Section 10.7 Graphs of Polar Equations

Solutions to Even-Numbered Exercises

2. Cardioid

4. Lemniscate

6. Limaçon

8. $r = 16 \cos 3\theta$

$\theta = \dfrac{\pi}{2}$: $-r = 16 \cos(3(-\theta))$

$-r = 16 \cos(-3\theta)$

$-r = 16 \cos 3\theta$

Not an equivalent equation

Polar axis: $r = 16 \cos(3(-\theta))$

$r = 16 \cos(-3\theta)$

$r = 16 \cos 3\theta$

Equivalent equation

Pole: $-r = 16 \cos 3\theta$

Not an equivalent equation

Answer: Symmetric with respect to polar axis

10. $r = \dfrac{4}{1 - \cos \theta}$

$\theta = \dfrac{\pi}{2}$: $-r = \dfrac{4}{1 - \cos(-\theta)}$

$-r = \dfrac{4}{1 - \cos \theta}$

Not an equivalent equation

$r = \dfrac{4}{1 - \cos(\pi - \theta)}$

$r = \dfrac{4}{1 - (\cos \pi \cos \theta + \sin \pi \sin \theta)}$

$r = \dfrac{4}{1 + \cos \theta}$

Not an equivalent equation

Polar axis: $r = \dfrac{4}{1 - \cos(-\theta)}$

$r = \dfrac{4}{1 - \cos \theta}$

Equivalent equation

Pole: $-r = \dfrac{4}{1 - \cos \theta}$ Not an equivalent equation

$r = \dfrac{4}{1 - \cos(\pi + \theta)}$

$r = \dfrac{4}{1 - (\cos \pi \cos \theta - \sin \pi \sin \theta)}$

$r = \dfrac{4}{1 + \cos \theta}$ Not an equivalent equation

Answer: Symmetric with respect to the polar axis

12. $r = 4 - \sin\theta$

$\theta = \dfrac{\pi}{2}$: $r = 4 - \sin(\pi - \theta)$

$r = 4 - (\sin\pi\cos\theta - \cos\pi\sin\theta)$

$r = 4 - \sin\theta$

Equivalent equation

Polar axis: $r = 4 - \sin(-\theta)$

$r = 4 + \sin\theta$

Not an equivalent equation

$-r = 4 - \sin(\pi - \theta)$

$-r = 4 - \sin\theta$

Not an equivalent equation

Pole: $-r = 4 - \sin\theta$ Not an equivalent equation

$r = 4 - \sin(\pi + \theta)$

$r = 4 - (\sin\pi\cos\theta + \sin\theta\cos\pi)$

$r = 4 + \sin\theta$

Not an equivalent equation

Answer: Symmetric with respect to $\theta = \dfrac{\pi}{2}$

14. $r = 2\csc\theta\cos\theta = 2\cot\theta$

$\theta = \dfrac{\pi}{2}$: $-r = 2\cot(-\theta)$

$r = 2\cot\theta$ Equivalent equation

Polar axis: $-r = 2\cot(\pi - \theta)$

$-r = 2\cot(-\theta)$

$r = 2\cot\theta$ Equivalent equation

Pole: $r = 2\cot(\pi + \theta)$

$r = 2\cot\theta$ Equivalent equation

Answer: Symmetric with respect to $\theta = \dfrac{\pi}{2}$, polar axis and pole.

16. $r^2 = 25\cos 4\theta$

$\theta = \dfrac{\pi}{2}$: $(-r)^2 = 25\cos[4(-\theta)]$

$r^2 = 25\cos 4\theta$

Equivalent equation

Polar axis: $r^2 = 25\cos[4(-\theta)]$

$r^2 = 25\cos 4\theta$ Equivalent equation

Pole: $(-r)^2 = 25\cos 4\theta$

$r^2 = 25\cos 4\theta$ Equivalent equation

Answer: Symmetric with respect to $\theta = \dfrac{\pi}{2}$, polar axis and pole.

18. $|r| = |6 + 12\cos\theta| \le |6| + |12\cos\theta|$

$= 6 + 12|\cos\theta| \le 18$

$\cos\theta = 1$

$\theta = 0$

Maximum: $|r| = 18$ when $\theta = 0$

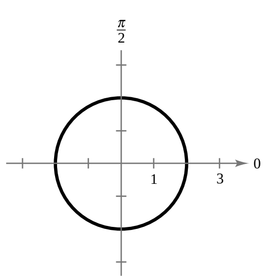
Zero: $r = 0$ when $\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

20. $|r| = |5\sin 2\theta|$

Maximum: $|r| = 5$ when $\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

Zero: $r = 0$ when $\theta = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi$

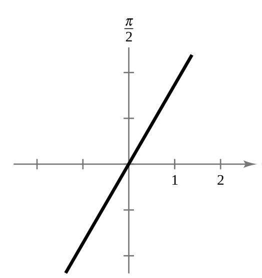
22. Circle: $r = 2$



24. $r = -\dfrac{5\pi}{3}$

Circle

Radius: $\dfrac{5\pi}{3}$

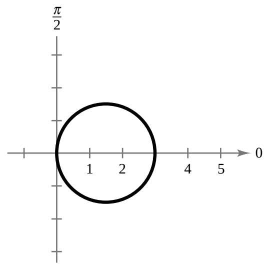


26. $r = 3\cos\theta$

Circle

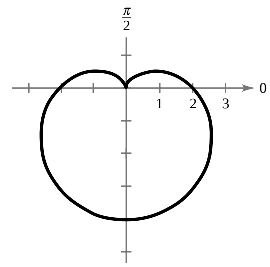
Radius: $\frac{3}{2}$

Center: $\left(\frac{3}{2}, 0\right)$



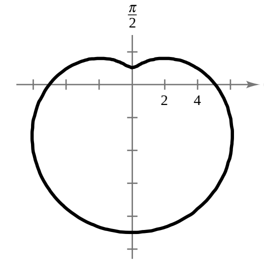
28. $r = 2 - 2\sin\theta$

Cardioid



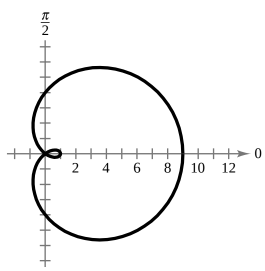
30. $r = 5 - 4\sin\theta$

Dimpled limaçon



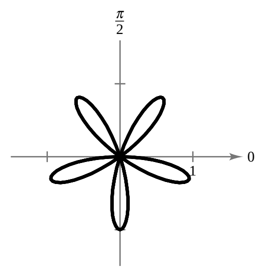
32. $r = 4 + 5\cos\theta$

Limaçon



34. $r = -\sin 5\theta$

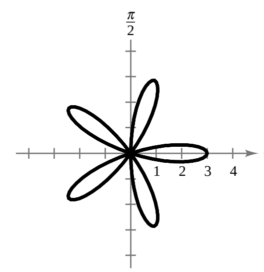
Rose curve



36. $r = 3\cos 5\theta$

Rose curve

5 petals

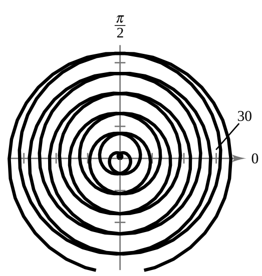


38. $r = \theta$

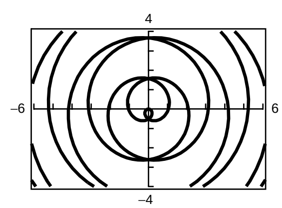
Symmetric with respect to

$\theta = \frac{\pi}{2}$

Spiral

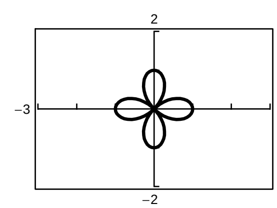


40.



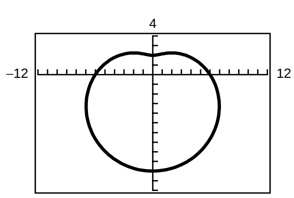
$-\frac{41\pi}{4} \le \theta \le \frac{41\pi}{4}$

42.



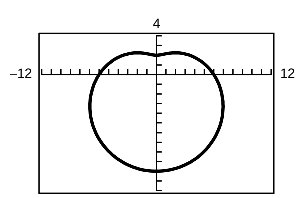
$0 \le \theta \le 2\pi$

44.



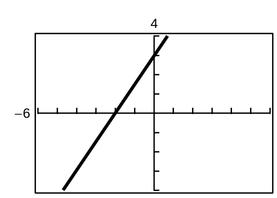
$0 \le \theta \le 2\pi$

46.



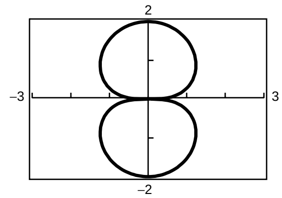
$0 \le \theta \le 2\pi$

48.



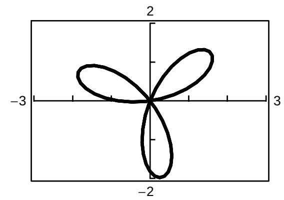
$0 \le \theta \le 2\pi$

50.



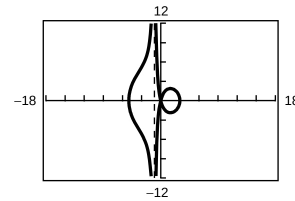
$0 \le \theta \le 2\pi$

52.



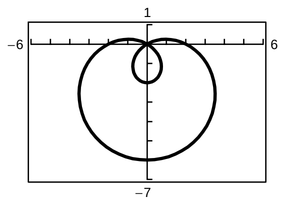
$0 \le \theta \le 2\pi$

54.



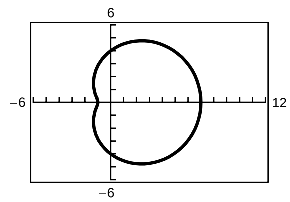
$0 \le \theta \le 2\pi$

56.



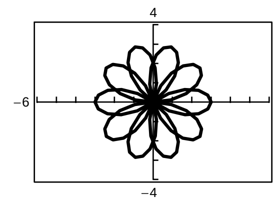
$0 \le \theta < 2\pi$

58.



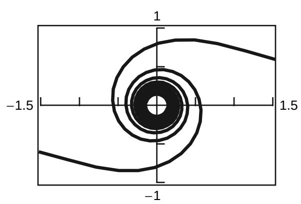
$0 \le \theta < 2\pi$

60.



$0 \le \theta < 4\pi$

62.



$0 < \theta < \infty$

64.

$$r = 2 + \csc\theta = 2 + \frac{1}{\sin\theta}$$

$$r\sin\theta = 2\sin\theta + 1$$

$$r(r\sin\theta) = 2r\sin\theta + r$$

$$\left(\pm\sqrt{x^2+y^2}\right)(y) = 2y + \left(\pm\sqrt{x^2+y^2}\right)$$

$$\left(\pm\sqrt{x^2+y^2}\right)(y-1) = 2y$$

$$\left(\pm\sqrt{x^2+y^2}\right) = \frac{2y}{y-1}$$

$$x^2 + y^2 = \frac{4y^2}{(y-1)^2}$$

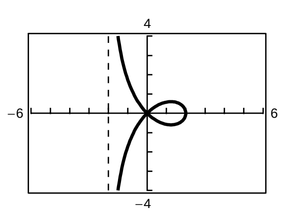
$$x^2 = \frac{y^2(3+2y-y^2)}{(y-1)^2}$$

$$x = \pm\sqrt{\frac{y^2(3+2y-y^2)}{(y-1)^2}}$$

$$= \pm\left|\frac{y}{y-1}\right|\sqrt{3+2y-y^2}$$

The graph has an asymptote at $y = 1$.

66.



68. False. It has 16 petals.

70. False. For example, let $r = \cos 3\theta$.

72. The graph of $r = f(\theta)$ is rotated about the pole through an angle ϕ. Let (r, θ) be any point on the graph of $r = f(\theta)$. Then $(r, \theta + \phi)$ is rotated through the angle ϕ, and since $r = f((\theta + \phi) - \phi) = f(\theta)$, it follows that $(r, \theta + \phi)$ is on the graph of $r = f(\theta - \phi)$.

74. (a) $r = 2 - \sin\left(\theta - \frac{\pi}{4}\right)$

$= 2 - \frac{\sqrt{2}}{2}(\sin\theta - \cos\theta)$

(b) $r = 2 - \sin\left(\theta - \frac{\pi}{2}\right)$

$= 2 + \cos\theta$

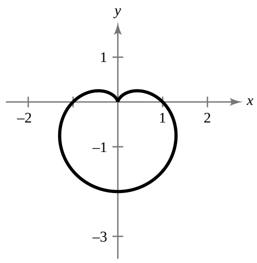
(c) $r = 2 - \sin(\theta - \pi)$

$= 2 + \sin\theta$

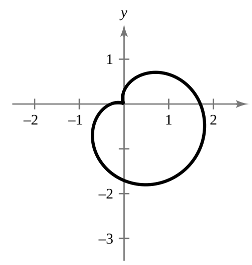
(d) $r = 2 - \sin\left(\theta - \frac{3\pi}{2}\right)$

$= 2 - \cos\theta$

76. (a) $r = 1 - \sin\theta$

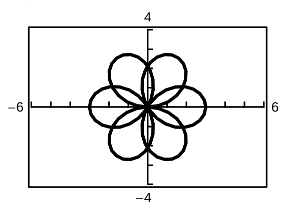


(b) $r = 1 - \sin\left(\theta - \frac{\pi}{4}\right)$



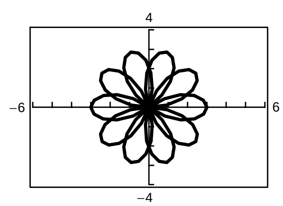
78. $r = 3\sin k\theta$

(a)



$k = 1.5$: $0 \le \theta < 4\pi$

(b)



$k = 2.5$: $0 \le \theta < 6\pi$

(c) Yes

80. $a_3 = a_1 + 2d$

$\frac{11}{6} = \frac{5}{2} + 2d \implies 2d = \frac{11}{6} - \frac{15}{6} = -\frac{4}{6} \implies d = -\frac{1}{3}$

$a_1 = \frac{5}{2}, a_2 = \frac{5}{2} - \frac{1}{3} = \frac{13}{6}, a_3 = \frac{13}{6} - \frac{1}{3} = \frac{11}{6}$

$a_4 = \frac{11}{6} - \frac{1}{3} = \frac{9}{6} = \frac{3}{2}, a_5 = \frac{3}{2} - \frac{1}{3} = \frac{7}{6}$

$a_n = a_1 + (n-1)\left(-\frac{1}{3}\right) = -\frac{1}{3}n + \frac{17}{6}$

82. $a_1 = 0.525$

$a_2 = 0.525 + 0.75 = 1.275$

$a_3 = 2.025$

$a_4 = 2.775$

$a_5 = 3.525$

$a_n = a_1 + (n-1)d = 0.525 + (n-1)(0.75) = 0.75n - 0.225$

84. $\sum_{n=1}^{50} 8n = \frac{8(50)(51)}{2} = 10{,}200$

86. $\sum_{n=1}^{200} (300 - n) = 300(200) - \frac{200(201)}{2} = 39{,}900$

88. $\sum_{n=1}^{\infty} 6(0.4)^{n-1} = 6\left(\frac{1}{1 - 0.4}\right) = 6\left(\frac{1}{0.6}\right) = 10$