88. Let $\alpha = \arcsin x$ and $\beta = \arccos x$. Then, $\sin \alpha = x$ and $\cos \beta = x$. Thus, $\sin \alpha = \cos \beta$ which implies that α and β are complementary angles and we have

$$\alpha + \beta = \frac{\pi}{2}$$

$$\arcsin x + \arccos x = \frac{\pi}{2}.$$

90. $840°$ is coterminal with $120°$. Quadrant II

$$\sin 840° = \sin 120° = \frac{\sqrt{3}}{2}$$

$$\cos 840° = \cos 120° = -\frac{1}{2}$$

$$\tan 840° = \tan 120° = -\sqrt{3}$$

92. $\frac{17\pi}{3}$ is coterminal with $\frac{5\pi}{3}$. Quadrant IV

$$\sin \frac{17\pi}{3} = \sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos \frac{17\pi}{3} = \cos \frac{5\pi}{3} = \frac{1}{2}$$

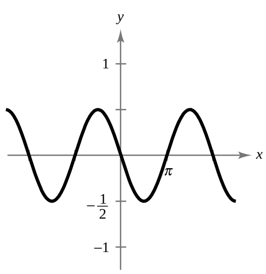
$$\tan \frac{17\pi}{3} = -\sqrt{3}$$

94. $y = \frac{1}{2} \sin (x + \pi)$

Amplitude $= \frac{1}{2}$

Period $= 2\pi$

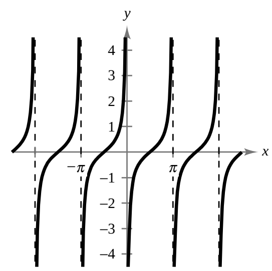
Phase shift: $-\pi$



96. $y = \frac{1}{2} \tan \left(x + \frac{\pi}{2}\right)$

Period $= \pi$

Asymptotes: $x = 0$, $x = \pi$



Section 4.8 Applications and Models

Solutions to Even-Numbered Exercises

2. $B = 56°, c = 15$

$$A = 90° - 56° = 34°$$

$$\sin B = \frac{b}{c} \Longrightarrow b = c \sin B = 5 \sin 56° \approx 12.44$$

$$\cos B = \frac{a}{c} \Longrightarrow a = c \cos B = 15 \cos 56° \approx 8.39$$

4. $A = 7.4°, a = 40.5$

$$B = 90° - 7.4° = 82.6°$$

$$\tan A = \frac{a}{b} \Longrightarrow b = \frac{a}{\tan A} = \frac{40.5}{\tan 7.4°} \approx 311.83$$

$$\sin A = \frac{a}{c} \Longrightarrow c = \frac{a}{\sin a} = \frac{40.5}{\sin 7.4°} \approx 314.45$$

6. $a = 25, c = 35$

$b = \sqrt{c^2 - a^2} = \sqrt{35^2 - 25^2} = \sqrt{600} \approx 24.49$

$\sin A = \frac{a}{c} \Longrightarrow A = \arcsin \frac{a}{c}$

$= \arcsin \frac{25}{35} \approx 45.58$

$\cos B = \frac{a}{c} \Longrightarrow B = \arccos \frac{a}{c}$

$= \arccos \frac{25}{35} \approx 44.42°$

8. $b = 1.72, c = 8.35$

$a = \sqrt{c^2 - b^2} = \sqrt{66.7641} \approx 8.17$

$\cos A = \frac{b}{c} \Longrightarrow A = \arccos\left(\frac{b}{c}\right) \approx \arccos\left(\frac{1.72}{8.35}\right) \approx 78.11°$

$\sin B = \frac{b}{c} \Longrightarrow B = \arcsin\left(\frac{b}{c}\right) \approx 11.89°$

10. $B = 65°\ 12', a = 14.2$

$A = 90° - B = 90° - 65°\ 12' = 24°\ 48'$

$\cos B = \frac{a}{c} \Longrightarrow c = \frac{a}{\cos B}$

$= \frac{14.2}{\cos 65^{o}\ 12'} \approx 33.85$

$\tan B = \frac{b}{a} \Longrightarrow b = a \tan B$

$= 14.2 \tan 65°\ 12'$
≈ 30.73

12. $\theta = 18°, b = 10$ meters

$\tan \theta = \frac{\text{altitude}}{b/2}$

$\text{altitude} = \frac{b}{2} \tan \theta$

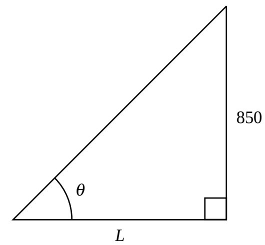
$= \frac{10}{2} \tan 18° \approx 1.62$ meters

14. $\theta = 72.94°, b = 3.36$ cm

$\tan \theta = \frac{\text{altitude}}{\left(\frac{b}{2}\right)}$

$\text{altitude} = \frac{b}{2} \tan \theta = \frac{3.36}{2} \tan 72.94° \approx 5.47$ cm

16. (a) $\tan \theta = \dfrac{850}{L} \Longrightarrow L = \dfrac{850}{\tan \theta} = 850 \cot \theta$

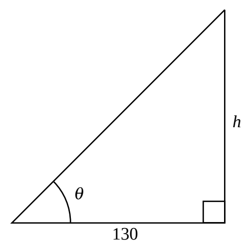


(b)

θ	$10°$	$20°$	$30°$	$40°$	$50°$
L	4821	2335	1472	1013	713

(c) No, the cotangent function is not a linear function.

18. (a) $\tan \theta = \dfrac{h}{130} \Longrightarrow h = 130 \tan \theta$



(b)

θ	$10°$	$15°$	$20°$	$25°$	$30°$
h	22.9	34.8	47.3	60.6	75.1

20. $\tan 28° = \dfrac{a}{100} \Longrightarrow a = 100 \tan 28°$

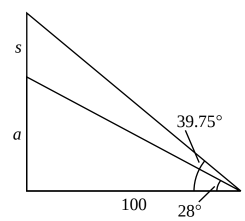
$$\tan 39.75° = \frac{a + s}{100}$$

$$a + s = 100 \tan 39.75°$$

$$s = 100 \tan 39.75° - a$$

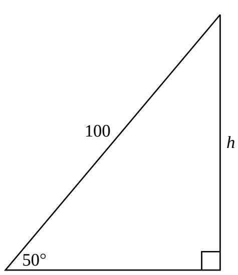
$$= 100 \tan 39.75 - 100 \tan 28°$$

$$\approx 30 \text{ feet}$$

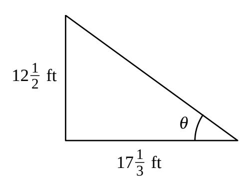


22. $\sin 50° = \dfrac{h}{100}$

$$h = 100 \sin 50° \approx 76.6 \text{ feet}$$



24. (a)



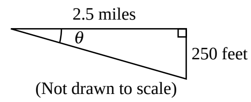
(b) $\tan \theta = \dfrac{12\frac{1}{2}}{17\frac{1}{3}}$

(c) $\theta = \arctan \dfrac{12\frac{1}{2}}{17\frac{1}{3}} \approx 35.8°$

26. $\tan\theta = \dfrac{250}{2.5(5280)}$

$$\theta = \arctan\left(\frac{250}{2.5(5280)}\right)$$

$$\approx 1.1°$$

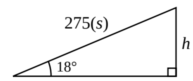


28. $\sin 18° = \dfrac{h}{275(s)}$

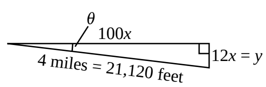
$$s = \frac{h}{275 \sin 18°}$$

If $h = 10{,}000$, $s = \dfrac{10{,}000}{275\sin 18°} \approx 117.7$ seconds

If $h = 16{,}000$, $s = \dfrac{16{,}000}{275\sin 18°} \approx 188.3$ seconds



30.



Angle of grade: $\tan\theta = \dfrac{12x}{100x}$

$\theta = \arctan 0.12 \approx 6.8°$

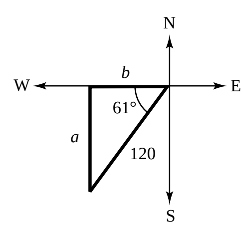
Change in elevation: $\sin\theta = \dfrac{y}{21{,}120}$

$y = 21{,}120 \sin\theta = 21{,}120 \sin(\arctan 0.12) \approx 2516.3$ feet

32. $90° - 29° = 61°$; $(20)(6) = 120$ nautical miles

$\sin 61° = \dfrac{a}{120} \Longrightarrow a \approx 104.95$ nautical miles

$\cos 61° = \dfrac{b}{120} \Longrightarrow b \approx 58.18$ nautical miles



34. $\tan 14° = \dfrac{d}{x} \Longrightarrow x = d\cot 14°$

$\tan 34° = \dfrac{d}{y} \Longrightarrow \dfrac{d}{30 - x} = \dfrac{d}{30 - d\cot 14°}$

$$\cot 34° = \frac{30 - d\cot 14°}{d}$$

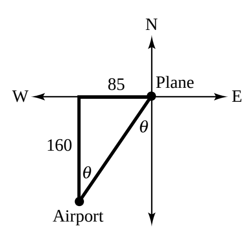
$$d\cot 34° = 30 - d\cot 14°$$

$$d = \frac{30}{\cot 34° + \cot 14°}$$

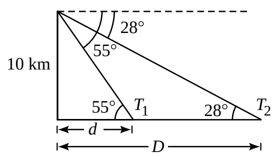
$$\approx 5.46 \text{ kilometers}$$

36. $\tan\theta = \dfrac{85}{160} \Longrightarrow \theta \approx 27.98°$

Bearing: S 27.98° W



38.



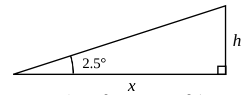
$$\cot 55° = \frac{d}{10} \Longrightarrow d \approx 7 \text{ kilometers}$$

$$\cot 28° = \frac{D}{10} \Longrightarrow D \approx 18.8 \text{ kilometers}$$

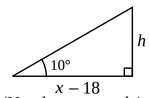
Distance between towns:

$$D - d = 18.8 - 7 = 11.8 \text{ kilometers}$$

40.



(Not drawn to scale)



(Not drawn to scale)

$$\tan 2.5° = \frac{h}{x}, \tan 10° = \frac{h}{x - 18}$$

$$x = \frac{h}{\tan 2.5°}, x = \frac{h}{\tan 10°} + 18$$

$$\frac{h}{\tan 2.5°} = \frac{h}{\tan 10°} + 18 = \frac{h + 18 \tan 10°}{\tan 10°}$$

$$h \tan 10° = h \tan 2.5° + 18(\tan 10°)(\tan 2.5°)$$

$$h = \frac{18(\tan 10°)(\tan 2.5°)}{\tan 10° - \tan 2.5°} \approx 1.04 \text{ miles} \approx 5518 \text{ feet}$$

42. $L_1 = 2x + y = 8 \Longrightarrow m_1 = -2$

$$L_2 = x - 5y = -4 \Longrightarrow m_2 = \frac{1}{5}$$

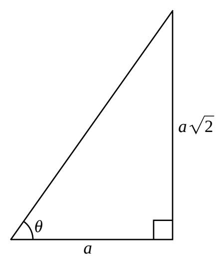
$$\tan \alpha = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

$$\alpha = \arctan\left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

$$= \arctan\left|\frac{\frac{1}{5} - (-2)}{1 + \frac{1}{5}(-2)}\right|$$

$$= \arctan\left(3\tfrac{2}{3}\right) \approx 74.7°$$

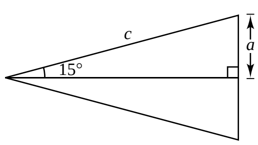
44.



$$\tan \theta = \frac{a\sqrt{2}}{a} = \sqrt{2}$$

$$\theta = \arctan \sqrt{2} \approx 54.7°$$

46.



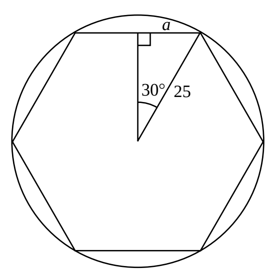
$$c = \frac{35}{2} = 17.5$$

$$\sin 15° = \frac{a}{c}$$

$$a = c \sin 15° = 17.5 \sin 15° \approx 4.53$$

Distance $= 2a \approx 9.06$ centimeters

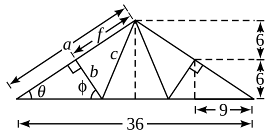
48.



$$\sin 30° = \frac{a}{24}$$

$$a = 24 \sin 30° = 12$$

Length of side $= 2a = 2(12) = 24$ inches

50. 

$$\tan\theta = \frac{12}{18}$$

$$\theta = \arctan\frac{2}{3} = 0.588 \text{ rad} \approx 33.7°$$

$$\cos\theta = \frac{18}{a}$$

$$a = \frac{18}{\cos\theta} \approx 21.6$$

$$f \approx \frac{21.6}{2} = 10.8$$

$$\phi \approx 90 - 33.7 = 56.3°$$

$$\sin\phi = \frac{6}{b}$$

$$b = \frac{6}{\sin\phi} \approx 7.2$$

$$c = \sqrt{10.8^2 + 7.2^2} \approx 12.98$$

52. $d = \frac{1}{2}\cos 20\pi t$

(a) Maximum displacement: $|a| = \left|\frac{1}{2}\right| = \frac{1}{2}$

(b) Frequency:

$$\frac{\omega}{2\pi} = \frac{20\pi}{2\pi} = 10$$

(c) Least positive value for t for which $d = 0$:

$$\frac{1}{2}\cos 20\pi t = 0$$

$$\cos 20\pi t = 0$$

$$20\pi t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2} \cdot \frac{1}{20\pi} = \frac{1}{40}$$

54. $d = \frac{1}{64}\sin 792\pi t$

(a) Maximum displacement:

$$|a| = \left|\frac{1}{64}\right| = \frac{1}{64}$$

(b) Frequency:

$$\frac{\omega}{2\pi} = \frac{792\pi}{2\pi} = 396$$

(c) Least positive value for t for which $d = 0$:

$$\frac{1}{64}\sin 792\pi t = 0$$

$$\sin 792\pi t = 0$$

$$792\pi t = \pi$$

$$t = \frac{\pi}{792\pi} = \frac{1}{792}$$

56. Displacement at $t = 0$ is $0 \Longrightarrow d = a\sin\omega t$

Amplitude: $|a| = 3$

Period: $\frac{2\pi}{\omega} = 6 \Longrightarrow \omega = \frac{\pi}{3}$

$$d = 3\sin\left(\frac{\pi t}{3}\right)$$

58. Displacement at $t = 0$ is $2 \Longrightarrow d = a\cos\omega t$

Amplitude: $|a| = 2$

Period: $\frac{2\pi}{\omega} = 10 \Longrightarrow \omega = \frac{\pi}{5}$

$$d = 2\cos\left(\frac{\pi t}{5}\right)$$

60. At $t = 0$, buoy is at its high point $\Longrightarrow d = a\cos\omega t$.

Distance from high to low $= 2|a| = 3.5$

$$|a| = \frac{7}{4}$$

Returns to high point every 10 seconds: Period $= \frac{2\pi}{\omega} = 10 \Longrightarrow \omega = \frac{\pi}{5}$

$$d = \frac{7}{4}\cos\frac{\pi t}{5}$$

62. (a)

θ	L_1	L_2	$L_1 + L_2$
0.1	$\frac{2}{\sin 0.1}$	$\frac{3}{\cos 0.1}$	23.0
0.2	$\frac{2}{\sin 0.2}$	$\frac{3}{\cos 0.2}$	13.1
0.3	$\frac{2}{\sin 0.3}$	$\frac{3}{\cos 0.3}$	9.9
0.4	$\frac{2}{\sin 0.4}$	$\frac{3}{\cos 0.4}$	8.4

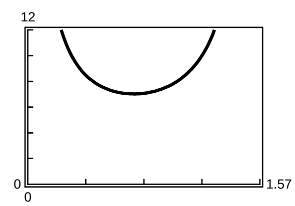
(b)

0.5	$\frac{2}{\sin 0.5}$	$\frac{3}{\cos 0.5}$	7.6
0.6	$\frac{2}{\sin 0.6}$	$\frac{3}{\cos 0.6}$	7.2
0.7	$\frac{2}{\sin 0.7}$	$\frac{3}{\cos 0.7}$	7.0
0.8	$\frac{2}{\sin 0.8}$	$\frac{3}{\cos 0.8}$	7.1

The minimum length of the elevator is 7.0 meters.

(c) $L = L_1 + L_2 = \frac{2}{\sin\theta} + \frac{3}{\cos\theta}$

(d)



From the graph, it appears that the minimum length is 7.0 meters, which agrees with the estimate of part (b).

64. (a) and (b)

Base 1	*Base 2*	*Altitude*	*Area*
8	$8 + 16\cos 10°$	$8\sin 10°$	22.1
8	$8 + 16\cos 20°$	$8\sin 20°$	42.5
8	$8 + 16\cos 30°$	$8\sin 30°$	59.7
8	$8 + 16\cos 40°$	$8\sin 40°$	72.7
8	$8 + 16\cos 50°$	$8\sin 50°$	80.5
8	$8 + 16\cos 60°$	$8\sin 60°$	83.1
8	$8 + 16\cos 70°$	$8\sin 70°$	80.7

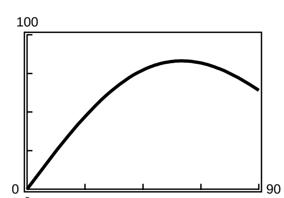
Maximum is 83.1°

(c) $A = \frac{1}{2}(b_1 + b_2)h$

$= \frac{1}{2}[8 + (8 + 16\cos\theta)]8\sin\theta$

$= 64(1 + \cos\theta)(\sin\theta)$

(d)



Maximum is 83.1°