

4. (a) $x = 3, y = 1$

$$r = \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$\sin \theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos \theta = \frac{x}{r} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\tan \theta = \frac{y}{x} = \frac{1}{3}$$

$$\csc \theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$$

$$\sec \theta = \frac{r}{x} = \frac{\sqrt{10}}{3}$$

$$\cot \theta = \frac{x}{y} = \frac{3}{1} = 3$$

(b) $x = 2, y = -4$

$$r = \sqrt{2^2 + (-4)^2} = 2\sqrt{5}$$

$$\sin \theta = \frac{y}{r} = \frac{-4}{2\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{-4}{2} = -2$$

$$\csc \theta = \frac{r}{y} = \frac{2\sqrt{5}}{-4} = -\frac{\sqrt{5}}{2}$$

$$\sec \theta = \frac{r}{x} = \frac{2\sqrt{5}}{2} = \sqrt{5}$$

$$\cot \theta = \frac{x}{y} = \frac{2}{-4} = -\frac{1}{2}$$

6. $x = 8, y = 15$

$$r = \sqrt{8^2 + 15^2} = 17$$

$$\sin \theta = \frac{y}{r} = \frac{15}{17}$$

$$\cos \theta = \frac{x}{r} = \frac{8}{17}$$

$$\tan \theta = \frac{y}{x} = \frac{15}{8}$$

$$\csc \theta = \frac{r}{y} = \frac{17}{15}$$

$$\sec \theta = \frac{r}{x} = \frac{17}{8}$$

$$\cot \theta = \frac{x}{y} = \frac{8}{15}$$

8. $x = -24, y = 10, r = \sqrt{(-24)^2 + (10)^2} = 26$

$$\sin \theta = \frac{y}{r} = \frac{10}{26} = \frac{5}{13}$$

$$\cos \theta = \frac{x}{r} = \frac{-24}{26} = -\frac{12}{13}$$

$$\tan \theta = \frac{y}{x} = \frac{10}{-24} = -\frac{5}{12}$$

$$\csc \theta = \frac{r}{y} = \frac{13}{5}$$

$$\sec \theta = \frac{r}{x} = -\frac{13}{12}$$

$$\cot \theta = \frac{x}{y} = -\frac{12}{5}$$

10. $x = -5, y = -6, r = \sqrt{(-5)^2 + (-6)^2} = \sqrt{61}$

$$\sin \theta = \frac{y}{r} = \frac{-6}{\sqrt{61}} = \frac{-6\sqrt{61}}{61}$$

$$\cos \theta = \frac{x}{r} = \frac{-5}{\sqrt{61}} = \frac{-5\sqrt{61}}{61}$$

$$\tan \theta = \frac{y}{x} = \frac{-6}{-5} = \frac{6}{5}$$

$$\csc \theta = \frac{r}{y} = -\frac{\sqrt{61}}{6}$$

$$\sec \theta = \frac{r}{x} = -\frac{\sqrt{61}}{5}$$

$$\cot \theta = \frac{x}{y} = \frac{5}{6}$$

12. $x = 6, y = -14,$

$$r = \sqrt{6^2 + (-14)^2} = \sqrt{232} = 2\sqrt{58}$$

$$\sin \theta = \frac{y}{r} = \frac{-14}{2\sqrt{58}} = \frac{-7}{\sqrt{58}} = \frac{-7\sqrt{58}}{58}$$

$$\cos \theta = \frac{x}{r} = \frac{6}{2\sqrt{58}} = \frac{3}{\sqrt{58}} = \frac{3\sqrt{58}}{58}$$

$$\tan \theta = \frac{y}{x} = \frac{-14}{6} = -\frac{7}{3}$$

$$\csc \theta = \frac{r}{y} = -\frac{\sqrt{58}}{7}$$

$$\sec \theta = \frac{r}{x} = \frac{\sqrt{58}}{3}$$

$$\cot \theta = \frac{x}{y} = \frac{-3}{7}$$

14. $\sin \theta > 0$ and $\cos \theta > 0$

$$\frac{y}{r} > 0 \text{ and } \frac{x}{r} > 0$$

Quadrant I

16. $\sec \theta > 0$ and $\cot \theta < 0$

$$\frac{r}{x} > 0 \text{ and } \frac{x}{y} < 0$$

Quadrant IV

18. $\tan \theta > 0$ and $\csc \theta < 0$

$$\frac{y}{x} > 0 \text{ and } \frac{r}{y} < 0$$

Quadrant III

20. $\cos \theta = \frac{x}{r} = \frac{-4}{5} \Rightarrow y = |3|$

$$\theta \text{ in Quadrant III} \Rightarrow y = -3$$

$$\sin \theta = \frac{y}{r} = -\frac{3}{5} \quad \csc \theta = -\frac{5}{3}$$

$$\cos \theta = \frac{x}{r} = -\frac{4}{5} \quad \sec \theta = -\frac{5}{4}$$

$$\tan \theta = \frac{y}{x} = \frac{3}{4} \quad \cot \theta = \frac{4}{3}$$

22. $\csc \theta = \frac{r}{y} = \frac{4}{1} \Rightarrow x = \pm\sqrt{15}$

$$\cot \theta < 0 \Rightarrow x = -\sqrt{15}$$

$$\sin \theta = \frac{y}{r} = \frac{1}{4} \quad \csc \theta = 4$$

$$\cos \theta = \frac{x}{r} = -\frac{\sqrt{15}}{4} \quad \sec \theta = -\frac{4\sqrt{15}}{15}$$

$$\tan \theta = \frac{y}{x} = -\frac{\sqrt{15}}{15} \quad \cot \theta = -\sqrt{15}$$

24. $\cot \theta$ is undefined $\Rightarrow \theta = n\pi$

$$\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \Rightarrow \theta = \pi, y = 0, x = -r$$

$$\sin \theta = \frac{y}{r} = \frac{0}{r} = 0 \quad \csc \theta = \frac{r}{y} \text{ is undefined.}$$

$$\cos \theta = \frac{x}{r} = \frac{-r}{r} = -1 \quad \sec \theta = \frac{r}{x} = -1$$

$$\tan \theta = \frac{y}{x} = \frac{0}{x} = 0 \quad \cot \theta = \frac{x}{y} \text{ is undefined.}$$

26. $\tan \theta$ is undefined $\Rightarrow \theta = n\pi + \frac{\pi}{2}$

$$\sin \theta = \frac{y}{r} = \frac{-r}{r} = -1 \quad \csc \theta = \frac{r}{y} = -1$$

$$\cos \theta = \frac{x}{r} = \frac{0}{r} = 0 \quad \sec \theta = \frac{r}{x} \text{ is undefined.}$$

$$\tan \theta = \frac{y}{x} \text{ is undefined.} \quad \cot \theta = \frac{x}{y} = \frac{0}{y} = 0$$

28. $(-x, -\frac{1}{3}x)$ Quadrant III

$$r = \sqrt{x^2 + \frac{1}{9}x^2} = \frac{\sqrt{10}x}{3}$$

$$\sin \theta = \frac{y}{r} = \frac{(1/3)x}{(\sqrt{10}x)/3} = -\frac{\sqrt{10}}{10}$$

$$\cos \theta = \frac{x}{r} = \frac{-x}{(\sqrt{10}x)/3} = -\frac{3\sqrt{10}}{10}$$

$$\tan \theta = \frac{y}{x} = \frac{(-1/3)x}{-x} = \frac{1}{3}$$

$$\csc \theta = \frac{r}{y} = \frac{(\sqrt{10}x)/3}{(-1/3)x} = -\sqrt{10}$$

$$\sec \theta = \frac{r}{x} = \frac{(\sqrt{10}x)/3}{-x} = -\frac{\sqrt{10}}{3}$$

$$\cot \theta = \frac{x}{y} = \frac{-x}{(-1/3)x} = 3$$

30. $4x + 3y = 0 \Rightarrow y = -\frac{4}{3}x$

$$(x, -\frac{4}{3}x) \text{ Quadrant IV}$$

$$r = \sqrt{x^2 + \frac{16}{9}x^2} = \frac{5}{3}x$$

$$\sin \theta = \frac{y}{r} = \frac{(-4/3)x}{(5/3)x} = -\frac{4}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{x}{(5/3)x} = \frac{3}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{(-4/3)x}{x} = -\frac{4}{3}$$

$$\csc \theta = -\frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\tan \theta = -\frac{3}{4}$$

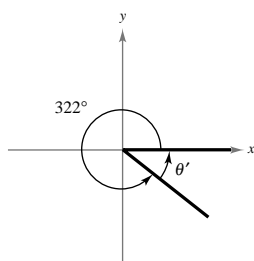
$$32. \tan \frac{\pi}{2} = \frac{y}{x} = \frac{1}{0} \implies \text{undefined}$$

since $\frac{\pi}{2}$ corresponds to $(0, 1)$.

$$36. \csc \frac{3\pi}{2} = \frac{1}{\sin \frac{3\pi}{2}} = \frac{1}{-1} = -1$$

$$40. \theta = 322^\circ$$

$$\theta' = 360^\circ - 322^\circ = 38^\circ$$



$$34. \csc \pi = \frac{r}{y} = \frac{1}{0} \implies \text{undefined}$$

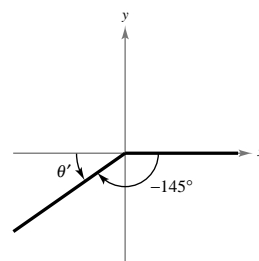
since π corresponds to $(-1, 0)$.

$$38. \csc \frac{\pi}{2} = \frac{1}{\sin \frac{\pi}{2}} = \frac{1}{1} = 1$$

$$42. \theta = -145^\circ$$

is coterminal with 215° .

$$\theta' = 215^\circ - 180^\circ = 35^\circ$$

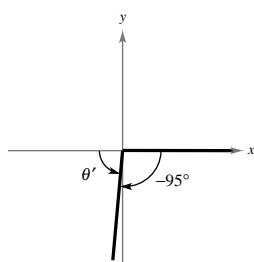


$$44. \theta = -95^\circ$$

is coterminal with

$$360^\circ - 95^\circ = 265^\circ.$$

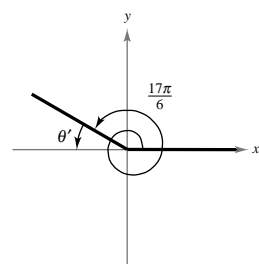
$$\theta' = 265^\circ - 180^\circ = 85^\circ$$



$$46. \theta = \frac{17\pi}{6}$$

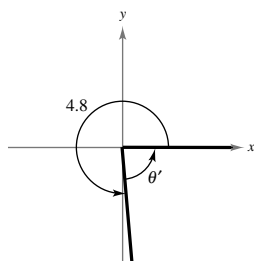
is coterminal with $\frac{5\pi}{6}$.

$$\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$$



$$48. \theta = 4.8$$

$$\theta' = 2\pi - 4.8 \approx 1.4832$$

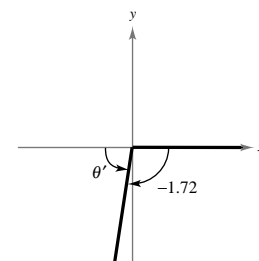


$$50. \theta = -1.72^\circ$$

is coterminal with

$$2\pi - 1.72 \approx 4.5632$$

$$\theta' = 4.5632 - \pi \approx 1.4216$$



$$52. \theta = 300^\circ, \theta' = 360^\circ - 300^\circ = 60^\circ, \text{ Quadrant IV}$$

$$\sin 300^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

$$\cos 300^\circ = \cos 60^\circ = \frac{1}{2}$$

$$\tan 300^\circ = -\tan 60^\circ = -\sqrt{3}$$

$$54. \theta = -495^\circ, \theta' = 45^\circ, \text{ Quadrant III}$$

$$\sin(-495^\circ) = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\cos(-495^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\tan(495^\circ) = \tan 45^\circ = 1$$

56. $\theta = -330^\circ$, $\theta' = 30^\circ$, Quadrant I

$$\sin(-330^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$\cos(-330^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan(-330^\circ) = \tan 30^\circ = \frac{\sqrt{3}}{3}$$

60. $\theta = -\frac{\pi}{2}$ is coterminal with $\frac{3\pi}{2}$.

$$\sin\left(-\frac{\pi}{2}\right) = \sin \frac{3\pi}{2} = -1$$

$$\cos\left(-\frac{\pi}{2}\right) = \cos \frac{3\pi}{2} = 0$$

$$\tan\left(-\frac{\pi}{2}\right) = \tan \frac{3\pi}{2} \text{ is undefined.}$$

64. $\theta = -\frac{20\pi}{3}$ is coterminal with $\frac{4\pi}{3}$ in Quadrant III.

$$\theta' = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

$$\sin \theta = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

$$\tan \theta = -\tan \frac{\pi}{3} = -\sqrt{3}$$

68. $\csc 330^\circ = \frac{1}{\sin 330^\circ} = -2.0000$

72. $\sin(-195^\circ) \approx 0.2588$

76. $\tan\left(-\frac{\pi}{9}\right) \approx -0.3640$

58. $\theta = \frac{\pi}{4}$, $\theta' = \frac{\pi}{4}$ in Quadrant I

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\tan \frac{\pi}{4} = 1$$

62. $\theta = \frac{10\pi}{3}$ is coterminal with $\frac{4\pi}{3}$.

$$\theta' = \frac{4\pi}{3} - \pi = \frac{\pi}{3} \text{ in Quadrant III.}$$

$$\sin \frac{10\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos \frac{10\pi}{3} = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

$$\tan \frac{10\pi}{3} = \tan \frac{\pi}{3} = \sqrt{3}$$

66. $\sec 225^\circ = \frac{1}{\cos 225^\circ} \approx -1.4142$

70. $\cot(-220^\circ) = \frac{1}{\tan(-220^\circ)} \approx -1.1918$

74. $\sin(-0.65) \approx -0.6052$

78. $\cos\left(\frac{-15\pi}{14}\right) \approx -0.9749$

$$80. (a) \cos \theta = \frac{\sqrt{2}}{2} \implies \text{reference angle is } 45^\circ \text{ or } \frac{\pi}{4}$$

and θ is in Quadrant I or IV.

Values in degrees: $45^\circ, 315^\circ$

Values in radians: $\frac{\pi}{4}, \frac{7\pi}{4}$

$$(b) \cos \theta = -\frac{\sqrt{2}}{2} \implies \text{reference angle is } 45^\circ \text{ or } \frac{\pi}{4}$$

and θ is in Quadrant II or III.

Values in degrees: $135^\circ, 225^\circ$

Values in radians: $\frac{3\pi}{4}, \frac{5\pi}{4}$

$$84. (a) \cot \theta = -\sqrt{3} \implies \frac{\cos \theta}{\sin \theta}$$

Reference angle is $\frac{\pi}{6}$ or 30° .

Values in degrees: $150^\circ, 330^\circ$

Values in radians: $\frac{5\pi}{6}, \frac{11\pi}{6}$

$$(b) \text{ Value in degrees: } 45^\circ \text{ or } 315^\circ$$

Value in radians: $\frac{\pi}{4}$ or $\frac{7\pi}{4}$

$$86. \cos \theta = 0.8746$$

Quadrant I: $\theta = \cos^{-1} 0.8746 \approx 29.00^\circ$

Quadrant IV: $\theta = 360^\circ - 29.00^\circ = 331.00^\circ$

$$90. \csc \theta = -1.0038 \implies \sin \theta \approx -0.9962$$

Quadrant IV: $\theta \approx \sin^{-1}(-0.9962) \approx -85.01^\circ$, or 274.99°

Quadrant III: $\theta = 180 + 85.01 = 265.01^\circ$

$$92. \tan \theta = -2.1832 \implies \tan^{-1}(-2.1832) \approx -65.39$$

Quadrant II: $\theta \approx 114.61^\circ$

Quadrant IV: $\theta \approx 294.61^\circ$

$$82. (a) \csc \theta = -\sqrt{2} \implies \sin \theta = \frac{-1}{\sqrt{2}}$$

Reference angle 45° or $\frac{\pi}{4}$

Values in degrees: $225^\circ, 315^\circ$

Values in radians: $\frac{5\pi}{4}, \frac{7\pi}{4}$

$$(b) \csc \theta = 2 \implies \sin \theta = \frac{1}{2}$$

Reference angle is $\frac{\pi}{6}$ or 30° .

Values in degrees: $30^\circ, 150^\circ$

Values in radians: $\frac{\pi}{6}, \frac{5\pi}{6}$

$$88. \cot \theta = 0.7521 \implies \tan \theta \approx 1.3296$$

Quadrant I: $\theta \approx \tan^{-1}(1.3296) \approx 53.05^\circ$

Quadrant III: $\theta \approx 180^\circ + 53.05^\circ \approx 233.05^\circ$

$$94. \sin \theta = 0.0175$$

Quadrant I: $\theta = \sin^{-1} 0.0175 \approx 1.00^\circ$

Quadrant II: $\theta = 180^\circ - 1.00^\circ = 179^\circ$

$$96. \cot \theta = 5.671 \Rightarrow \tan \theta = \frac{1}{5.671}$$

$$\text{Quadrant I: } \theta = \tan^{-1}\left(\frac{1}{5.671}\right) \approx 10.00^\circ$$

$$\text{Quadrant III: } \theta = 180^\circ + 10^\circ = 190^\circ$$

$$100. \csc \theta = -2$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\cot^2 \theta = \csc^2 \theta - 1$$

$$\cot^2 \theta = (-2)^2 - 1$$

$$\cot^2 \theta = 3$$

$\cot \theta < 0$ in Quadrant IV.

$$\cot \theta = -\sqrt{3}$$

$$98. \cot \theta = -3$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$1 + (-3)^2 = \csc^2 \theta$$

$$10 = \csc^2 \theta$$

$\csc \theta > 0$ in Quadrant II.

$$\sqrt{10} = \csc \theta$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sin \theta = \frac{1}{\csc \theta} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$102. \sec \theta = -\frac{9}{4}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

$$\tan^2 \theta = \left(-\frac{9}{4}\right)^2 - 1$$

$$\tan^2 \theta = \frac{65}{16}$$

$\tan \theta > 0$ in Quadrant III.

$$\tan \theta = \frac{\sqrt{65}}{4}$$

$$104. S = 23.1 + 0.442t + 4.3 \sin \frac{\pi t}{6}$$

$$(a) \text{ February 2000} \Rightarrow t = 2$$

$$S = 23.1 + 0.442(2) + 4.3 \sin \frac{2\pi}{6}$$

$$\approx 27.7 \text{ thousand or } 27,700 \text{ units}$$

$$(c) \text{ September 2000} \Rightarrow t = 9$$

$$S = 23.1 + 0.442(9) + 4.3 \sin \frac{9\pi}{6}$$

$$\approx 22.8 \text{ thousand or } 22,800 \text{ units}$$

$$(b) \text{ February 2001} \Rightarrow t = 14$$

$$S = 23.1 + 0.442(14) + 4.3 \sin \frac{14\pi}{6}$$

$$\approx 33.0 \text{ thousand or } 33,000 \text{ units}$$

$$(d) \text{ September 2001} \Rightarrow t = 21$$

$$S = 23.1 + 0.442(21) + 4.3 \sin \frac{21\pi}{6}$$

$$\approx 28.1 \text{ thousand or } 28,100 \text{ units}$$

$$106. \text{ False. } \tan 24^\circ = -\tan 156^\circ$$

$$108. \text{ False. } -\cot\left(\frac{3\pi}{4}\right) = -(-1) = 1 \text{ and } \cot\left(-\frac{\pi}{4}\right) = -1$$

110. (a)

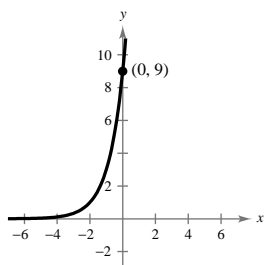
θ	0°	20°	40°	60°	80°
$\sin \theta$	0	0.3420	0.6428	0.8660	0.9848
$\sin(180^\circ - \theta)$	0	0.3420	0.6428	0.8660	0.9848

(b) It appears that $\sin \theta = \sin(180^\circ - \theta)$ for all θ .

112. (a) $\sin 0.75 = y \approx 0.7$

(b) $\cos 2.5 = x \approx -0.8$

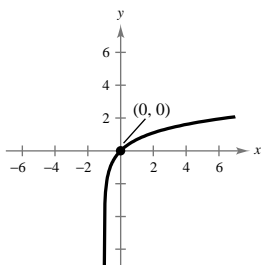
114. $y = 3^{x+2}$



Intercept: $(0, 9)$

Horizontal asymptote: $y = 0$

116. $y = \ln(x + 1)$



Intercept: $(0, 0)$

Asymptote: $x = -1$

118. $\frac{4500}{4 + e^x} = 50$

$$90 = 4 + e^{2x}$$

$$86 = e^{2x}$$

$$2x = \ln 86$$

$$x = \frac{1}{2} \ln 86 \approx 2.227$$

120. $\ln \sqrt{x + 10} = \frac{1}{2} \ln(x + 10) = 1 \Rightarrow \ln(x + 10) = 2$

$$\Rightarrow x + 10 = e^2 \Rightarrow x = e^2 - 10 \approx -2.611$$

122. $\sec \theta = 8$

$$b = \sqrt{8^2 - 1^2} = \sqrt{63}$$

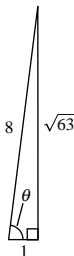
$$\cos \theta = \frac{1}{8}$$

$$\sin \theta = \frac{\sqrt{63}}{8}$$

$$\tan \theta = \frac{\sqrt{63}}{1}$$

$$\cot \theta = \frac{1}{\sqrt{63}} = \frac{\sqrt{63}}{63}$$

$$\csc \theta = \frac{8}{\sqrt{63}} = \frac{8\sqrt{63}}{63}$$



126. False. $-\cot\left(\frac{3\pi}{4}\right) = -(-1) = 1$ and $\cot\left(-\frac{\pi}{4}\right) = -1$

Section 4.5 Graphs of Sine and Cosine Functions

Solutions to Even-Numbered Exercises

2. $y = 2 \cos 3x$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{3}$$

$$\text{Amplitude} = |a| = 2$$

6. $y = \frac{3}{2} \cos \frac{\pi x}{2}$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{(\pi/2)} = 4$$

$$\text{Amplitude} = |a| = \frac{3}{2}$$

10. $y = \frac{1}{3} \sin 10x$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{10} = \frac{\pi}{5}$$

$$\text{Amplitude: } |a| = \frac{1}{3}$$

4. $y = -3 \sin \frac{x}{3}$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{(1/3)} = 6\pi$$

$$\text{Amplitude} = |a| = |-3| = 3$$

8. $y = -\cos \frac{2x}{5}$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{\left(\frac{2}{5}\right)} = 5\pi$$

$$\text{Amplitude} = |a| = |-1| = 1$$

12. $y = \frac{5}{2} \cos \frac{x}{4}$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{(1/4)} = 8\pi$$

$$\text{Amplitude} = |a| = \frac{5}{2}$$