



4. Polar coordinates: $\left(2, -\dfrac{\pi}{4}\right)$

$$x = 2\cos\left(-\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

$$y = 2\sin\left(-\frac{\pi}{4}\right) = 2\left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$$

Rectangular coordinates: $\left(\sqrt{2}, -\sqrt{2}\right)$