# C H A P T E R   6
# Additional Topics in Trigonometry

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## Section 6.1    Law of Sines

**Solutions to Even-Numbered Exercises**

**2.**



Given: $B = 45°, C = 120°, c = 15$

$A = 180° - B - C = 15°$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{15 \sin 15°}{\sin 120°} \approx 4.48$

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{15(\sin 45°)}{\sin 120°} \approx 12.25$

**4.**



Given: $B = 10°, C = 135°, c = 45$

$A = 180° - B - C = 35°$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{45 \sin 35°}{\sin 135°} \approx 36.50$

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{45 \sin 35°}{\sin 135°} \approx 11.05$

**6.** Given: $A = 60°, a = 9, c = 10$

$\sin C = \dfrac{c \sin A}{a} = \dfrac{10 \sin 60°}{9} \approx 0.9623 \implies C \approx 74.21°$ or $C \approx 105.79°$

Case 1

$C \approx 74.21°$

$B = 180° - A - C \approx 45.79°$

$b = \dfrac{a}{\sin A}(\sin B) \approx \dfrac{9 \sin 45.79°}{\sin 60°} \approx 7.45$

Case 2

$C \approx 105.79°$

$B = 180° - A - C \approx 14.21°$

$b = \dfrac{a}{\sin A}(\sin B) \approx \dfrac{9 \sin 14.21°}{\sin 60°} \approx 2.55$

**8.** Given: $A = 24.3°, C = 54.6°, c = 2.68$

$B = 180° - A - C = 101.1°$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{2.68 \sin 24.3°}{\sin 54.6°} \approx 1.35$

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{2.68 \sin 101.1°}{\sin 54.6°} \approx 3.23$

**10.** Given: $A = 5° \, 40', B = 8° \, 15', b = 4.8$

$C = 180° - A - B = 166° \, 5'$

$a = \dfrac{b}{\sin B}(\sin A) = \dfrac{4.8 \sin 5° \, 40'}{\sin 8° \, 15'} \approx 3.30$

$c = \dfrac{b}{\sin B}(\sin C) = \dfrac{4.8 \sin 166° \, 5'}{\sin 8° \, 15'} \approx 8.05$

**12.** Given: $C = 85° \, 20', a = 35, c = 50$

$\sin A = \dfrac{a \sin C}{c} = \dfrac{35 \sin 85° \, 20'}{50} \approx 0.6977 \implies A \approx 44.24°$

$B = 180° - A - C \approx 50.43°$

$b = \dfrac{c}{\sin C}(\sin B) \approx \dfrac{50 \sin 50.43°}{\sin 85° \, 20'} \approx 38.67$
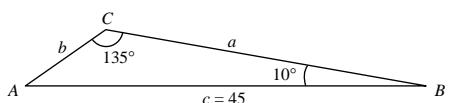
**14.** Given: $B = 2° \, 45', b = 6.2, c = 5.8$

$\sin C = \dfrac{c \sin B}{b} = \dfrac{5.8 \sin 2° \, 45'}{6.2} \approx 0.04488 \implies C \approx 2.57°$

$A = 180° - B - C \approx 174.68°$

$a = \dfrac{b}{\sin B}(\sin A) \approx \dfrac{6.2 \sin 174.68°}{\sin 2° \, 45'} \approx 11.99$

**16.** Given: $A = 94°$, $a = 14.6$, $b = 14.6$. $A$ obtuse and $a \leq b \implies$ no triangle exists

**18.** Given: $a = 4.5$, $b = 5$, $A = 58°$

$$\sin B = \frac{b \sin A}{a} = \frac{5 \sin 58°}{4.5} \approx 0.9423 \implies B = 70.4° \text{ or } B = 109.6°$$

| Case 1 | Case 2 |
|---|---|
| $B \approx 70.4°$ | $B \approx 109.6°$ |
| $C \approx 180° - 70.4° - 58° = 51.6°$ | $C \approx 180° - 109.6° - 58° = 12.4°$ |
| $c \approx \frac{4.5}{\sin 58°}(\sin 51.6°) \approx 4.16$ | $c \approx \frac{4.5}{\sin 58°}(\sin 12.4°) \approx 1.14$ |

**20.** $A = 140°$, $a = 48$, $b = 46$

$$\sin B = \frac{\sin A}{a} b = \frac{\sin 140°}{48}(46) = 0.6160 \implies B \approx 38.0$$

$$C = 180° - A - B = 2°$$

$$c = \frac{a}{\sin A} \sin C = \frac{48}{\sin 140°} \sin 2° \approx 2.6$$

**22.** Given: $A = 60°$, $a = 10$

    (a) One solution if $b \leq 10$ or $b = \dfrac{10}{\sin 60°}$.

    (b) Two solutions if $10 < b < \dfrac{10}{\sin 60°}$.

    (c) No solutions if $b > \dfrac{10}{\sin 60°}$.

**24.** Area $= \dfrac{1}{2} ac \sin B = \dfrac{1}{2}(103)(58) \sin 74.5° \approx 2878.4$ sq. units
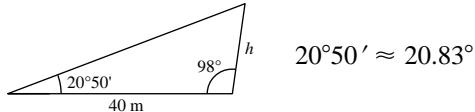
**26.** $A = 5° 15'$, $b = 4.5$, $c = 22$

    Area $= \frac{1}{2}bc \sin A$

        $= \left(\frac{1}{2}\right)(4.5)(22) \sin 5.25° \approx 4.529$ sq. units

**28.** $C = 84° 30'$, $a = 16$, $b = 20$

    Area $= \frac{1}{2}ab \sin C$

        $= \left(\frac{1}{2}\right)(16)(20) \sin 84.5° \approx 159.3$ sq. units

**30.** (a)



$20°50' \approx 20.83°$

    (b) $A = 180 - 98° - 20.83° = 61.17°$ (or $61°10'$)

$$\frac{40}{\sin A} = \frac{h}{\sin (20° 50')} \implies h = \frac{40 \sin (20° 50')}{\sin 61.17°}$$
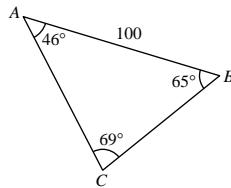
    (c) $h \approx 16.2$ m

**32.** Given: $A = 74° - 28° = 46°$,
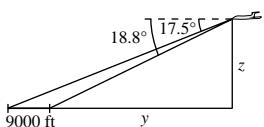
$B = 180° - 41° - 74° = 65°$, $c = 100$

$C = 180° - 46° - 65° = 69°$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{100}{\sin 69°}(\sin 46°) \approx 77$ meters



**34.** (a)



(b) $\dfrac{x}{\sin 17.5°} = \dfrac{9000}{\sin 1.3°}$

$x \approx 119289.1261$ feet $\approx 22.6$ miles

(c) $\dfrac{y}{\sin 71.2°} = \dfrac{x}{\sin 90°}$

$y = x \sin 71.2° \approx 119289.1261 \sin 71.2°$

$\approx 112924.963$ feet $\approx 21.4$ miles
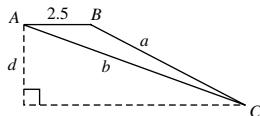
(d) $= 119289.1261 \sin 18.8° \approx 38,442.8$ feet

**36.** $A = 20$, $B = 90° + 63° = 153°$, $c = 10\left(\dfrac{1}{4}\right) = 2.5$

$C = 180° - 20° - 153° = 7°$

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{2.5 \sin 153°}{\sin 7°} \approx 9.31$

$d \approx b \sin A \approx 9.31 \sin 20° \approx 3.2$ miles
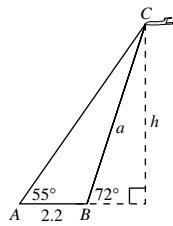


**38.** $A = 55°$, $B = 180° - 72 = 108°$, $c = 2.2$

$C = 180° - 55° - 108° = 17°$

$a = \dfrac{c}{\sin C}(\sin A)$

$= \dfrac{2.2}{\sin 17°}(\sin 55°) \approx 6.16$

$h = 6.16 \sin 72° \approx 5.86$ miles



**40.** $\dfrac{\sin(42° - \theta)}{10} = \dfrac{\sin 48°}{17}$

$\sin(42° - \theta) \approx 0.43714$
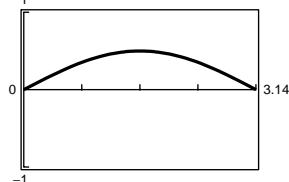
$\theta \approx 16.1°$

**42.** (a) $\dfrac{\sin \alpha}{9} = \dfrac{\sin \beta}{18}$

$\sin \alpha = 0.5 \sin \beta$

$\alpha = \arcsin(0.5 \sin \beta)$

(b)
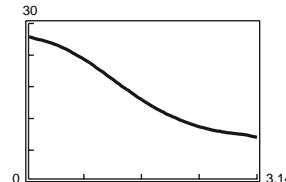
Domain: $0 < \beta < \pi$

Range: $0 < \alpha \leq \pi/6$

(c)    $\gamma = \pi - \alpha - \beta = \pi - \beta - \arcsin(0.5 \sin \beta)$

$\dfrac{c}{\sin \gamma} = \dfrac{18}{\sin \beta}$

$c = \dfrac{18 \sin \gamma}{\sin \beta} = \dfrac{18 \sin[\pi - \beta - \arcsin(0.5 \sin \beta)]}{\sin \beta}$

(d)

Domain: $0 < \beta < \pi$

Range: $9 < c < 27$
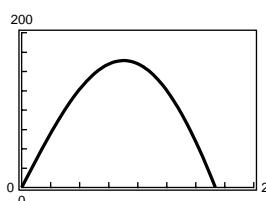
(e)

| $\beta$ | 0 | 0.4 | 0.8 | 1.2 | 1.6 | 2.0 | 2.4 | 2.8 |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 0 | 0.1960 | 0.3669 | 0.4848 | 0.5234 | 0.4720 | 0.3445 | 0.1683 |
| $c$ | Undef. | 25.95 | 23.07 | 19.19 | 15.33 | 12.29 | 10.31 | 9.27 |

As $\beta \to 0$, $c \to 27$.

As $\beta \to \pi$, $c \to 9$.

**44.** True. No angle could be 90°.

**46.** (a) $A = \dfrac{1}{2}(30)(20) \sin\left(\theta + \dfrac{\theta}{2}\right) - \dfrac{1}{2}(8)(20) \sin \dfrac{\theta}{2} - \dfrac{1}{2}(8)(30) \sin \theta$

$= 300 \sin \dfrac{3\theta}{2} - 80 \sin \dfrac{\theta}{2} - 120 \sin \theta$

$= 20\left[15 \sin \dfrac{3\theta}{2} - 4 \sin \dfrac{\theta}{2} - 6 \sin \theta\right]$

(b)

(c) Domain: $0 \leq \theta \leq 1.6690$

The domain would increase in length and the area would increase if the 8 centimeter line segment were decreased.

**48.** $\cot \theta = \dfrac{9}{2}$

$\sin \theta = -\dfrac{2}{\sqrt{85}} = -\dfrac{2\sqrt{85}}{85}$

$\cos \theta = \cot \theta \cdot \sin \theta = \dfrac{9}{2}\left(-\dfrac{2}{\sqrt{85}}\right) = -\dfrac{9}{\sqrt{85}} = -\dfrac{9\sqrt{85}}{85}$

$\sec \theta = -\dfrac{\sqrt{85}}{9}$