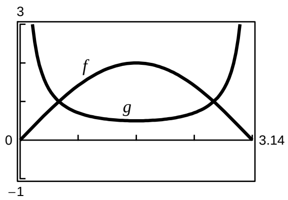
**84.** $f(x) = 2 \sin x$

$g(x) = \dfrac{1}{2} \csc x$

(a)




(b) $f > g$ on the interval, $\dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$

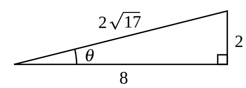
(c) As $x \longrightarrow \pi$, (from the left) $f(x) = 2 \sin x \longrightarrow 0$ and $g(x) = \dfrac{1}{2} \csc x \longrightarrow \infty$ since $g(x)$ is the reciprocal of $f(x)$.

**86.** Not one-to-one

**88.** One-to-one

$$f(x) = \sqrt{3x - 14}$$
$$y = \sqrt{3x - 14}$$
$$x = \sqrt{3y - 14}$$
$$x^2 = 3y - 14$$
$$y = \frac{x^2 + 14}{3}$$
$$f^{-1}(x) = \frac{x^2 + 14}{3}, x \geq 0$$

**90.** Third side $= \sqrt{68 - 4} = 8$




$$\sin \theta = \frac{2}{2\sqrt{17}} = \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}$$
$$\cos \theta = \frac{8}{2\sqrt{17}} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$$
$$\tan \theta = \frac{2}{8} = \frac{1}{4}$$
$$\cot \theta = 4$$
$$\csc \theta = \sqrt{17}$$
$$\sec \theta = \frac{\sqrt{17}}{4}$$

# Section 4.7 Inverse Trigonometric Functions

### Solutions to Even-Numbered Exercises

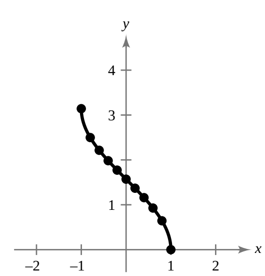
**2.** $y = \arccos x$

(a)

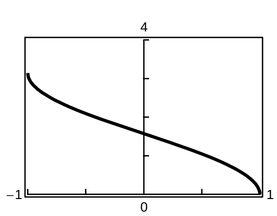
| $x$ | −1 | −0.8 | −0.6 | −0.4 | −0.2 |
|---|---|---|---|---|---|
| $y$ | 3.1416 | 2.4981 | 2.2143 | 1.9823 | 1.7722 |

| $x$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|---|
| $y$ | 1.5708 | 1.3694 | 1.1593 | 0.9273 | 0.6435 | 0 |

(b)




(c)




(d) Intercepts are $\left(0, \dfrac{\pi}{2}\right)$ and $(1, 0)$.
No symmetry

**4.** $\sin\frac{\pi}{6} = \frac{1}{2} \Longrightarrow \arcsin\frac{1}{2} = \frac{\pi}{6}$

**6.** $\arccos\frac{\sqrt{2}}{2} = \frac{\pi}{4} \Longrightarrow \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

**8.** (a) $\arcsin\frac{1}{2} = \frac{\pi}{6} \approx 0.524$

(b) $\arcsin 0 = 0$

**10.** (a) $\arctan\frac{\sqrt{3}}{3} = \frac{\pi}{6} \approx 0.524$

(b) $\arctan 1 = \frac{\pi}{4} \approx 0.785$

**12.** (a) $y = \arctan\left(-\sqrt{3}\right) \Longrightarrow \tan y = \sqrt{3}$ for $-\frac{\pi}{2} < y < \frac{\pi}{2} \Longrightarrow y = -\frac{\pi}{3}$

(b) $y = \arctan\sqrt{3} \Longrightarrow \tan y = \sqrt{3} \Longrightarrow y = \frac{\pi}{3}$

**14.** (a) $y = \arcsin -\frac{\sqrt{3}}{2} \Longrightarrow \sin y = -\frac{\sqrt{3}}{2}$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Longrightarrow y = -\frac{\pi}{3}$

(b) $y = \arctan\left(\frac{-\sqrt{3}}{3}\right) \Longrightarrow \tan y = \frac{-\sqrt{3}}{3} \Longrightarrow y = -\frac{\pi}{6}$

**16.** $y = \arctan x \leftrightarrow \tan y = x$

$\left(-\sqrt{3}, -\frac{\pi}{3}\right), \left(-\frac{\sqrt{3}}{3}, -\frac{\pi}{6}\right), \left(1, \frac{\pi}{4}\right)$

**18.** (a) $\arccos 0.22 \approx 1.35$

(b) $\arcsin 0.45 \approx 0.47$

**20.** (a) $\arctan(-6) \approx -1.41$

(b) $\arctan 18 \approx 1.52$

**22.** (a) $\arccos(-0.51) \approx 2.11$

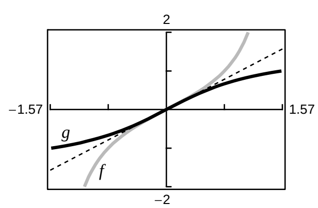
(b) $\arcsin(-0.125) \approx -0.13$

**24.** $f(x) = \tan x$ and $g(x) = \arctan x$

Graph: $y_1 = \tan x$

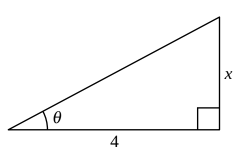
$y_2 = \tan^{-1} x$

$y_3 = x$



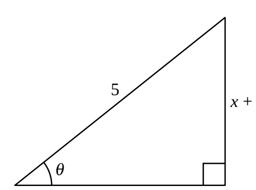

**26.** $\tan\theta = \frac{x}{4}$

$\theta = \arctan\frac{x}{4}$




**28.** $\sin\theta = \frac{x+2}{5}$

$\theta = \arcsin\left(\frac{x+2}{5}\right)$




**30.** $\sin(\arcsin 0.7) = 0.7$

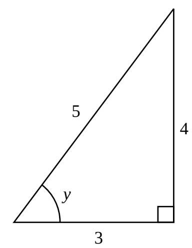
**32.** $\cos[\arccos(-0.3)] = -0.3$

**34.** $\arcsin(\sin 3\pi) = \arcsin(0) = 0$

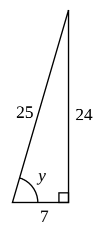
Note: $3\pi$ is not in the range of the arcsine function.

**36.** $\arctan\left(\tan\frac{11\pi}{6}\right) = \arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

**38.** Let $y = \arctan\frac{4}{3}$. Then $\tan y = \frac{4}{3}$, $0 < y < \frac{\pi}{2}$, and $\sin y = \frac{4}{5}$.



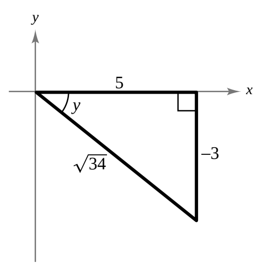

**40.** Let $y = \arcsin\frac{24}{25}$. Then $\sin y = \frac{24}{25}$, and $\cos y = \frac{7}{25}$.




**42.** Let $y = \arctan\left(-\frac{3}{5}\right)$. Then,

$\tan y = -\frac{3}{5}$, $-\frac{\pi}{2} < y < 0$

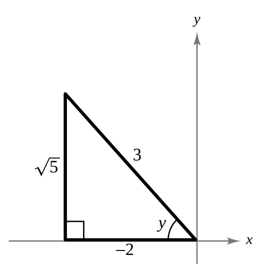
and $\sec y = \frac{\sqrt{34}}{5}$.




**44.** Let $y = \arccos\left(-\frac{2}{3}\right)$. Then,

$\cos y = -\frac{2}{3}$, $-\frac{\pi}{2} < y < \pi$

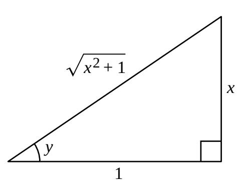
and $\sin y = \frac{\sqrt{5}}{3}$.




**46.** Let $y = \arctan x$. Then,

$\tan y = x$

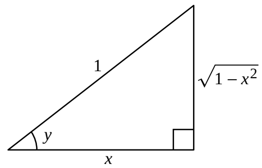
and $\cot y = \frac{1}{x}$.



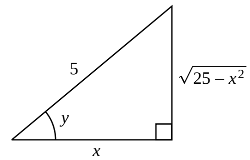
**48.** Let $y = \arccos x$. Then,

$\cos y = x = \dfrac{x}{1}$

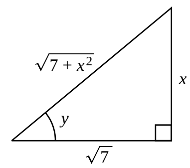
and $\sin y = \sqrt{1 - x^2}$.



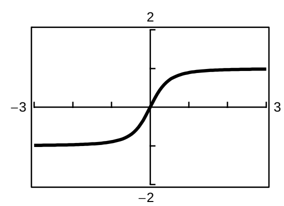

**50.** Let $y = \arccos \dfrac{x}{5}$. Then $\cos y = \dfrac{x}{5}$, and $\tan y = \dfrac{\sqrt{25 - x^2}}{x}$.




**52.** Let $y = \arctan \dfrac{x}{\sqrt{7}}$. Then $\tan y = \dfrac{x}{\sqrt{7}}$ and $\csc y = \dfrac{\sqrt{7 + x^2}}{x}$.




**54.** $f(x) = \sin(\arctan 2x)$, $g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$



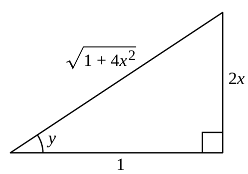

Let $y = \arctan 2x$. Then,

$\tan y = 2x = \dfrac{2x}{1}$

and $\sin y = \dfrac{2x}{\sqrt{1 + 4x^2}}$.

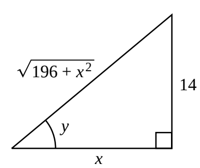
$g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}} = f(x)$

The graph has horizontal asymptotes at $y = \pm 1$.




**56.** Let $y = \arctan \dfrac{14}{x}$. Then $\tan y = \dfrac{14}{x}$ and $\sin y = \dfrac{14}{\sqrt{196 + x^2}}$.

Thus $y = \arcsin\left(\dfrac{14}{\sqrt{196 + x^2}}\right)$.




**58.** Let $y = \arccos \dfrac{3}{\sqrt{x^2 - 2x + 10}}$. Then,

$\cos y = \dfrac{3}{\sqrt{x^2 - 2x + 10}} = \dfrac{3}{\sqrt{(x-1)^2 + 9}}$

and $\sin y = \dfrac{|x - 1|}{\sqrt{(x-1)^2 + 9}}$.

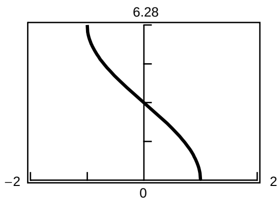
Thus, $y = \arcsin \dfrac{|x - 1|}{\sqrt{(x-1)^2 + 9}} = \arcsin \dfrac{|x - 1|}{\sqrt{x^2 - 2x + 10}}$.

**60.** $y = 2 \arccos x$

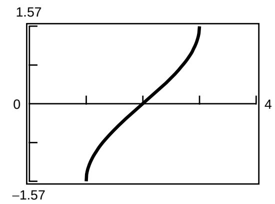
Domain: $-1 \le x \le 1$

Range: $0 \le y \le 2\pi$

Vertical stretch of $f(x) = \arccos x$



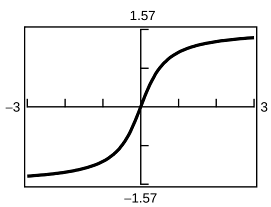

**62.** The graph of $f(x) = \arcsin(x - 2)$ is a horizontal translation of the graph of $y = \arcsin x$ by two units.



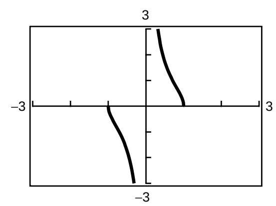

**64.** $f(x) = \arctan 2x$

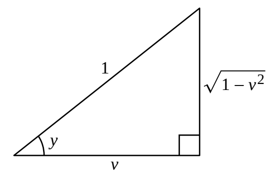
Domain: all real numbers

Range: $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$




**66.** $h(v) = \tan(\arccos v) = \dfrac{\sqrt{1 - v^2}}{v}$



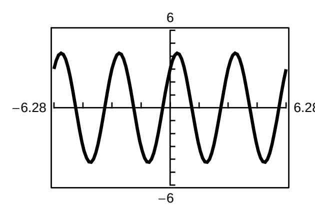

Domain: $-1 \le v \le 1, v \ne 0$

Range: all real numbers

**68.** $f(t) = 3 \cos 2t + 3 \sin 2t = \sqrt{3^2 + 3^2} \sin\left(2t + \arctan \dfrac{3}{3}\right)$

$$= 3\sqrt{2} \sin(2t + \arctan 1)$$

$$= 3\sqrt{2} \sin\left(2t + \frac{\pi}{4}\right)$$



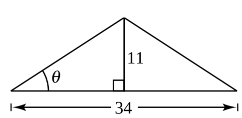

The graphs are the same.

**70.** (a) $\sin \theta = \dfrac{10}{s} \Longrightarrow \theta = \arcsin\left(\dfrac{10}{s}\right)$

(b) $s = 52$: $\theta = \arcsin\left(\dfrac{10}{52}\right) \approx 0.1935 \quad (\approx 11.1°)$

$s = 26$: $\theta = \arcsin\left(\dfrac{10}{26}\right) \approx 0.3948 \quad (\approx 22.6°)$

**72.** (a) $\tan \theta = \dfrac{11}{17} \Longrightarrow \theta \approx 0.5743$ or $32.9°$

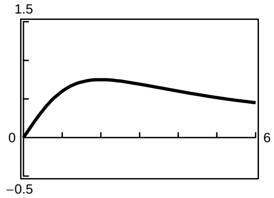



(b) If base diameter = 40 feet,

$\tan \theta = \dfrac{h}{20} \Longrightarrow h = 20 \cdot \tan \theta \approx 12.9$ feet.

**74.** $\beta = \arctan \dfrac{3x}{x^2 + 4}$

(a)




(b) $\beta$ is maximum when $x = 2$.

(c) The graph has a horizontal asymptote at $\beta = 0$. As $x$ increases, $\beta$ decreases.

**76.** (a) $\tan \theta = \dfrac{5}{x}$

$\theta = \arctan \dfrac{5}{x}$

(b) $x = 10$: $\theta = \arctan \dfrac{5}{10} \approx 26.6° = 0.46$ rad

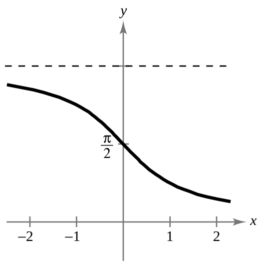
$x = 3$: $\theta = \arctan \dfrac{5}{3} \approx 59.0° = 1.03$ rad

**78.** False. $\arcsin \dfrac{1}{2} = \dfrac{\pi}{6}$

**80.** $y = \text{arccot}\, x$ if and only if $\cot y = x$.

Domain: $-\infty < x < \infty$

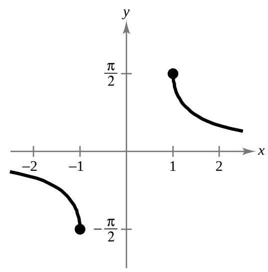
Range: $0 < x < \pi$




**82.** $y = \text{arccsc}\, x$ if and only if $\csc y = x$.

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[-\dfrac{\pi}{2}, 0\right) \cup \left(0, \dfrac{\pi}{2}\right]$




**84.** Let $y = \arcsin(-x)$. Then,

$$\begin{aligned} \sin y &= -x \\ -\sin y &= x \\ \sin(-y) &= x \\ -y &= \arcsin x \\ y &= -\arcsin x. \end{aligned}$$

Therefore, $\arcsin(-x) = -\arcsin x$.

**86.**

$$\begin{aligned} y &= \pi - \arccos x \\ \cos y &= \cos(\pi - \arccos x) \\ \cos y &= \cos \pi \cos(\arccos x) + \sin \pi \sin(\arccos x) \\ \cos y &= -x \\ y &= \arccos(-x) \end{aligned}$$