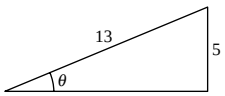
Section 4.3 Right Triangle Trigonometry

Solutions to Even-Numbered Exercises

2.



$b = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{12}{13}$

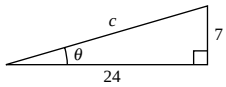
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{5}{12}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{13}{5}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{13}{12}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{12}{5}$

4.



$c = \sqrt{24^2 + 7^2} = 25$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{7}{25}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{24}{25}$

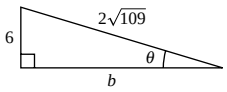
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{7}{24}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{25}{7}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{25}{24}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{24}{7}$

6.



$b = \sqrt{\left(2\sqrt{109}\right)^2 - 6^2} = 20$

$\sin\theta = \dfrac{6}{2\sqrt{109}} = \dfrac{3\sqrt{109}}{109}$

$\cos\theta = \dfrac{20}{2\sqrt{109}} = \dfrac{10\sqrt{109}}{109}$

$\tan\theta = \dfrac{6}{20} = \dfrac{3}{10}$

$\csc\theta = \dfrac{\sqrt{109}}{3}$

$\sec\theta = \dfrac{\sqrt{109}}{10}$

$\cot\theta = \dfrac{10}{3}$

8.

$\text{hyp} = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

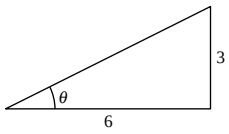
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1}{2}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{\sqrt{5}}{1} = \sqrt{5}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{\sqrt{5}}{2}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{2}{1} = 2$



$\text{hyp} = \sqrt{3^2 + 6^2} = 3\sqrt{5}$

$\sin\theta = \dfrac{3}{3\sqrt{5}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

$\cos\theta = \dfrac{6}{3\sqrt{5}} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$

$\tan\theta = \dfrac{3}{6} = \dfrac{1}{2}$

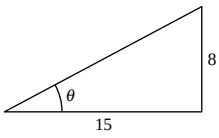
$\csc\theta = \dfrac{3\sqrt{5}}{3} = \sqrt{5}$

$\sec\theta = \dfrac{3\sqrt{5}}{6} = \dfrac{\sqrt{5}}{2}$

$\cot\theta = \dfrac{6}{3} = 2$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

10.



$\text{adj} = \sqrt{15^2 - 8^2} = \sqrt{161}$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{8}{15}$

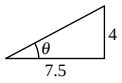
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{161}}{15}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{8}{\sqrt{161}} = \dfrac{8\sqrt{161}}{161}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{15}{8}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{15}{\sqrt{161}} = \dfrac{15\sqrt{161}}{161}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{\sqrt{161}}{8}$



$\text{adj} = \sqrt{7.5^2 - 4^2} = \dfrac{\sqrt{161}}{2}$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{4}{7.5} = \dfrac{8}{15}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{161}}{2 \cdot 7.5} = \dfrac{\sqrt{161}}{15}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{4}{\left(\sqrt{161}/2\right)} = \dfrac{8}{\sqrt{161}} = \dfrac{8\sqrt{161}}{161}$

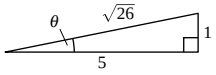
$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{7.5}{4} = \dfrac{15}{8}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{7.5}{\left(\sqrt{161}/2\right)} = \dfrac{15}{\sqrt{161}} = \dfrac{15\sqrt{161}}{161}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{\sqrt{161}}{2 \cdot 4} = \dfrac{\sqrt{161}}{8}$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

12. $\text{opp} = \sqrt{5^2 + 1^2} = \sqrt{26}$



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{1}{\sqrt{26}} = \dfrac{\sqrt{26}}{26}$

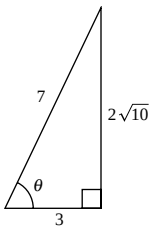
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{5}{\sqrt{26}} = \dfrac{5\sqrt{26}}{26}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1}{5}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{\sqrt{26}}{1} = \sqrt{26}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{\sqrt{26}}{5}$

14. $\text{opp} = \sqrt{7^2 - 3^2} = \sqrt{40} = 2\sqrt{10}$



$\sin\theta = \dfrac{2\sqrt{10}}{7}$

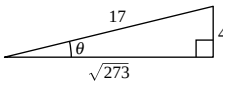
$\tan\theta = \dfrac{2\sqrt{10}}{3}$

$\csc\theta = \dfrac{7}{2\sqrt{10}} = \dfrac{7\sqrt{10}}{20}$

$\sec\theta = \dfrac{7}{3}$

$\cot\theta = \dfrac{3}{2\sqrt{10}} = \dfrac{3\sqrt{10}}{20}$

16. $\text{adj} = \sqrt{17^2 - 4^2} = \sqrt{237}$



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{4}{17}$

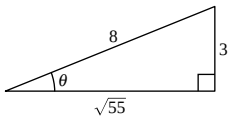
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{273}}{17}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{4}{\sqrt{273}} = \dfrac{4\sqrt{273}}{273}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{17}{\sqrt{273}} = \dfrac{17\sqrt{273}}{273}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\sqrt{273}}{4}$

18. $\text{adj} = \sqrt{8^2 - 3^2} = \sqrt{55}$



$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{55}}{8}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{3}{\sqrt{55}} = \dfrac{3\sqrt{55}}{55}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{8}{3}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{8}{\sqrt{55}} = \dfrac{8\sqrt{55}}{55}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\sqrt{55}}{3}$

20. $\sin 30^\circ = \dfrac{1}{2}$, $\tan 30^\circ = \dfrac{\sqrt{3}}{3}$

(a) $\csc 30^\circ = \dfrac{1}{\sin 30^\circ} = 2$

(b) $\cot 60^\circ = \tan(90^\circ - 60^\circ) = \tan 30^\circ = \dfrac{\sqrt{3}}{3}$

(c) $\cos 30^\circ = \dfrac{\sin 30^\circ}{\tan 30^\circ} = \dfrac{(1/2)}{\left(\sqrt{3}/3\right)} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

(d) $\cot 30^\circ = \dfrac{1}{\tan 30^\circ} = \dfrac{3}{\sqrt{3}} = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

22. $\sec\theta = 5$, $\tan\theta = 2\sqrt{6}$

(a) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{5}$

(b) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{2\sqrt{6}} = \dfrac{\sqrt{6}}{12}$

(c) $\cot(90^\circ - \theta) = \tan\theta = 2\sqrt{6}$

(d) $\sin\theta = \tan\theta\cos\theta = \left(2\sqrt{6}\right)\left(\dfrac{1}{5}\right) = \dfrac{2\sqrt{6}}{5}$

24. $\tan\beta = 5$ (β lies in Quadrant I or III)

(a) $\cot\beta = \dfrac{1}{\tan\beta} = \dfrac{1}{5}$

(b) $\sec^2\beta = 1 + \tan^2\beta \Rightarrow \cos\beta = \pm\dfrac{1}{\sqrt{1+\tan^2\beta}} = \pm\dfrac{1}{\sqrt{1+25}} = \pm\dfrac{1}{\sqrt{26}} = \pm\dfrac{\sqrt{26}}{26}$

(c) $\tan(90^\circ - \beta) = \cot\beta = \dfrac{1}{5}$

(d) $\csc\beta = \pm\sqrt{1+\cot^2\beta} = \pm\sqrt{1+\frac{1}{25}} = \pm\dfrac{\sqrt{26}}{5}$

26. $\csc\theta\tan\theta = \dfrac{1}{\sin\theta}\cdot\dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta} = \sec\theta$

28. $\cot\alpha\sin\alpha = \dfrac{\cos\alpha}{\sin\alpha}\sin\alpha = \cos\alpha$

30. $(\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = \csc^2\theta - \cot^2\theta = 1$

32.
$$\frac{\tan\beta + \cot\beta}{\tan\beta} = \frac{\tan\beta}{\tan\beta} + \frac{\cot\beta}{\tan\beta}$$
$$= 1 + \frac{\cot\beta}{(1/\cot\beta)}$$
$$= 1 + \cot^2\beta = \csc^2\beta$$

34. (a) $\csc 30^\circ = \dfrac{1}{\sin 30^\circ} = \dfrac{1}{(1/2)} = 2$

(b) $\sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

36. (a) $\sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

(b) $\csc 45^\circ = \dfrac{1}{\sin 45^\circ} = \dfrac{1}{\left(\sqrt{2}/2\right)} = \sqrt{2}$

38. (a) $\tan\dfrac{\pi}{3} = \dfrac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

(b) $\cot 30^\circ = \dfrac{\cos 30^\circ}{\sin 30^\circ} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

40. (a) $\tan 18.5^\circ \approx 0.3346$

(b) $\cot 71.5^\circ = \dfrac{1}{\tan 71.5^\circ} \approx 0.3346$

42. (a) $\cos(8^\circ\,50'\,25'') = \cos\left(8 + \dfrac{50}{60} + \dfrac{25}{3600}\right)$
$\approx \cos(8.840278) \approx 0.9881$

(b) $\sec(8^\circ\,50'\,25'') = \dfrac{1}{\cos(8^\circ\,50'\,25'')} \approx 1.0120$

44. (a) $\sec(1.25) = \dfrac{1}{\cos 1.25} \approx 3.1714$

(b) $\cos(1.25) \approx 0.3153$

46. (a) $\sec\left(\dfrac{\pi}{2} - 1\right) = \dfrac{1}{\cos\left(\dfrac{\pi}{2} - 1\right)} \approx 1.1884$

(b) $\cot\left(\dfrac{\pi}{2} - \dfrac{1}{2}\right) = \dfrac{1}{\tan\left(\dfrac{\pi}{2} - \dfrac{1}{2}\right)} \approx 0.5463$

48. (a) $\cos\theta = \dfrac{\sqrt{2}}{2} \Longrightarrow \theta = 45° = \dfrac{\pi}{4}$

(b) $\tan\theta = 1 \Longrightarrow \theta = 45° = \dfrac{\pi}{4}$

50. (a) $\tan\theta = \sqrt{3} \Longrightarrow \theta = 60° = \dfrac{\pi}{3}$

(b) $\cos\theta = \dfrac{1}{2} \Longrightarrow \theta = 60° = \dfrac{\pi}{3}$

52. (a) $\cot\theta = \dfrac{\sqrt{3}}{3}$

$\tan\theta = \dfrac{3}{\sqrt{3}} = \sqrt{3} \Longrightarrow \theta = 60° = \dfrac{\pi}{3}$

(b) $\sec\theta = \sqrt{2}$

$\cos\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \Longrightarrow \theta = 45° = \dfrac{\pi}{4}$

54. (a) $\cos\theta = 0.9848 \Longrightarrow \theta \approx 10° \approx 0.175$

(b) $\cos\theta = 0.8746 \Longrightarrow \theta \approx 29° \approx 0.506$

56. (a) $\sin\theta = 0.3746 \Longrightarrow \theta \approx 22° \approx 0.384$

(b) $\cos\theta = 0.3746 \Longrightarrow \theta \approx 68° \approx 1.187$

58. $\cos 60° = \dfrac{x}{16}$

$x = 16\cos 60° = 8$

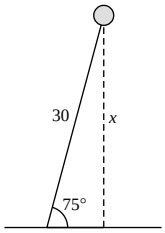
60. $\sin 45° = \dfrac{20}{r}$

$r = \dfrac{20}{\sin 45°} = \dfrac{20}{\sqrt{2}/2} = 20\sqrt{2}$

62 $\cos 75° = \dfrac{25}{r}$

$r = \dfrac{25}{\cos 75°} \approx 96.6$

64. (a)



(b) $\sin 75° = \dfrac{x}{30}$

(c) $x = 30\sin 75° \approx 28.98$ meters

66. Let x = distance from the boat to the shoreline.

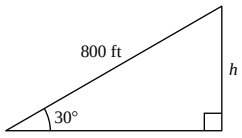
$\tan 4.5° = \dfrac{60}{x}$

$x = \dfrac{60}{\tan 4.5°} \approx 762.37$ feet

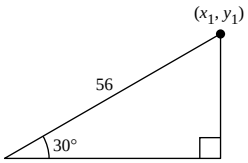
68. (a) $\sin 30° = \dfrac{h}{800}$

$h = 800\sin 30° = 400$ ft

(b) Vertical speed $= \dfrac{1}{2}(320) = 160$ ft/min.



70.



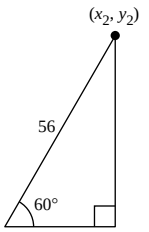
$$\sin 30° = \frac{y_1}{56}$$

$$y_1 = (\sin 30°)(56) = \left(\frac{1}{2}\right)(56) = 28$$

$$\cos 30° = \frac{x_1}{56}$$

$$x_1 = \cos 30°(56) = \frac{\sqrt{3}}{2}(56) = 28\sqrt{3}$$

$$(x_1, y_1) = \left(28\sqrt{3}, 28\right)$$



$$\sin 60° = \frac{y_2}{56}$$

$$y_2 = \sin 60°(56) = \left(\frac{\sqrt{3}}{2}\right)(56) = 28\sqrt{3}$$

$$\cos 60° = \frac{x_2}{56}$$

$$x_2 = (\cos 60°)(56) = \left(\frac{1}{2}\right)(56) = 28$$

$$(x_2, y_2) = \left(28, 28\sqrt{3}\right)$$

72. $x \approx 9.397,\ y \approx 3.420$

$$\sin\theta = \frac{y}{10} \approx 0.34$$

$$\cos\theta = \frac{x}{10} \approx 0.94$$

$$\tan\theta = \frac{y}{x} \approx 0.36$$

$$\cot\theta = \frac{x}{y} \approx 2.75$$

$$\sec\theta = \frac{10}{x} \approx 1.06$$

$$\csc\theta = \frac{10}{y} \approx 2.92$$

74. True.

$$\sin 60° \csc 60°$$

$$= \sin 60° \frac{1}{\sin 60°} = 1$$

76. False.

$$\sin 45° + \cos 45°$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} \neq 1$$

78. (a)

θ	0°	20°	40°	60°	80°
$\sin\theta$	0	0.3420	0.6428	0.8660	0.9848
$\cos\theta$	1	0.9397	0.7660	0.5000	0.1736
$\tan\theta$	0	0.3640	0.8391	1.7321	5.6713

(b) Sine and tangent are increasing; cosine is decreasing.

(c) In each case, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

80. $\tan\theta = \dfrac{0.672s^2}{3000}$

(a)

s	10	20	30	40	50	60
θ	1.28°	5.12°	11.40°	19.72°	29.25°	38.88°

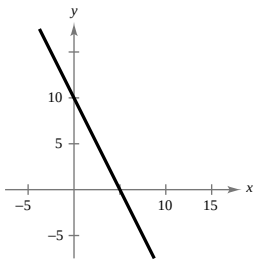
(b) θ increases at an increasing rate. The function is not linear.

82. $2x + y = 10$

$y = -2x + 10$

x-intercept: $(5, 0)$

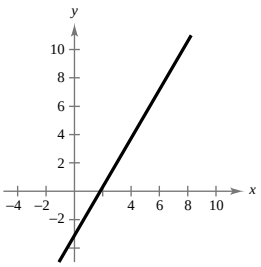
y-intercept: $(0, 10)$



84. $2x - 7y = 22$

x-intercept: $(11, 0)$

y-intercept: $\left(0, -\frac{22}{7}\right)$



86. $-290.8°$ is coterminal with $69.2°$ and lies in Quadrant I.

88. $-122° 50'$ lies in Quadrant III.

Section 4.4 Trigonometric Functions of Any Angle

Solutions to Even-Numbered Exercises

2. (a) $x = 12, y = -5$

$r = \sqrt{12^2 + (-5)^2} = 13$

$\sin\theta = \frac{y}{r} = \frac{-5}{13} = -\frac{5}{13}$

$\cos\theta = \frac{x}{r} = \frac{12}{13}$

$\tan\theta = \frac{y}{x} = \frac{-5}{12} = -\frac{5}{12}$

$\csc\theta = \frac{r}{y} = \frac{13}{-5} = -\frac{13}{5}$

$\sec\theta = \frac{r}{x} = \frac{13}{12}$

$\cot\theta = \frac{x}{y} = \frac{12}{-5} = -\frac{12}{5}$

(b) $x = -1, y = 1$

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x} = \frac{1}{-1} = -1$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

$\cot\theta = \frac{x}{y} = \frac{-1}{1} = -1$