50. $\theta = \pi + k\pi$ since $\cot\theta$ undefined

$\theta = \pi$ since $\cos\theta < 0$

$\sin\theta = 0$

$\csc\theta$ undefined

$\tan\theta = 0$

$\sec\theta = -1$

$\cos\theta = -1$

52. $\csc\theta\tan\theta - \dfrac{1-\sin^2\theta}{\cos\theta} = \dfrac{1}{\sin\theta}\cdot\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos^2\theta}{\cos\theta} = \dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1-\cos^2\theta}{\cos\theta}$

54. $2\cos 2\theta\cos 5\theta = 2\frac{1}{2}[\cos(2\theta - 5\theta) + \cos(2\theta + 5\theta)] = \cos 3\theta + \cos 7\theta$

56. $\dfrac{5}{2}\sin\dfrac{3\pi}{4}\sin\dfrac{5\pi}{6} = \dfrac{5}{2}\cdot\dfrac{1}{2}\left[\cos\left(\dfrac{3\pi}{4} - \dfrac{5\pi}{6}\right) - \cos\left(\dfrac{3\pi}{4} + \dfrac{5\pi}{6}\right)\right] = \dfrac{5}{4}\left[\cos\left(-\dfrac{\pi}{12}\right) - \cos\left(\dfrac{19\pi}{12}\right)\right]$

Section 6.2 Law of Cosines

Solutions to Even-Numbered Exercises

2. Given: $a = 9, b = 3, c = 11$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{3^2 + 11^2 - 9^2}{2(3)(11)} \approx 0.7424 \Longrightarrow A \approx 42.1°$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{9^2 + 3^2 - 11^2}{2(9)(3)} \approx -0.5741 \Longrightarrow C \approx 125.0°$

$B = 180° - A - C \approx 12.9°$

4. Given: $C = 108°, a = 10, b = 6.5$

$c^2 = a^2 + b^2 - 2ab\cos C = 10^2 + 6.5^2 - 2(10)(6.5)\cos 108° \approx 182.42 \Longrightarrow c \approx 13.5$

$\sin B = \dfrac{\sin C}{c}b = \dfrac{\sin 108°}{13.5}(6.5) \approx 0.4579 \Longrightarrow B \approx 27.3°$

$A = 180° - B - C = 44.7°$

6. Given: $a = 45, b = 30, c = 72$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{45^2 + 30^2 - 72^2}{2(45)(30)} \approx -0.8367 \Longrightarrow C \approx 146.8°$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac} = \dfrac{45^2 + 72^2 - 30^2}{2(45)(72)} \approx -0.9736 \Longrightarrow B \approx 13.2°$

$A = 180° - B - C = 20.0°$

8. Given: $a = 1.42, b = 0.75, c = 1.25$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(0.75)^2 + (1.25)^2 - (1.42)^2}{2(0.75)(1.25)} = 0.05792 \implies A \approx 86.7°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(1.42)^2 + (1.25)^2 - (0.75)^2}{2(1.42)(1.25)} \approx 0.8497 \implies B \approx 31.8°$$

$180° - 86.7° - 31.8° \approx 61.5°$

10. Given: $B = 75°\,20', a = 6.2, c = 9.5$

$b^2 = a^2 + c^2 - 2ac\cos B = (6.2)^2 + (9.5)^2 - 2(6.2)(9.5)\cos 75°\,20' \approx 98.8636 \implies b \approx 9.94$

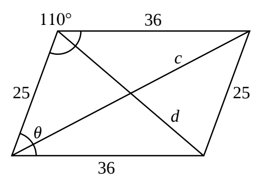
$$\sin A = \frac{a \sin B}{b} \approx \frac{6.2 \sin 75°\,20'}{9.94} \approx 0.6034 \implies A \approx 37.1°$$

$C \approx 180° - 75°\,20' - 37.1° \approx 67.6°$

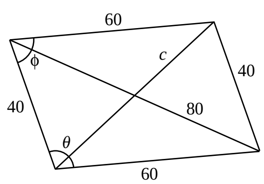
12. $c^2 = 25^2 + 36^2 - 2(25)(36)\cos 110° \approx 2536.6 \implies c \approx 50.4$

$2\theta = 360° - 2(110°) = 140 \implies \theta = 70°$

$d^2 = 25^2 + 36^2 - 2(25)(36)\cos 70° \approx 1305.4 \implies d \approx 36.1$



14.



$$\cos\theta = \frac{40^2 + 60^2 - 80^2}{2(40)(60)} \approx -\frac{1}{4} \implies \theta \approx 104.5°$$

$2\phi \approx 360° - 2(104.5°) = 151° \implies \phi = 75.5°$

$c^2 \approx 40^2 + 60^2 - 2(40)(60)\cos 75.5° = 4000$

$c \approx 63.25$

16. $$\cos\alpha = \frac{25^2 + 17.5^2 - 25^2}{2(25)(17.5)}$$

$\alpha \approx 69.512°$

$\beta \approx 180 - \alpha \approx 110.488°$

$a^2 = 17.5^2 + 25^2 - 2(17.5)(25)\cos 110.488°$

$a \approx 35.18$

$z = 180 - 2\alpha \approx 40.976$

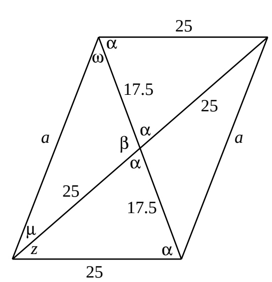
$$\cos\mu = \frac{25^2 + 35.18^2 - 17.5^2}{2(25)(35.18)}$$

$\mu \approx 27.771°$

$\theta = \mu + z \approx 68.7°$

$\omega = 180° - \mu - \beta \approx 41.741°$

$\phi = \omega + \alpha \approx 111.3°$



18. $s = \frac{a + b + c}{2} = \frac{14 + 17 + 7}{2} = 19$

Area $= \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{19(5)(2)(12)} \approx 47.7$ sq. units

20. Given: $a = 75.4, b = 52, c = 52$

$s = \frac{75.4 + 52 + 52}{2} = 89.7$

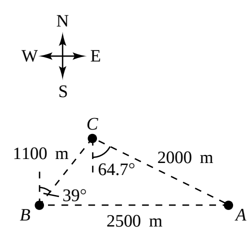
Area $= \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{89.7(14.3)(37.7)(37.7)} \approx 1350$ sq. units

22. $s = \frac{a + b + c}{2} = \frac{4.45 + 1.85 + 3.00}{2} = 4.65$

Area $= \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{4.65(0.2)(2.8)(1.65)} \approx 2.07$ sq. units

24. $\cos B = \frac{1100^2 + 2500^2 - 2000^2}{2(1100)(2500)} = 0.6291$



$B \approx 51.0°$

$90 - B \approx 39.$

Bearing at B is approximately N 39° E.

$\cos C = \frac{1100^2 + 2000^2 - 2500^2}{2(1100)(2000)} = -0.2364$

$C \approx 103.7°$

$C - (90° - 51.0°) = 64.7°$

Bearing at C is approximately S 64.7° E.

26. $\cos\theta = \frac{2^2 + 3^2 - (4.5)^2}{2(2)(3)} \approx -0.60417$

$\theta \approx 127.2°$

28. The angles at the base of the tower are 96° and 84°. The longer guy wire g_1 is given by:

${g_1}^2 = 75^2 + 100^2 - 2(75)(100)\cos 96° \approx 17{,}192.9 \implies g_1 \approx 131.1$ feet

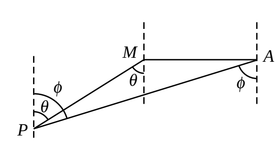
The shorter guy wire g_2 is given by:

${g_2}^2 = 75^2 + 100^2 - 2(75)(100)\cos 84° \approx 14{,}057.1 \implies g_2 \approx 118.6$ feet

30. Bearing of M from P: N θ E
Bearing of A from P: N ϕ E

Since M is due west of A, it follows that $\theta = M - 90°$ and $\phi = 90° - A$.

$\cos M = \frac{165^2 + 216^2 - 368^2}{2(165)(216)} \approx -0.8634 \implies M \approx 149.7°$



$\cos A = \frac{165^2 + 368^2 - 216^2}{2(165)(368)} \approx 0.95515 \implies A \approx 17.2°$

$\theta \approx 149.7° - 90° \approx 59.7° \implies$ Bearing of Minneapolis from Phoenix: N 59.7° E

$\phi \approx 90° - 17.2° \approx 72.8° \implies$ Bearing of Minneapolis from Phoenix: N 72.8° E

32. $x^2 = 330^2 + 420^2 - 2(330)(420)\cos 6°$

≈ 9618.5

$x \approx 98.1$ feet

34. $s = \dfrac{a+b+c}{2} = \dfrac{145+257+290}{2} = 346$

Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{346(201)(89)(56)} \approx 18{,}617.7$ sq. ft

36. (a) $d^2 = 10^2 + 7^2 - 2(10)(7)\cos\theta \Longrightarrow d = \sqrt{149 - 140\cos\theta}$

(b) $\theta = \arccos\left[\dfrac{10^2+7^2-d^2}{2(10)(7)}\right] = \arccos\left[\dfrac{149-d^2}{140}\right]$

(c) $s = \dfrac{360° - \theta}{360°}(2\pi r) = \dfrac{(360°-\theta)\pi}{45}$

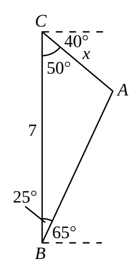
(d)

d (inches)	9	10	12	13	14	15	16
θ (degrees)	60.9°	69.5°	88.0°	98.2°	109.6°	122.9°	139.8°
s (inches)	20.88	20.28	18.99	18.28	17.48	16.55	15.37

38. $A = 180° - 50° - 25° = 105°$

$\dfrac{x}{\sin 25°} = \dfrac{7}{\sin 105°}$

$x = \dfrac{7\sin 25°}{\sin 105°} \approx 3.06$ feet



40. True. The third side is found by the Law of Cosines. The other angles are determined by the Law of Sines.

42. $a = 25,\ b = 55,\ c = 72,\ s = \dfrac{25+55+72}{2} = 76$

(a) $A = \sqrt{(76)(76-25)(76-55)(76-72)} \approx 570.60$ sq. units

(b) $\cos C = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{25^2+55^2-72^2}{2(25)(55)} \Longrightarrow C \approx 123.905°$

$2R = \dfrac{c}{\sin C} \approx \dfrac{72}{\sin 123.905°} \Longrightarrow R \approx 43.3754$

$A = \pi R^2 \approx 5910.67$ sq. units

(c) $r = \sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}} = \sqrt{\dfrac{(51)(21)(4)}{76}} \approx 7.5079$

$A = \pi r^2 \approx 177.09$ sq. units