Section 4.6 Graphs of Other Trigonometric Functions

Solutions to Even-Numbered Exercises

2. $y = \tan \frac{x}{2}$

Period $= \frac{\pi}{b} = \frac{\pi}{(1/2)} = 2\pi$

Asymptotes: $x = -\pi, x = \pi$

Matches graph (d).

4. $y = 2 \csc x$

Period $= \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Asymptotes: $x = 0, x = \pi$

Matches graph (a).

6. $y = \frac{1}{2} \sec \frac{\pi x}{2}$

Period $= \frac{2\pi}{b} = \frac{2\pi}{(\pi/2)} = 4$

Asymptotes: $x = -1, x = 1$

Matches graph (h).

8. $y = -2 \sec 2\pi x$

Period $= \frac{2\pi}{2\pi} = 1$

Asymptotes: $x = -\frac{1}{4}, x = \frac{1}{4}$

Reflected in x-axis

Matches graph (c).

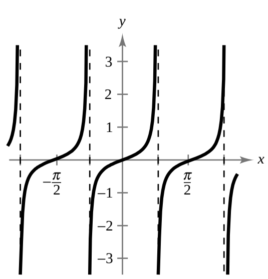
10. $y = \frac{1}{4} \tan 2x$

Period $= \frac{\pi}{2}$

Asymptotes: $2x = -\frac{\pi}{2} \Rightarrow x = -\frac{\pi}{4}$

$2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$

x	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$
y	$-\frac{1}{4}$	0	$\frac{1}{4}$

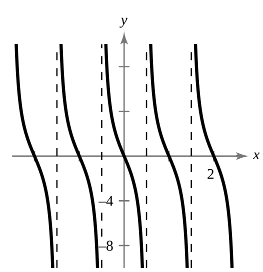


12. $y = -3 \tan \pi x$

Period $= \frac{\pi}{\pi} = 1$

Asymptotes: $x = -\frac{1}{2}, x = \frac{1}{2}$

x	$-\frac{1}{4}$	0	$\frac{1}{4}$
y	3	0	-3

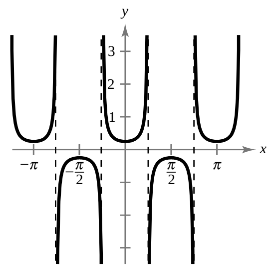


14. $y = \frac{1}{4} \sec 2x$

Period $= \pi$

Asymptotes: $-\frac{\pi}{4}, \frac{\pi}{4}$

x	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{2}$
y	0.354	$\frac{1}{4}$	0.354	$-\frac{1}{4}$

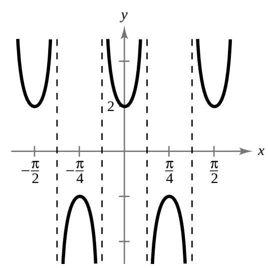


16. $y = 2 \sec 4x$

Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

Asymptotes: $x = -\frac{\pi}{8}, x = \frac{\pi}{8}$

x	$-\frac{\pi}{16}$	0	$\frac{\pi}{16}$
y	2.828	2	2.828

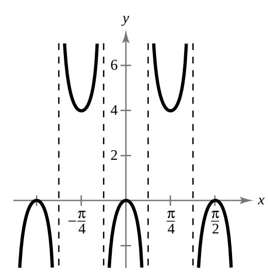


18. $y = -2 \sec 4x + 2$

Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

Asymptotes: $x = -\frac{\pi}{8}, x = \frac{\pi}{8}$

x	$-\frac{\pi}{16}$	0	$\frac{\pi}{16}$
y	-0.828	0	-0.828

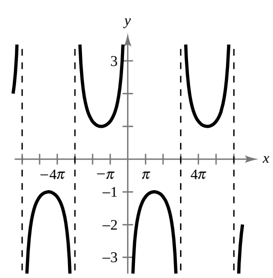


20. $y = -\csc \frac{x}{3}$

Period $= \frac{2\pi}{(1/3)} = 6\pi$

Asymptotes: $x = 0, x = 3\pi$

x	π	2π	4π
y	-1.155	-1.155	1.155

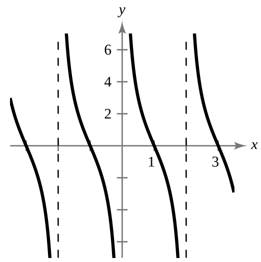


22. $y = 3 \cot \frac{\pi x}{2}$

Period $= \frac{\pi}{(\pi/2)} = 2$

Asymptotes: $x = 0, x = 2$

x	$\frac{1}{4}$	1	$\frac{3}{2}$
y	7.243	0	-3

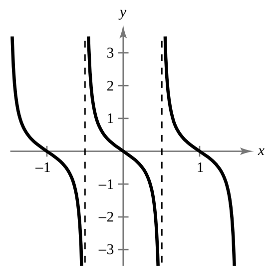


24. $y = -\frac{1}{2} \tan \pi x$

Period $= 1$

Asypmtotes: $x = -\frac{1}{2}, x = \frac{1}{2}$

x	$-\frac{1}{4}$	0	$\frac{1}{4}$
y	$\frac{1}{2}$	0	$-\frac{1}{2}$

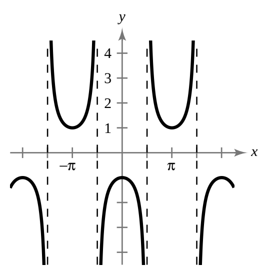


26. $y = \sec(x + \pi)$

Period $= 2\pi$

Asymptotes: $x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	-1.414	-1	-1.414

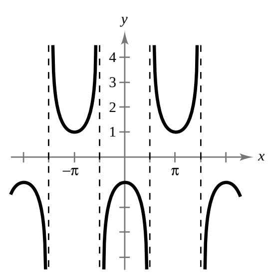


28. $y = \sec(\pi - x)$

Period $= 2\pi$

Asymptotes: $x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	-1.414	-1	-1.414

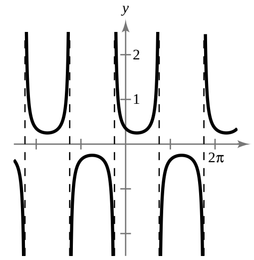


30. $y = \frac{1}{4}\csc\left(x + \frac{\pi}{4}\right)$

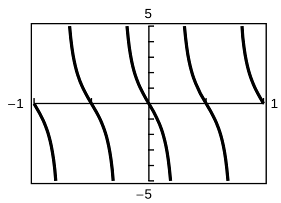
Period: 2π

Asymptotes: $x = -\frac{\pi}{4}, \frac{3\pi}{4}$

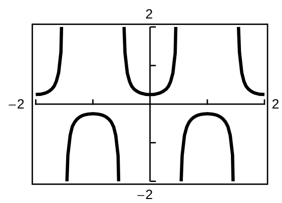
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
y	$\frac{\sqrt{2}}{4}$	$\frac{1}{4}$	$\frac{\sqrt{2}}{4}$



32. $y = -2\tan 2\pi x$

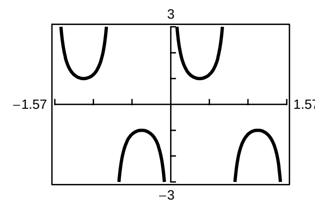


34. $y = \frac{1}{4}\sec \pi x = \frac{1}{4\cos \pi x}$

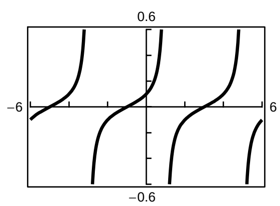


36. $y = -\csc(4x - \pi)$

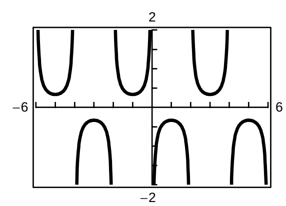
$y = \frac{-1}{\sin(4x - \pi)}$



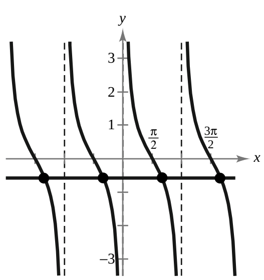
38. $y = 0.1\tan\left(\frac{\pi x}{4} + \frac{\pi}{4}\right)$



40. $y = \frac{1}{3}\sec\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) \Rightarrow y = \frac{1}{3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)}$



42.

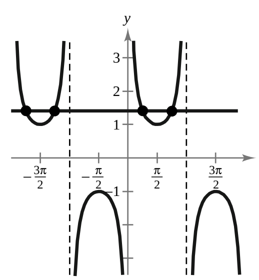


The solutions appear to be:

$x = -\frac{7\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

(or in decimal form: $-3.665, -0.524, 2.618, 5.760$)

44.



The solutions appear to be:

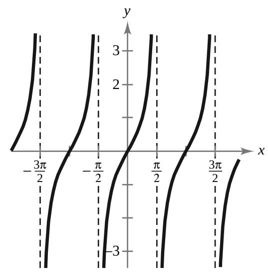
$-\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

(or in decimal form: $-5.498, -3.927, 0.785, 2.356$)

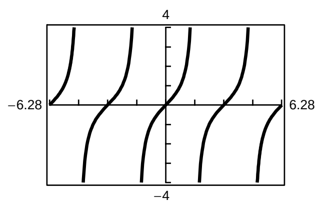
46. $f(x) = \tan x$

$\tan(-x) = -\tan x$

Thus, the function is odd and the graph of $y = \tan x$ is symmetric with the origin.



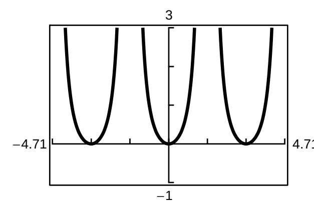
48. $y_1 = \sin x \sec x, y_2 = \tan x$



It appears that $y_1 = y_2$.

$$\sin x \sec x = \sin x \frac{1}{\cos x} = \frac{\sin x}{\cos x} = \tan x$$

50. $y_1 = \sec^2 x - 1, y_2 = \tan^2 x$



It appears that $y_1 = y_2$.

$$1 + \tan^2 x = \sec^2 x$$
$$\tan^2 x = \sec^2 x - 1$$

52. $f(x) = |x \sin x|$

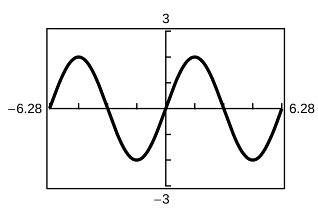
Matches graph (a) as $x \longrightarrow 0, f(x) \longrightarrow 0$.

54. $g(x) = |x| \cos x$. Even function

Matches graph (c) as $x \longrightarrow 0, g(x) \longrightarrow 0$.

56. $f(x) = \sin x - \cos\left(x + \frac{\pi}{2}\right)$

$g(x) = 2 \sin x$

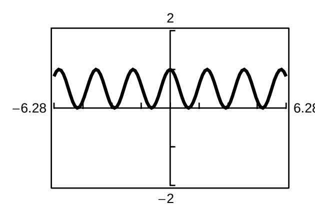


It appears that $f(x) = g(x)$. That is, that

$$\sin x - \cos\left(x + \frac{\pi}{2}\right) = 2 \sin x.$$

58. $f(x) = \cos^2 \frac{\pi x}{2}$

$g(x) = \frac{1}{2}(1 + \cos \pi x)$



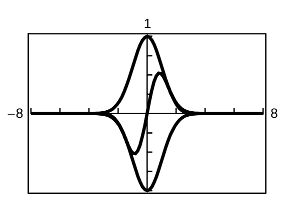
It appears that $f(x) = g(x)$. That is, that

$$\cos^2 \frac{\pi x}{2} = \frac{1}{2}(1 + \cos \pi x).$$

60. $g(x) = e^{-x^2/2} \sin x$

Damping factor: $y = e^{-x^2/2}$

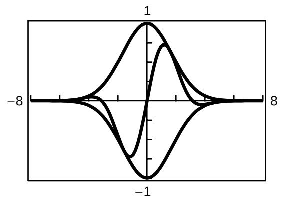
$-e^{-x^2/2} \le g(x) \le e^{-x^2/2}$



As $x \to \infty, g \to 0$

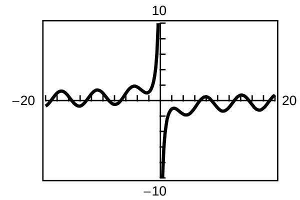
62. $h(x) = 2^{-x^2/4} \sin x$

Damping factor: $2^{-x^2/4}$



As $x \longrightarrow \infty$, $h(x) \longrightarrow 0$.

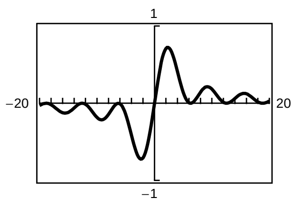
64. $f(x) = \sin x - \dfrac{4}{x}$



As $x \to 0$ from the left, $f(x) \to \infty$

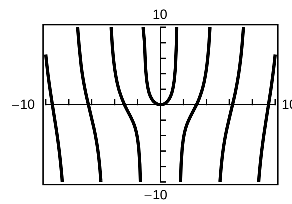
As $x \to 0$ from the right, $f(x) \to -\infty$

66. $f(x) = \dfrac{1 - \cos x}{x}$



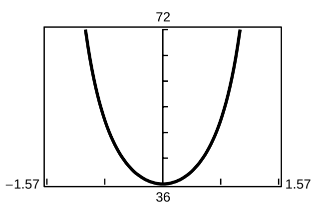
As $x \to 0$, $f(x) \to 0$

68. $f(x) = \dfrac{x}{\cot x} = x \tan x$



As $x \to 0$, $f(x) \to 0$

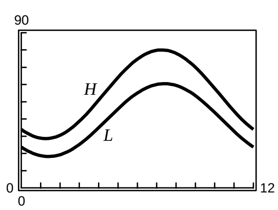
70. $\cos x = \dfrac{36}{d} \Longrightarrow d = \dfrac{36}{\cos x} = 36 \sec x$



72. $H(t) = 54.33 - 20.38 \cos \dfrac{\pi t}{6} - 15.69 \sin \dfrac{\pi t}{6}$

$L(t) = 39.36 - 15.70 \cos \dfrac{\pi t}{6} - 14.16 \sin \dfrac{\pi t}{6}$

(a)



Period of $\cos \dfrac{\pi t}{6}$: $\dfrac{2\pi}{(\pi/6)} = 12$

Period of $\sin \dfrac{\pi t}{6}$: $\dfrac{2\pi}{(\pi/6)} = 12$

Period of $H(t)$: 12

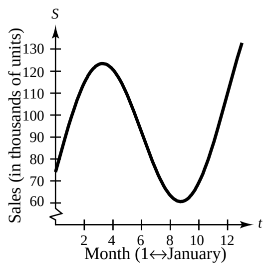
Period of $L(t)$: 12

(b) From the graph, it appears that the greatest difference between high and low temperatures occurs in summer. The smallest difference occurs in winter.

(c) The highest high and low temperatures appear to occur around the middle of July, roughly one month after the time when the sun is northernmost in the sky.

74. $S = 74 + 3t + 40 \sin \dfrac{\pi t}{6}$

(a)



(b) Maximum: $t \approx 3.3$ (March)

Minimum: $t \approx 8.7$ (August)

76. (a) Yes. For each t there corresponds one and only one value of y.

(b) One way to determine the frequency is to note that the time between the first and second maximum points is $t = 0.7622 - 0 = 0.7622$. Thus, the frequency is approximately $(0.7622)^{-1} = 1.3$ oscillation per second.

(c) One way to find such a model is to first fit an exponential model $y = ab^t$ to the data points $(0, 12)$, $(0.7622, 3.76)$, $(1.5476, 1.16)$.

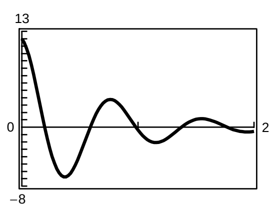
This yields

$$y = 12(0.2210)^t$$

Using $\dfrac{2\pi}{0.7622} \approx 8.2$ for the cosine term, we have $y = 12(0.2210)^t \cos(8.2t)$

(d) $\ln 0.221 \approx -1.51 \Rightarrow y = 12e^{-1.5t} \cos(8.2t)$

(e)



78. (a) 850 rev/min

(b) The direction of the saw is reversed.

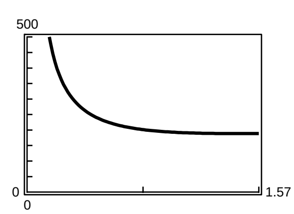
(c) $L = 60\left[\left(\dfrac{\pi}{2} + \phi\right) + \cot \phi\right],\ 0 < \phi < \dfrac{\pi}{2}$

(d)

ϕ	0.3	0.6	0.9	1.2	1.5
L	306.2	217.9	195.9	189.6	188.5

(e) Straight line lengths change faster.

(f)



80. True. $-2 \csc\left(-\dfrac{\pi}{3} + \dfrac{\pi}{3}\right)$ is not defined.

82. As $x \to \dfrac{\pi}{2}$ from the left, $\tan x \to \infty$

As $x \to \dfrac{\pi}{2}$ from the right, $\tan x \to -\infty$