14. $y = \frac{3}{4}\cos\frac{\pi x}{12}$

Period $= \frac{2\pi}{b} = \frac{2\pi}{\left(\frac{\pi}{12}\right)} = 24$

Amplitude: $|a| = \frac{3}{4}$

16. $f(x) = \cos x$, $g(x) = \cos(x + \pi)$

g is a horizontal shift of f π units to the left.

18. $f(x) = \sin 3x$, $g(x) = \sin(-3x)$

g is a reflection of f about the y-axis.
(or, about the x-axis)

20. $f(x) = \sin x$, $g(x) = \sin 3x$

The period of g is one-third the period of f.

22. $f(x) = \cos 4x$, $g(x) = -6 + \cos 4x$

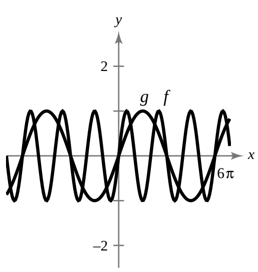
g is a vertical shift of f six units downward.

24. The period of g is one-half the period of f.

26. Shift the graph of f two units upward to obtain the graph of g.

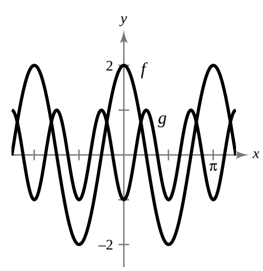
28. $f(x) = \sin x$, $g(x) = \sin\frac{x}{3}$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0
$\sin\frac{x}{3}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$



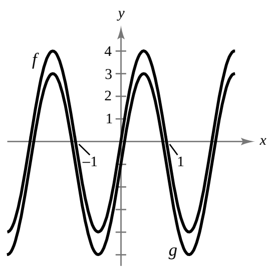
30. $f(x) = 2\cos 2x$, $g(x) = -\cos 4x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2\cos 2x$	2	0	-2	0	2
$-\cos 4x$	-1	1	-1	1	-1



32. $f(x) = 4 \sin \pi x, g(x) = 4 \sin \pi x - 1$

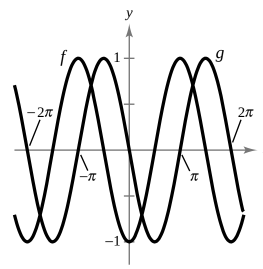
x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	4	0	-4	0
$g(x)$	-1	3	-1	-5	-1



34. $f(x) = -\cos x$

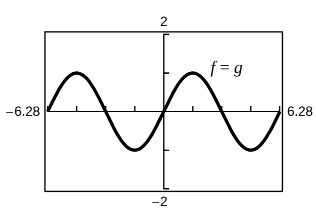
$g(x) = -\cos\left(x - \frac{\pi}{2}\right)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$-\cos x$	-1	0	1	0	-1
$-\cos(x - \pi)$	0	-1	0	1	0



36. $f(x) = \sin x, g(x) = -\cos\left(x + \frac{\pi}{2}\right)$

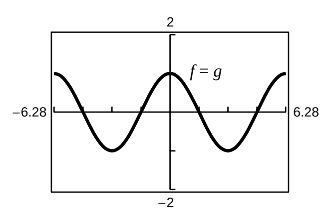
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0
$-\cos\left(x - \frac{\pi}{2}\right)$	0	1	0	-1	0



Conjecture: $\sin x = -\cos\left(x + \frac{\pi}{2}\right)$

38. $f(x) = \cos x, g(x) = -\cos(x - \pi)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x$	1	0	-1	0	1
$-\cos(x - \pi)$	1	0	-1	0	1



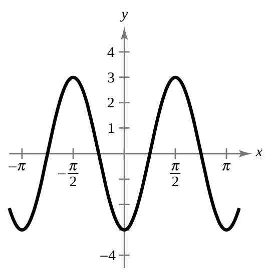
Conjecture: $\cos x = -\cos(x - \pi)$

40. $y = -3 \cos 2x$

Period $= \dfrac{2\pi}{2} = \pi$

Amplitude $= 3$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	-3	0	3	0	-3

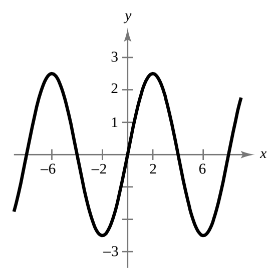


42. $y = \dfrac{5}{2} \sin \dfrac{\pi x}{4}$

Period $= \dfrac{2\pi}{(\pi/4)} = 8$

Amplitude $= \dfrac{5}{2}$

x	0	2	4	6	8
y	0	$\frac{5}{2}$	0	$-\frac{5}{2}$	0

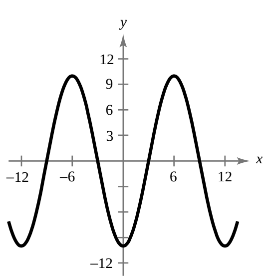


44. $y = -10 \cos \dfrac{\pi x}{6}$

Period $= \dfrac{2\pi}{(\pi/6)} = 12$

Amplitude $= 10$

x	0	3	6	9	12
y	-10	0	10	0	-10

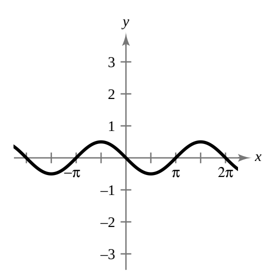


46. $y = \dfrac{1}{2} \sin(x - \pi)$

Period $= 2\pi$

Amplitude $= \dfrac{1}{2}$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	$-\frac{3}{2}$	0	$\frac{3}{2}$	0

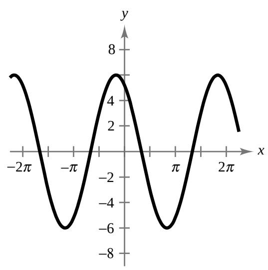


48. $y = 6\cos\left(x + \frac{\pi}{6}\right)$

Period $= 2\pi$

Amplitude $= 6$

x	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{5\pi}{6}$
y	6	$3\sqrt{3}$	3	0	-6

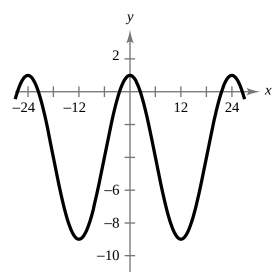


50. $y = -4 + 5\cos\frac{\pi t}{12}$

Period $= \frac{2\pi}{(\pi/12)} = 24$

Amplitude $= 5$

t	0	6	12	18	24
y	1	-4	-9	-4	1

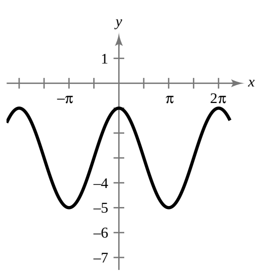


52. $y = 2\cos x - 3$

Period $= 2\pi$

Amplitude $= 2$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	-1	-3	-5	-3	-1

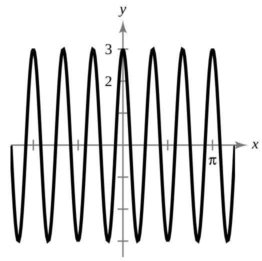


54. $y = -3\cos(6x + \pi)$

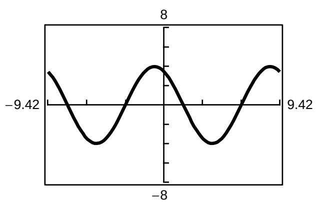
Period $= \dfrac{2\pi}{6} = \dfrac{\pi}{3}$

Amplitude $= 3$

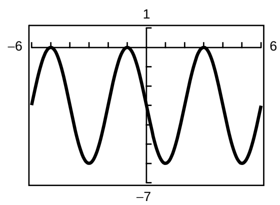
x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{3\pi}{12}$	$\frac{\pi}{3}$
y	3	0	-3	0	3



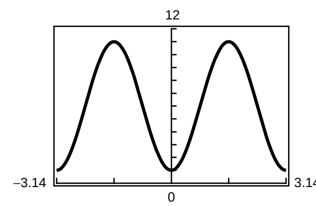
56. $y = -4\sin\left(\dfrac{2}{3}x - \dfrac{\pi}{3}\right)$



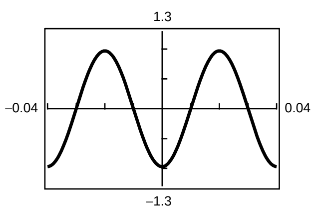
58. $y = 3\cos\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right) - 3$



60. $y = 5\cos(\pi - 2x) + 6$



62. $y = -\dfrac{97}{100}\cos(50\pi t)$



64. $f(x) = a\cos x + d$

Amplitude $= \dfrac{-2 - (-4)}{2} = 1$

Reflected in the x-axis: $a = -1$

$-4 = -1\cos 0 + d$

$d = -3$

$y = -3 - \cos x$

66. $y = a\cos x + d$

Amplitude $= \dfrac{1}{2}$

Period $= 2\pi$

Reflected in x-axis, $a = -1$

$d = -4$

$y = -4 - \dfrac{1}{2}\cos x$

68. $y = a\sin(bx - c)$

Amplitude $= 2 \Longrightarrow a = 2$

Period $= 4\pi$

$\dfrac{2\pi}{b} = 4\pi \Longrightarrow b = \dfrac{1}{2}$

Phase shift: $c = 0$

$y = 2\sin\left(\dfrac{x}{2}\right)$

70. $y = a\sin(bx - c)$

Amplitude $= 2 \Longrightarrow a = 2$

Period $= 4$

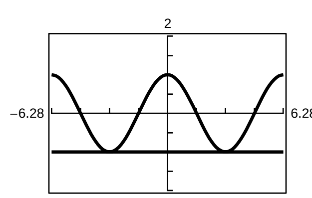
$\dfrac{2\pi}{b} = 4 \Longrightarrow b = \dfrac{\pi}{2}$

Phase shift: $\dfrac{c}{b} = -1 \Longrightarrow c = -\dfrac{\pi}{2}$

$y = 2\sin\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right)$

72. $y_1 = \cos x$

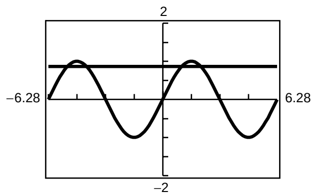
$y_2 = -1$



$y_1 = y_2$ when $x = \pi, -\pi$.

74. $y_1 = \sin x$

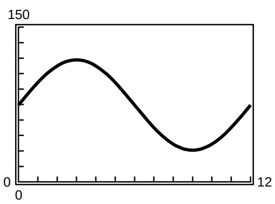
$y_2 = \dfrac{\sqrt{3}}{2}$



$y_1 = y_2$ when $x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, -\dfrac{4\pi}{3}, -\dfrac{5\pi}{3}$.

76. The period of the model would change because the time for a respiratory cycle would decrease.

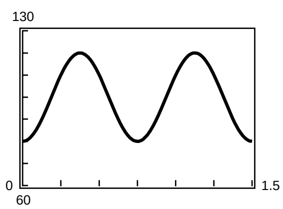
78. $S = 74.50 + 43.75 \sin \dfrac{\pi t}{6}$



Maximum sales: March ($t = 3$)

Minimum sales: September ($t = 9$)

80. $P = 100 - 20 \cos \dfrac{8\pi}{3}\text{t}$



$$\text{period} = \frac{2\pi}{\left(\frac{8\pi}{3}\right)} = \frac{3}{4}$$

$$\frac{1\text{ heartbeat}}{\left(\frac{3}{4}\right)} \Longrightarrow \frac{4}{3}\text{ heartbeats/second} = 80\text{ heartbeats/min}$$

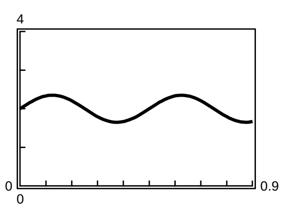
82. (a) Yes, y is a function of t because for each value of t there corresponds one and only one value of y.

(b) The period is approximately $2(0.375 - 0.125) = 0.5$ seconds.

The amplitude is approximately $\frac{1}{2}(2.35 - 1.65) = 0.35$ centimeters.

(c) One model is $y = 0.35 \sin 4\pi t + 2$.

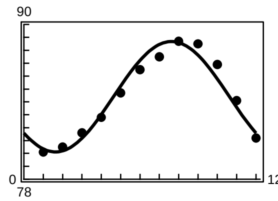
(d)



84. (a) A model for Chicago is

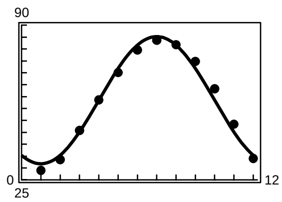
$$C(t) = 56.35 + 27.35 \sin\left(\frac{\pi t}{6} + 4.19\right).$$

(b)



The model is a good fit for most months.

(c)



The model is a good fit.

(d) Use the constant term of each model to estimate the average annual temperature.

Honolulu: 84.40°

Chicago: 56.35°

(e) Each model has a period of 12. This corresponds to the 12 months in a year.

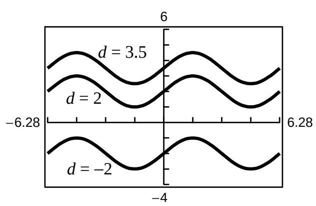
(f) Chicago has a greater variability in temperatures during the year. The amplitude of each model indicates this variability.

86. True

88. $y = 2 + \sin x$

$y = 3.5 + \sin x$

$y = -2 + \sin x$

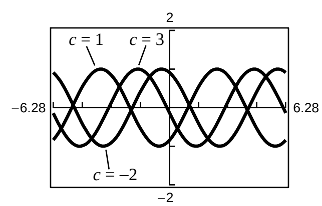


Each value of d produces a vertical shift of $y = \sin x$ upward (or downward) by d units.

90. $y = \sin(x - 1)$

$y = \sin(x - 3)$

$y = \sin(x - (-2))$



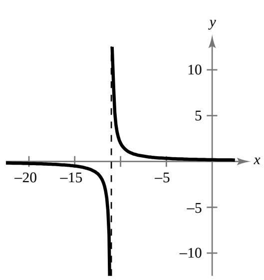
Each value of c produces a horizontal shift of $y = \sin x$ to the left (or right) by c units.

92. (a) In Exercise 91, $f(x) = \cos x$ is even and we saw that $h(x) = \cos^2 x$ is even. Therefore, for $f(x)$ even and $h(x) = [f(x)]^2$, we make the conjecture that $h(x)$ is even.

(b) In Exercise 91, $g(x) = \sin x$ is odd and we saw that $h(x) = \sin^2 x$ is even. Therefore, for $g(x)$ odd and $h(x) = [g(x)]^2$, we make the conjecture that $h(x)$ is even.

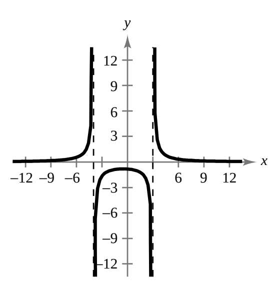
(c) From part (c) of 91, we conjecture that the product of an even function and an odd function is odd.

94.



Asymptotes: $x = -11, y = 0$

96. $f(x) = \dfrac{10}{(x + 4)(x - 3)}$



Asymptotes: $x = -4, x = 3, y = 0$

98. $\dfrac{13\pi}{2} = \dfrac{13\pi}{2}\left(\dfrac{180}{\pi}\right) = 1170°$

100. $8.5\pi = 8.5\pi\left(\dfrac{180}{\pi}\right) = 1530°$