CHAPTER 4
Trigonometric Functions

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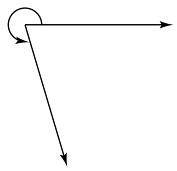
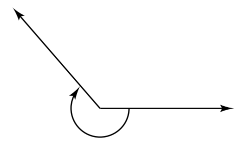
Section 4.1 Radian and Degree Measure

Solutions to Even-Numbered Exercises

2. 

The angle shown is approximately 5 radians.

4. 

The angle shown is approximately -4 radians.

6. (a) Since $\frac{3\pi}{2} < \frac{7\pi}{4} < 2\pi$, $\frac{7\pi}{4}$ lies in Quadrant IV.

(b) Since $\frac{5\pi}{2} < \frac{11\pi}{4} < 3\pi$, $\frac{11\pi}{4}$ lies in Quadrant II.

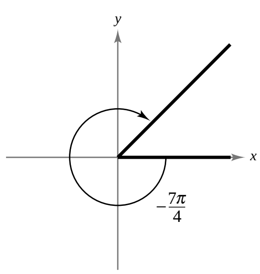
8. (a) Since $-\frac{\pi}{2} < -1 < 0$; -1 lies in Quadrant IV.

(b) Since $-\pi < -2 < -\frac{\pi}{2}$; -2 lies in Quadrant III.

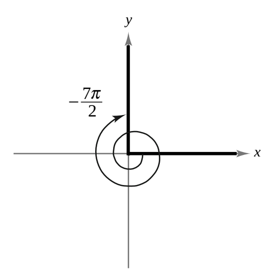
10. (a) Since $\frac{3\pi}{2} < 5.63 < 2\pi$; 5.63 lies in Quadrant IV.

(b) Since $-\pi < -2.25 < -\frac{\pi}{2}$; -2.25 lies in Quadrant III.

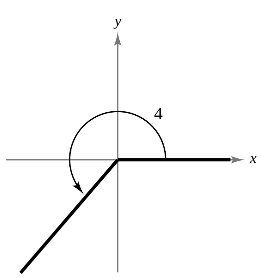
12. (a) $\frac{-7\pi}{4}$



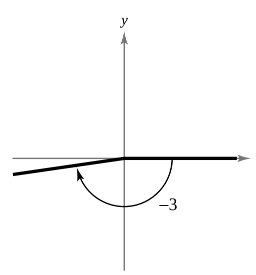
(b) $\frac{-7\pi}{2}$



14. (a) 4



(b) -3



16. (a) $\frac{7\pi}{6} + 2\pi = \frac{19\pi}{6}$

$\frac{7\pi}{6} - 2\pi = -\frac{5\pi}{6}$

(b) $-\frac{11\pi}{6} + 2\pi = \frac{\pi}{6}$

$-\frac{11\pi}{6} - 2\pi = -\frac{23\pi}{6}$

18. (a) $\frac{7\pi}{8} + 2\pi = \frac{23\pi}{8}$

$\frac{7\pi}{8} - 2\pi = \frac{-9\pi}{8}$

(b) $\frac{8\pi}{35} + 2\pi = \frac{78\pi}{35}$

$\frac{8\pi}{35} - 2\pi = \frac{-62\pi}{35}$

20. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

Supplement: $\pi - \frac{\pi}{12} = \frac{11\pi}{12}$

(b) Complement does not exist

Supplement: $\pi - \frac{11\pi}{12} = \frac{\pi}{12}$

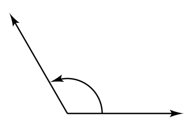
22. (a) Complement does not exist.

Supplement: $\pi - 3 \approx 0.1416$

(b) Complement: $\frac{\pi}{2} - 1.5 \approx 0.0708$

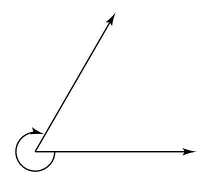
Supplement: $\pi - 1.5 \approx 1.6416$

24.



The angle shown is approximately 120°.

26.



The angle shown is approximately −300°.

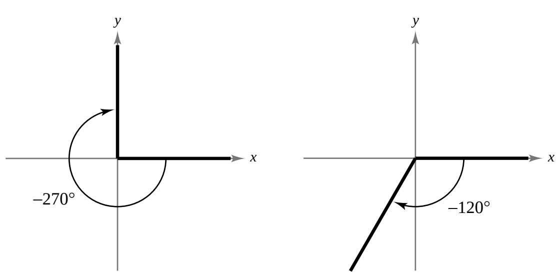
28. (a) Since $0° < 7.9° < 90°$, 7.9° lies in Quadrant I.

(b) Since $180° < 257.5° < 270°$, 257.5° lies in Quadrant III.

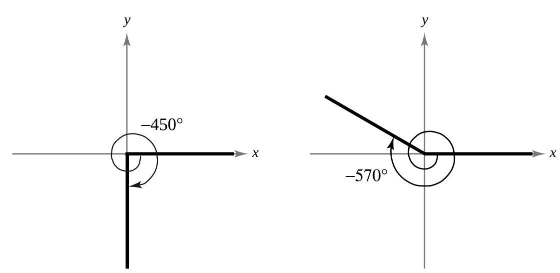
30. (a) Since $-270° < -260.25° < -180°$, −260.25° lies in Quadrant II.

(b) Since $-90° < -2.4° < 0°$, −2.4° lies in Quadrant IV.

32. (a) −270° (b) −120°



34. (a) −450° (b) −570°



36. (a) $114° + 360° = 474°$

$114° - 360° = -246°$

(b) $-390° + 720° = 330°$

$-390° + 360° = -30°$

38. (a) $-445° + 720° = 275°$

$-445° + 360° = -85°$

(b) $-740° + 1080° = 340°$

$-740° + 720° = -20°$

40. (a) Complement: $90° - 87° = 3°$

Supplement: $180° - 87° = 93°$

(b) Complement: none $(167° > 90°)$

Supplement: $180° - 167° = 13°$

42. (a) Complement: none $(130° > 90°)$

Supplement: $180° - 130° = 50°$

(b) Complement: none $(170° > 90°)$

Supplement: $180° - 170° = 10°$

44. (a) $315° = 315°\left(\frac{\pi}{180°}\right) = \frac{7\pi}{4}$

(b) $120° = 120°\left(\frac{\pi}{180°}\right) = \frac{2\pi}{3}$

46. (a) $-270° = -270°\left(\frac{\pi}{180°}\right) = -\frac{3\pi}{2}$

(b) $144° = 144°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{5}$

48. $83.7° = 83.7°\left(\frac{\pi}{180°}\right) \approx 1.461$ radians

50. $-46.52° = -46.52°\left(\frac{\pi}{180°}\right) \approx -0.812$ radians

52. $0.54° = 0.54°\left(\frac{\pi}{180°}\right) \approx 0.009$ radians

54. $395° = 395°\left(\frac{\pi}{180°}\right) \approx 6.894$ radians

56. (a) $-\frac{7\pi}{12} = -\frac{7\pi}{12}\left(\frac{180°}{\pi}\right) = -105°$

(b) $\frac{\pi}{9} = \frac{\pi}{9}\left(\frac{180°}{\pi}\right) = 20°$

58. (a) $\frac{15\pi}{6} = \frac{15\pi}{6}\left(\frac{180°}{\pi}\right) = 450°$

(b) $\frac{28\pi}{15} = \frac{28\pi}{15}\left(\frac{180°}{\pi}\right) = 336°$

60. $\frac{8\pi}{13} = \frac{8\pi}{13}\left(\frac{180°}{\pi}\right) \approx 110.769°$

62. $6.5\pi = 6.5\pi\left(\frac{180°}{\pi}\right) = 1170°$

64. $4.8 = 4.8\left(\frac{180°}{\pi}\right) \approx 275.020°$

66. $-0.48 = -0.48\left(\frac{180°}{\pi}\right) \approx -27.502°$

68. (a) $275°\,10' = 275° + \left(\frac{10}{60}\right)^{\circ} \approx 275° + 0.167° = 275.167°$

(b) $9°\,12' = 9° + \left(\frac{12}{60}\right)^{\circ} = 9° + 0.2° = 9.2°$

70. (a) $-125°\,36'' = -125° - \left(\frac{36}{3600}\right)^{\circ} = -125° - 0.01° = -125.01°$

(b) $330°\,25'' = 330° + \left(\frac{25}{3600}\right)^{\circ} \approx 330° + 0.00694 = 330.00694° \approx 330.007°$

72. (a) $-345.12° = -345°\,7'\,12''$

(b) $310.75° = 310°\,45'$

74. (a)
$$\begin{aligned}-0.355 &= -0.355\left(\frac{180}{\pi}\right)^{\circ}\\ &\approx -20.34\\ &= -(20° + (0.34)(60'))\\ &= -(20° + 20' + 0.4(60''))\\ &= -20°\,20'\,24''\end{aligned}$$

(b)
$$\begin{aligned}0.7865 &= 0.7865\left(\frac{180}{\pi}\right)^{\circ}\\ &\approx 45.0631\\ &= 45° + (0.0631)(60')\\ &= 45° + 3' + 0.786(60'')\\ &\approx 45°\,3'\,47''\end{aligned}$$

76.
$$\begin{aligned}s &= r\theta\\ 31 &= 12\theta\\ \theta &= \tfrac{31}{12} = 2\tfrac{7}{12} \text{ radians}\end{aligned}$$

78.
$$\begin{aligned}s &= r\theta\\ 60 &= 75\theta\\ \theta &= \tfrac{60}{75} = \tfrac{4}{5} \text{ radians}\end{aligned}$$

Because the angle represented is clockwise, this angle is $-\frac{4}{5}$ radians.

80. $\theta = \frac{s}{r} = \frac{10}{22} = \frac{5}{11}$ radian

82. $r = 80$ kilometers, $s = 160$ kilometers

$$\theta = \frac{s}{r} = \frac{160}{80} = 2 \text{ radians}$$

84. $r = 9$ feet, $\theta = 60° = \dfrac{\pi}{3}$

$s = r\theta = 9\left(\dfrac{\pi}{3}\right) = 3\pi$ feet

86. $r = 40$ centimeters, $\theta = \dfrac{3\pi}{4}$

$s = r\theta = 40\left(\dfrac{3\pi}{4}\right) = 30\pi$ centimeters

88. $r = 4000$ miles

$\theta = 31°\,47' + 26°\,10' = 57°\,57'$

≈ 1.011 radian

$s = r\theta \approx (4000)(1.011) \approx 4044$ miles

90. $r = 6378$ kilometers, $s = 800$ kilometers

$\theta = \dfrac{s}{r} = \dfrac{800}{6378} \approx 0.125 \text{ radians} = 0.125\left(\dfrac{180°}{\pi}\right)$

$\approx 7.19°$

92. $\theta = \dfrac{s}{r} = \dfrac{12}{5} = 2.4 \text{ radians} = 2.4\left(\dfrac{180°}{\pi}\right) \approx 137.5°$

94. Linear speed $= \dfrac{s}{t} = \dfrac{r\theta}{t} = \dfrac{(6400 + 1250)2\pi}{90} = 170\pi \approx 534.07$

96. (a) $\dfrac{\text{Revolutions}}{\text{Second}} = \dfrac{2400}{60} = 40$ rev/sec

Angular speed $= (2\pi)(40) = 80\pi$ rad/sec

(b) Radius of saw blade $= \dfrac{7.5}{2} = 3.75$ in.

Radius in feet $= \dfrac{3.75 \text{ in.}}{12 \text{ in./ft}} = 0.3125$ ft

Speed $= \dfrac{s}{t} = \dfrac{r\theta}{t} = r\dfrac{\theta}{t}$

$= r(\text{angular speed})$

$= 0.3125(80\pi) = 78.54$ ft/sec

98. (a) Arc length of larger sprocket in feet: $s = r\theta = \left(\dfrac{4}{12}\right)(2\pi) = \dfrac{2\pi}{3}$ feet

The angle θ of the smaller sprocket is $\theta = \dfrac{s}{r} = \dfrac{(2\pi/3) \text{ ft}}{(2/12) \text{ ft}} = 4\pi.$

The arc length of the tire is $s = r\theta = \left(\dfrac{14}{12}\right)(4\pi) = \dfrac{14\pi}{3}$ feet.

Speed $= \dfrac{s}{t} = \dfrac{(14\pi/3) \text{ ft}}{1 \text{ sec}} = \dfrac{14\pi}{3}$ ft/sec

(b) $\left(\dfrac{14\pi}{3} \text{ ft/sec}\right)(3600 \text{ sec/hr})\left(\dfrac{1 \text{ mile}}{5280 \text{ ft}}\right) \approx 10$ mi/hr

100. No, $-1260°$ is coterminal with $180°$

102. If θ is constant, the length of the arc is proportional to the radius ($s = r\theta$), and hence increasing.

104. Let A be the area of a circular sector of radius r and central angle θ. Then

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi} \implies A = \frac{1}{2}r^2\theta.$$

106. Because $s = r\theta$, $\theta = \frac{12}{15}$. Hence, $A = \frac{1}{2}r^2\theta = \frac{1}{2}15^2\left(\frac{12}{15}\right) = 90 \text{ ft}^2$.