

$$\begin{aligned}
 44. \quad \frac{1}{2}bc(1 + \cos A) &= \frac{1}{2}bc \left[1 + \frac{b^2 + c^2 - a^2}{2bc} \right] \\
 &= \frac{1}{2}bc \left[\frac{2bc + b^2 + c^2 - a^2}{2bc} \right] \\
 &= \frac{1}{4}[(b + c)^2 - a^2] \\
 &= \frac{1}{4}[(b + c) + a][(b + c) - a] \\
 &= \frac{b + c + a}{2} \cdot \frac{b + c - a}{2} \\
 &= \frac{a + b + c}{2} \cdot \frac{-a + b + c}{2}
 \end{aligned}$$

$$\begin{aligned}
 46. \quad 2 \csc^2 x - 3 &= \csc^2 x - 1 \\
 \csc^2 x &= 2 \\
 \sin^2 x &= \frac{1}{2} \\
 \sin x &= \pm \frac{\sqrt{2}}{2} \\
 x &= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}
 \end{aligned}$$

$$48. \quad \cos x \cot x - \cos x = 0$$

$$\cos x(\cot x - 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad \cot x = 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$50. \quad \sin\left(x - \frac{\pi}{2}\right) - \sin\left(x + \frac{\pi}{2}\right) = -\cos x - \cos x = -2 \cos x$$

Section 6.3 Vectors in the Plane

Solutions to Even-Numbered Exercises

$$2. \quad \mathbf{u} = \langle 0 - (-3), 4 - (-4) \rangle = \langle 3, 8 \rangle$$

$$\mathbf{v} = \langle 3 - 0, 3 - (-5) \rangle = \langle 3, 8 \rangle$$

$$\mathbf{u} = \mathbf{v}$$

$$6. \quad \text{Initial point: } (-1, -1)$$

$$\text{Terminal point: } (3, 5)$$

$$\mathbf{v} = \langle 3 - (-1), 5 - (-1) \rangle = \langle 4, 6 \rangle$$

$$\|\mathbf{v}\| = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$$

$$10. \quad \text{Initial point: } (3.4, 0)$$

$$\text{Terminal point: } (0, 5.8)$$

$$\mathbf{v} = \langle 0 - 3.4, 5.8 - 0 \rangle = \langle -3.4, 5.8 \rangle$$

$$\|\mathbf{v}\| = \sqrt{(-3.4)^2 + (5.8)^2} \approx 6.7$$

$$4. \quad \text{Initial point: } (0, 0)$$

$$\text{Terminal point: } (4, -2)$$

$$\mathbf{v} = \langle 4 - 0, -2 - 0 \rangle = \langle 4, -2 \rangle$$

$$\|\mathbf{v}\| = \sqrt{4^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$8. \quad \text{Initial point: } (-4, -1)$$

$$\text{Terminal point: } (3, -1)$$

$$\mathbf{v} = \langle 3 - (-4), -1 - (-1) \rangle = \langle 7, 0 \rangle$$

$$\|\mathbf{v}\| = \sqrt{7^2 + 0^2} = 7$$

$$12. \quad \text{Initial point: } (-3, 11)$$

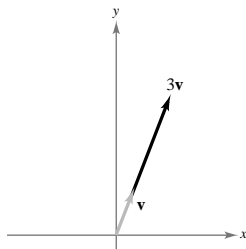
$$\text{Terminal point: } (9, 40)$$

$$\mathbf{v} = \langle 9 - (-3), 40 - 11 \rangle = \langle 12, 29 \rangle$$

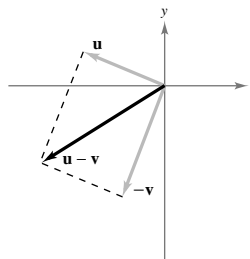
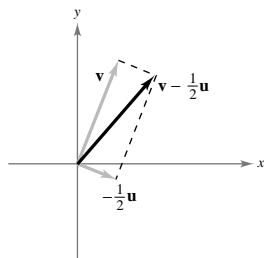
$$\|\mathbf{v}\| = \sqrt{12^2 + 29^2} = \sqrt{985}$$

$$14. \mathbf{v} = \langle -2.33 - 1.64, 3.86 - 7.21 \rangle = \langle -3.97, -3.35 \rangle$$

$$\|\mathbf{v}\| = \sqrt{(-3.97)^2 + (-3.35)^2} = \sqrt{26.9834} \approx 5.19$$

 16. $3\mathbf{v}$


18.


 20. $\mathbf{v} - \frac{1}{2}\mathbf{u}$


$$22. (a) \mathbf{u} + \mathbf{v} = \langle 5, 3 \rangle + \langle -4, 0 \rangle = \langle 1, 3 \rangle$$

$$(b) \mathbf{u} - \mathbf{v} = \langle 5, 3 \rangle - \langle -4, 0 \rangle = \langle 9, 3 \rangle$$

$$(c) 2\mathbf{u} - 3\mathbf{v} = 2\langle 5, 3 \rangle - 3\langle -4, 0 \rangle = \langle 22, 6 \rangle$$

$$(d) \mathbf{v} + 4\mathbf{u} = \langle -4, 0 \rangle + 4\langle 5, 3 \rangle = \langle 16, 12 \rangle$$

$$24. (a) \mathbf{u} + \mathbf{v} = \langle 0, -9 \rangle + \langle -6, 10 \rangle = \langle -6, 1 \rangle$$

$$(b) \mathbf{u} - \mathbf{v} = \langle 0, -9 \rangle - \langle -6, 10 \rangle = \langle 6, -19 \rangle$$

$$(c) 2\mathbf{u} - 3\mathbf{v} = 2\langle 0, -9 \rangle - 3\langle -6, 10 \rangle = \langle 18, -48 \rangle$$

$$(d) \mathbf{v} + 4\mathbf{u} = \langle -6, 10 \rangle + 4\langle 0, -9 \rangle = \langle -6, -26 \rangle$$

$$26. \mathbf{u} = 2\mathbf{i} - \mathbf{j}, \mathbf{v} = -\mathbf{i} + \mathbf{j}$$

$$(a) \mathbf{u} + \mathbf{v} = \mathbf{i}$$

$$(b) \mathbf{u} - \mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$$

$$(c) 2\mathbf{u} - 3\mathbf{v} = (4\mathbf{i} - 2\mathbf{j}) - (-3\mathbf{i} + 3\mathbf{j}) = 7\mathbf{i} - 5\mathbf{j}$$

$$(d) \mathbf{v} + 4\mathbf{u} = 7\mathbf{i} - 3\mathbf{j}$$

 28. $\mathbf{v} = \langle 0, -3 \rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{0^2 + (-3)^2}}\langle 0, -3 \rangle$$

$$= \frac{1}{3}\langle 0, -3 \rangle$$

$$= \langle 0, -1 \rangle$$

 30. $\mathbf{v} = \langle 5, -12 \rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{5^2 + (-12)^2}}\langle 5, -12 \rangle$$

$$= \frac{1}{13}\langle 5, -12 \rangle$$

$$= \left\langle \frac{5}{13}, -\frac{12}{13} \right\rangle$$

 32. $\mathbf{v} = \langle 8, -20 \rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{64 + 400}}\langle 8, -20 \rangle = \frac{1}{\sqrt{29}}\langle 2, -5 \rangle = \left\langle \frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}} \right\rangle$$

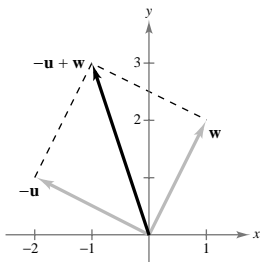
$$34. \mathbf{w} = \mathbf{i} - 2\mathbf{j}$$

$$\begin{aligned} \mathbf{u} &= \frac{1}{\|\mathbf{w}\|}\mathbf{w} \\ &= \frac{1}{\sqrt{1^2 + (-2)^2}}(\mathbf{i} - 2\mathbf{j}) \\ &= \frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j}) \\ &= \frac{\sqrt{5}}{5}\mathbf{i} - \frac{2\sqrt{5}}{5}\mathbf{j} \end{aligned}$$

$$\begin{aligned} 38. \mathbf{v} &= 3\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) \\ &= 3\left(\frac{1}{\sqrt{4^2 + (-4)^2}}\langle 4, -4 \rangle\right) \\ &= 3\left(\frac{1}{4\sqrt{2}}\langle 4, -4 \rangle\right) \\ &= \left\langle \frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}} \right\rangle \end{aligned}$$

$$\begin{aligned} 42. \mathbf{v} &= 4\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) \\ &= 4\left(\frac{1}{5}\langle 0, 5 \rangle\right) \\ &= \langle 0, 4 \rangle = 4\mathbf{j} \end{aligned}$$

$$\begin{aligned} 46. \mathbf{v} &= -\mathbf{u} + \mathbf{w} \\ &= -(2\mathbf{i} - \mathbf{j}) + (\mathbf{i} + 2\mathbf{j}) \\ &= -\mathbf{i} + 3\mathbf{j} = \langle -1, 3 \rangle \end{aligned}$$



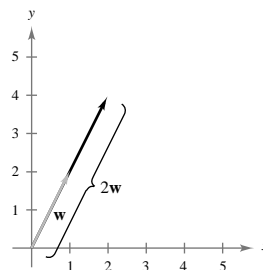
$$\begin{aligned} 50. \mathbf{v} &= 8(\cos 135^\circ \mathbf{i} + \sin 135^\circ \mathbf{j}) \\ \|\mathbf{v}\| &= 8, \theta = 135^\circ \end{aligned}$$

$$36. \mathbf{w} = -3\mathbf{i} \quad \|\mathbf{w}\| = 3$$

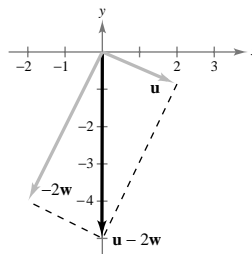
$$\mathbf{u} = \frac{1}{3}(-3\mathbf{i}) = -\mathbf{i}$$

$$\begin{aligned} 40. \mathbf{v} &= 10\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) \\ &= 10\left(\frac{1}{\sqrt{4+9}}\langle 2, -3 \rangle\right) \\ &= \left\langle \frac{20}{\sqrt{13}}, -\frac{30}{\sqrt{13}} \right\rangle = \frac{20}{\sqrt{13}}\mathbf{i} - \frac{30}{\sqrt{13}}\mathbf{j} \end{aligned}$$

$$44. \mathbf{v} = 2\mathbf{w} = 2\langle 1, 2 \rangle = \langle 2, 4 \rangle$$



$$\begin{aligned} 48. \mathbf{v} &= \mathbf{u} - 2\mathbf{w} \\ &= (2\mathbf{i} - \mathbf{j}) - 2(\mathbf{i} + 2\mathbf{j}) \\ &= -5\mathbf{j} = \langle 0, -5 \rangle \end{aligned}$$



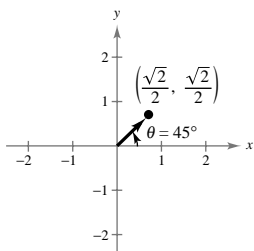
52. $\mathbf{v} = -4\mathbf{i} - 7\mathbf{j}$ Quadrant III

$$\|\mathbf{v}\| = \sqrt{(-4)^2 + 7^2} = \sqrt{65}$$

$$\tan \theta = \frac{-7}{-4} = \frac{7}{4} \Rightarrow \theta \approx 240.3^\circ$$

56. $\mathbf{v} = \langle \cos 45^\circ, \sin 45^\circ \rangle$

$$= \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$



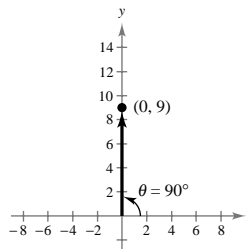
54. $\mathbf{v} = 12\mathbf{i} + 15\mathbf{j}$ Quadrant I

$$\|\mathbf{v}\| = \sqrt{12^2 + 15^2} = \sqrt{369} = 3\sqrt{41}$$

$$\tan \theta = \frac{15}{12} \Rightarrow \theta \approx 51.3^\circ$$

58. $\mathbf{v} = \langle 9 \cos 90^\circ, 9 \sin 90^\circ \rangle$

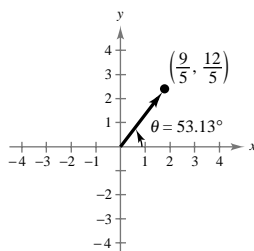
$$= \langle 0, 9 \rangle$$



60. $\mathbf{v} = 3 \left(\frac{1}{\sqrt{3^2 + 4^2}} \right) (3\mathbf{i} + 4\mathbf{j})$

$$= \frac{3}{5} (3\mathbf{i} + 4\mathbf{j})$$

$$= \frac{9}{5}\mathbf{i} + \frac{12}{5}\mathbf{j} = \left\langle \frac{9}{5}, \frac{12}{5} \right\rangle$$



62. $\mathbf{u} = \langle 2 \cos 30^\circ, 2 \sin 30^\circ \rangle = \langle \sqrt{3}, 1 \rangle$

$$\mathbf{v} = \langle 2 \cos 90^\circ, 2 \sin 90^\circ \rangle = \langle 0, 2 \rangle$$

$$\mathbf{u} + \mathbf{v} = \langle \sqrt{3}, 3 \rangle$$

64. $\mathbf{u} = \langle 35 \cos 25^\circ, 35 \sin 25^\circ \rangle = \langle 31.72, 14.79 \rangle$

$$\mathbf{v} = \langle 50 \cos 120^\circ, 50 \sin 120^\circ \rangle = \langle -25, 25\sqrt{3} \rangle$$

$$\mathbf{u} + \mathbf{v} \approx \langle 6.72, 58.09 \rangle$$

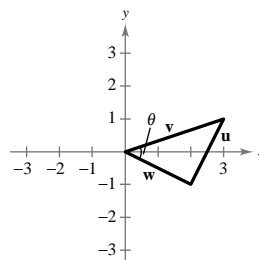
66. $\mathbf{v} = 3\mathbf{i} + \mathbf{j}$

$$\mathbf{w} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{u} = \mathbf{v} - \mathbf{w} = \mathbf{i} + 2\mathbf{j}$$

$$\cos \theta = \frac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{10 + 5 - 5}{2\sqrt{10}\sqrt{5}} = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$



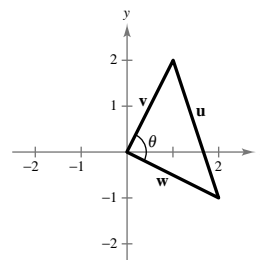
68. $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

$$\mathbf{w} = 2\mathbf{i} - \mathbf{j}$$

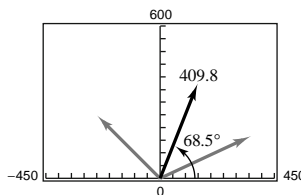
$$\mathbf{u} = \mathbf{v} - \mathbf{w} = -\mathbf{i} + 3\mathbf{j}$$

$$\cos \theta = \frac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{5 + 5 - 10}{2\sqrt{5}\sqrt{5}} = 0$$

$$\theta = 90^\circ$$



70. Analytically: $\mathbf{v} = 400\langle \cos 25^\circ, \sin 25^\circ \rangle$
 $\mathbf{u} = 300\langle \cos 135^\circ, \sin 135^\circ \rangle$
 $\mathbf{u} + \mathbf{v} = \langle 150.39, 381.18 \rangle$
 $\|\mathbf{u} + \mathbf{v}\| = 409.8$
 $\theta = \arctan\left(\frac{381.18}{150.39}\right) = 68.5^\circ$



72. Force One: $\mathbf{u} = 3000\mathbf{i}$
Force Two: $\mathbf{v} = 1000 \cos \theta \mathbf{i} + 1000 \sin \theta \mathbf{j}$
Resultant Force: $\mathbf{u} + \mathbf{v} = (3000 + 1000 \cos \theta)\mathbf{i} + 1000 \sin \theta \mathbf{j}$
 $\|\mathbf{u} + \mathbf{v}\| = \sqrt{(3000 + 1000 \cos \theta)^2 + (1000 \sin \theta)^2} = 3750$
 $9,000,000 + 6,000,000 \cos \theta + 1,000,000 = 14,062,500$
 $6,000,000 \cos \theta = 4,062,500$
 $\cos \theta = \frac{4,062,500}{6,000,000} \approx 0.6771$
 $\theta \approx 47.4^\circ$

74. (a) $\mathbf{u} = (70 \cos 30^\circ)\mathbf{i} - (70 \sin 30^\circ)\mathbf{j} \approx 60.62\mathbf{i} - 35\mathbf{j}$
 $\mathbf{v} = (40 \cos 45^\circ)\mathbf{i} + (40 \sin 45^\circ)\mathbf{j} \approx 28.28\mathbf{i} + 28.28\mathbf{j}$
 $\mathbf{w} = (60 \cos 135^\circ)\mathbf{i} + (60 \sin 135^\circ)\mathbf{j} \approx -42.43\mathbf{i} + 42.43\mathbf{j}$
 $\mathbf{u} + \mathbf{v} + \mathbf{w} = 46.47\mathbf{i} + 35.71\mathbf{j}$
 $\|\mathbf{u} + \mathbf{v} + \mathbf{w}\| \approx 58.61$ pounds
 $\tan \theta \approx \frac{35.71}{46.47} \approx 0.7684$
 $\theta \approx 37.5^\circ$

(b) $\mathbf{u} = (75 \cos 30^\circ)\mathbf{i} + (75 \sin 30^\circ)\mathbf{j} \approx 64.95\mathbf{i} + 37.5\mathbf{j}$
 $\mathbf{v} = (100 \cos 45^\circ)\mathbf{i} + (100 \sin 45^\circ)\mathbf{j} \approx 70.71\mathbf{i} + 70.71\mathbf{j}$
 $\mathbf{w} = (125 \cos 120^\circ)\mathbf{i} + (125 \sin 120^\circ)\mathbf{j} \approx -62.5\mathbf{i} + 108.3\mathbf{j}$
 $\mathbf{u} + \mathbf{v} + \mathbf{w} = 73.16\mathbf{i} + 216.5\mathbf{j}$
 $\|\mathbf{u} + \mathbf{v} + \mathbf{w}\| = 228.5$ pounds
 $\tan \theta = \frac{216.5}{73.16} \approx 2.9592 \approx 71.3^\circ$

76. Horizontal component of velocity: $1200 \cos 4^\circ \approx 1197.1$ ft/sec

Vertical component of velocity: $1200 \sin 4^\circ \approx 83.7$ ft/sec

78. Left cable: $\mathbf{u} = \|\mathbf{u}\|(\cos 155.7^\circ \mathbf{i} + \sin 155.7^\circ \mathbf{j})$

Right cable: $\mathbf{v} = \|\mathbf{v}\|(\cos 44.5^\circ \mathbf{i} + \sin 44.5^\circ \mathbf{j})$

$$\mathbf{u} + \mathbf{v} = 20,240\mathbf{j} \Rightarrow \|\mathbf{u}\| \cos 155.7^\circ + \|\mathbf{v}\| \cos 44.5^\circ = 0$$

$$\|\mathbf{u}\| \sin 155.7^\circ + \|\mathbf{v}\| \sin 44.5^\circ = 20,240$$

$$\Rightarrow -0.9114 \|\mathbf{u}\| + 0.7133 \|\mathbf{v}\| = 0$$

$$0.4115 \|\mathbf{u}\| + 0.7009 \|\mathbf{v}\| = 20,240$$

Solving this system for $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$ yields

$$\|\mathbf{u}\| = 15,485 \text{ pounds} \quad \text{Tension of left cable}$$

$$\|\mathbf{v}\| = 19,786 \text{ pounds} \quad \text{Tension of right cable}$$

80. (a) Rope 1: $\mathbf{u} = \|\mathbf{u}\|(\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{j})$

Rope 2: $\mathbf{v} = \|\mathbf{v}\|(\cos 120^\circ \mathbf{i} + \sin 120^\circ \mathbf{j})$

$$\mathbf{u} + \mathbf{v} = 100\mathbf{j} \Rightarrow \|\mathbf{u}\| \cos 60^\circ + \|\mathbf{v}\| \cos 120^\circ = 0$$

$$\|\mathbf{u}\| \sin 60^\circ + \|\mathbf{v}\| \sin 120^\circ = 100\mathbf{j}$$

$$\frac{1}{2} \|\mathbf{u}\| - \frac{1}{2} \|\mathbf{v}\| = 0 \Rightarrow \|\mathbf{u}\| = \|\mathbf{v}\|$$

$$\frac{\sqrt{3}}{2} \|\mathbf{u}\| + \frac{\sqrt{3}}{2} \|\mathbf{v}\| = \sqrt{3} \|\mathbf{u}\| = 100 \Rightarrow \|\mathbf{u}\| = \|\mathbf{v}\| = 57.7 \text{ pounds}$$

(b) $\mathbf{u} = \|\mathbf{u}\|(\cos(90 - \theta) \mathbf{i} + \sin(90 - \theta) \mathbf{j})$

$$\mathbf{v} = \|\mathbf{v}\|(\cos(90 + \theta) \mathbf{i} + \sin(90 + \theta) \mathbf{j})$$

$$\mathbf{u} + \mathbf{v} = 100 \mathbf{j} \Rightarrow \|\mathbf{u}\| \cos(90 - \theta) + \|\mathbf{v}\| \cos(90 + \theta) = 0$$

$$\Rightarrow \|\mathbf{u}\| = \|\mathbf{v}\|, \text{ and}$$

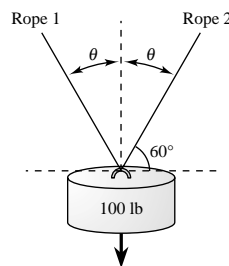
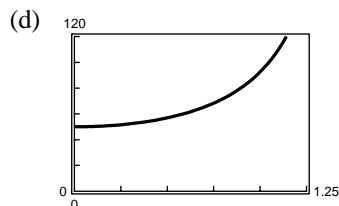
$$100 = \|\mathbf{u}\| \sin(90 - \theta) + \|\mathbf{u}\| \sin(90 + \theta)$$

$$= 2\|\mathbf{u}\| \cos \theta$$

$$\text{Hence, } \|\mathbf{u}\| = T = \frac{50}{\cos \theta} = 50 \sec \theta > \text{Domain: } 0^\circ \leq \theta \leq 90^\circ$$

(c)

θ	10°	20°	30°	40°	50°	60°
T	50.8	53.2	57.7	65.3	77.8	100



(e) The vertical component of the vectors decreases as θ increases.

82. Plane: $\mathbf{u} = (580 \cos 150^\circ)\mathbf{i} + (580 \sin 150^\circ)\mathbf{j} \approx -502.3\mathbf{i} + 290\mathbf{j}$

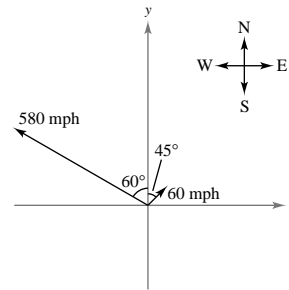
Wind: $\mathbf{v} = (60 \cos 45^\circ)\mathbf{i} + (60 \sin 45^\circ)\mathbf{j} \approx 42.4\mathbf{i} + 42.4\mathbf{j}$

$\mathbf{u} + \mathbf{v} \approx -459.9\mathbf{i} + 332.4\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(-459.9)^2 + (332.4)^2} \approx 567.4$

$\tan \theta \approx -\frac{332.4}{459.9} \approx -0.7229 \Rightarrow \theta \approx 144.1^\circ$

The ground speed is 567.4 miles per hour and the heading is N 54.1° W.



84. (a) Horizontal force: $\mathbf{u} = \|\mathbf{u}\|\mathbf{i}$

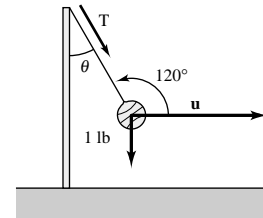
Weight: $\mathbf{w} = -\mathbf{j}$

Rope Tension: $\mathbf{T} = \|\mathbf{T}\|(\cos 120^\circ\mathbf{i} + \sin 120^\circ\mathbf{j})$

$\mathbf{u} + \mathbf{w} + \mathbf{T} = \mathbf{0} \Rightarrow \|\mathbf{u}\| + \|\mathbf{T}\| \cos 120^\circ = 0$

$-1 + \|\mathbf{T}\| \sin 120^\circ = 0$

Hence, $\|\mathbf{T}\| \frac{\sqrt{3}}{2} = 1 \Rightarrow \|\mathbf{T}\| = \frac{2}{\sqrt{3}} \approx 1.15$ lbs and $\|\mathbf{u}\| = \|\mathbf{T}\| \left(\frac{1}{2}\right) \approx 0.58$ lbs.



(b) $\mathbf{T} = \|\mathbf{T}\|(\cos(90 + \theta)\mathbf{i} + \sin(90 + \theta)\mathbf{j})$

$\mathbf{u} + \mathbf{w} + \mathbf{T} = \mathbf{0} \Rightarrow \|\mathbf{u}\| + \|\mathbf{T}\| \cos(90 + \theta) = 0$

$\|\mathbf{u}\| - \|\mathbf{T}\| \sin \theta = 0$

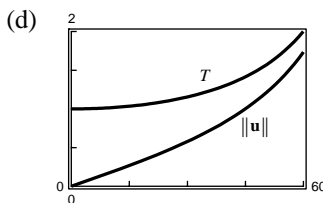
$-1 + \|\mathbf{T}\| \sin(90 + \theta) = 0$

$-1 + \|\mathbf{T}\| \cos \theta = 0 \Rightarrow \|\mathbf{T}\| = \sec \theta, 0 \leq \theta < \frac{\pi}{2}$

Hence, $\|\mathbf{u}\| = \|\mathbf{T}\| \sin \theta = \sec \theta \sin \theta = \tan \theta, 0 \leq \theta < \pi/2$.

(c)

θ	0°	10°	20°	30°	40°	50°	60°
\mathbf{T}	1	1.02	1.06	1.15	1.31	1.56	2
$\ \mathbf{u}\ $	0	0.18	0.36	0.58	0.84	1.19	1.73



(e) Both \mathbf{T} and $\|\mathbf{u}\|$ increases as θ increases, and approach each other in magnitude.

86. True, $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$

88. True

90. $\mathbf{F}_1 = \langle 10, 0 \rangle, \mathbf{F}_2 = 5\langle \cos \theta, \sin \theta \rangle$

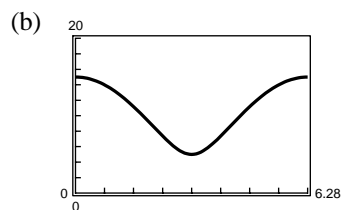
(a) $\mathbf{F}_1 + \mathbf{F}_2 = \langle 10 + 5 \cos \theta, 5 \sin \theta \rangle$

$$\begin{aligned} \|\mathbf{F}_1 + \mathbf{F}_2\| &= \sqrt{(10 + 5 \cos \theta)^2 + (5 \sin \theta)^2} \\ &= \sqrt{100 + 100 \cos \theta + 25 \cos^2 \theta + 25 \sin^2 \theta} \\ &= 5\sqrt{4 + 4 \cos \theta + \cos^2 \theta + \sin^2 \theta} \\ &= 5\sqrt{4 + 4 \cos \theta + 1} \\ &= 5\sqrt{5 + 4 \cos \theta} \end{aligned}$$

(c) Range: $[5, 15]$

Maximum is 15 when $\theta = 0$.

Minimum is 5 when $\theta = \pi$.



(d) The magnitude of the resultant is never 0 because the magnitudes of \mathbf{F}_1 and \mathbf{F}_2 are not the same.

92. The following program is written for a TI-82 or TI-83 graphing calculator. The program sketches two vectors $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ and $\mathbf{v} = c\mathbf{i} + d\mathbf{j}$ in standard position, and then sketches the vector difference $\mathbf{u} - \mathbf{v}$ using the parallelogram law.

PROGRAM: SUBVECT

```
:Input "ENTER A", A
:Input "ENTER B", B
:Input "ENTER C", C
:Input "ENTER D", D
:Line (0, 0, A, B)
:Line (0, 0, C, D)
:Pause
:A-C→E
:B-D→F
:Line (A, B, C, D)
:Line (A, B, E, F)
:Line (0, 0, E, F)
:Pause
:ClrDraw
:Stop
```

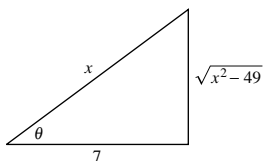
94. $\mathbf{u} = \langle 80 - 10, 80 - 60 \rangle = \langle 70, 20 \rangle$

$\mathbf{v} = \langle -20 - (-100), 70 - 0 \rangle = \langle 80, 70 \rangle$

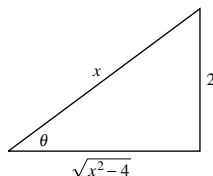
$\mathbf{u} - \mathbf{v} = \langle 70 - 80, 20 - 70 \rangle = \langle -10, -50 \rangle$

$\mathbf{v} - \mathbf{u} = \langle 80 - 70, 70 - 20 \rangle = \langle 10, 50 \rangle$

96. $\sqrt{x^2 - 49} = 7 \tan \theta$



98. $\sqrt{x^2 - 4} = 2 \cot \theta$



100. $A = 180^\circ - B - C = 52^\circ$

$b = \frac{c}{\sin C} \sin B \approx 13.2$

$a = \frac{c}{\sin C} \sin A \approx 27.9$

102. $c^2 = a^2 + b^2 - 2ab \cos C \approx 257.27 \Rightarrow c \approx 16.0$

$\sin B = \frac{\sin C}{c} \cdot b \approx 0.4040 \Rightarrow B \approx 23.8^\circ$

$A = 180^\circ - B - C = 60.2^\circ$