Section 6.4 Vectors and Dot Products

Solutions to Even-Numbered Exercises

2. $\mathbf{u} = \langle 5, 12 \rangle, \mathbf{v} = \langle -3, 2 \rangle$

$\mathbf{u} \cdot \mathbf{v} = 5(-3) + 12(2) = 9$

4. $\mathbf{u} \cdot \mathbf{v} = (3\mathbf{i} + 9\mathbf{j}) \cdot (10\mathbf{i} - 3\mathbf{j}) = 3(10) + 9(-3) = 3$

6. $\mathbf{u} = \langle 2, 2 \rangle$

$$\begin{aligned}\|\mathbf{u}\| - 2 &= \sqrt{\mathbf{u} \cdot \mathbf{u}} - 2\\ &= \sqrt{2(2) + 2(2)} - 2\\ &= \sqrt{8} - 2\\ &= 2\sqrt{2} - 2, \text{ scalar}\end{aligned}$$

8. $(\mathbf{w} \cdot \mathbf{u})\mathbf{v} = (\langle 1, -4 \rangle \cdot \langle 2, 2 \rangle)\langle -3, 4 \rangle = -6\langle -3, 4 \rangle = \langle 18, -24 \rangle$, vector

10. $4\mathbf{u} \cdot \mathbf{v} = 4\langle 2, 2 \rangle \cdot \langle -3, 4 \rangle = 4(-6 + 8) = 8$, scalar

12. $\mathbf{u} = \langle 2, -4 \rangle$

$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{\mathbf{u} \cdot \mathbf{u}}\\ &= \sqrt{2(2) + (-4)(-4)}\\ &= \sqrt{20} = 2\sqrt{5}\end{aligned}$$

14. $\mathbf{u} = 16\mathbf{i} - 10\mathbf{j}$

$$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{16(16) + (-10)(-10)} = \sqrt{356} = 2\sqrt{89}$$

16. $\mathbf{u} = 9\mathbf{i}$

$$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{9(9) + 0} = \sqrt{81} = 9$$

18. $\mathbf{u} = \langle 4, 4 \rangle, \mathbf{v} = \langle -2, 0 \rangle$

$$\begin{aligned}\cos\theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}\\ &= \frac{4(-2) + 4(0)}{(4\sqrt{2})(2)}\\ &= -\frac{\sqrt{2}}{2}\\ \theta &= 135^\circ\end{aligned}$$

20. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j}$

$$\begin{aligned}\cos\theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}\\ &= \frac{2(1) + (-3)(-2)}{\sqrt{2^2 + 3^2}\sqrt{1^2 + 2^2}}\\ &= \frac{8}{\sqrt{65}} \approx 0.992278\\ \theta &\approx 7.13^\circ\end{aligned}$$

22. $\mathbf{u} \cdot \mathbf{v} = \langle 0, 4\rangle \cdot \langle -3, 0\rangle = 0 \Longrightarrow \theta = 90°$

24. $\mathbf{u} = \cos\left(\frac{\pi}{4}\right)\mathbf{i} + \sin\left(\frac{\pi}{4}\right)\mathbf{j} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

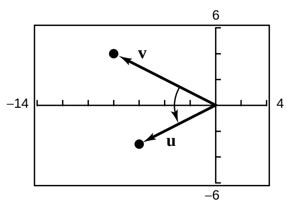
$\mathbf{v} = \cos\left(\frac{2\pi}{3}\right)\mathbf{i} + \sin\left(\frac{2\pi}{3}\right)\mathbf{j} = -\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\| \ \|\mathbf{v}\|} = \frac{-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}{(1)(1)} = \frac{\sqrt{6} - \sqrt{2}}{4} \Longrightarrow \theta = 75°$$

26. $\mathbf{u} = -6\mathbf{i} - 3\mathbf{j}, \mathbf{v} = -8\mathbf{i} + 4\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-6(-8) + (-3)(-4)}{\sqrt{45}\sqrt{80}} = \frac{36}{60} = 0.6$$

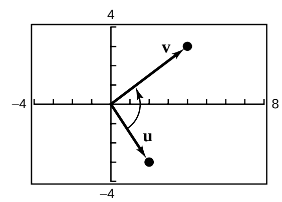
$\theta \approx 53.13°$



28. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}, \mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{2(4) + (-3)(3)}{\sqrt{13}\sqrt{25}} \approx -0.0555$$

$\theta \approx 93.18°$



30. $P = (-3, 0), Q = (2, 2), R = (0, 6)$

$\overrightarrow{PQ} = \langle 5, 2\rangle, \overrightarrow{QR} = \langle -2, 4\rangle, \overrightarrow{PR} = \langle 3, 6\rangle,$

$\overrightarrow{QP} = \langle -5, -2\rangle$

$$\cos\alpha = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\|\overrightarrow{PR}\|\,\|\overrightarrow{PR}\|} = \frac{27}{(\sqrt{29})(\sqrt{45})} \Longrightarrow \alpha \approx 41.6°$$

$$\cos B = \frac{\overrightarrow{QR}\cdot\overrightarrow{QP}}{\|\overrightarrow{QR}\|\,\|\overrightarrow{QP}\|} = \frac{2}{(\sqrt{20})(\sqrt{29})} \Longrightarrow \alpha \approx 85.2°$$

$\phi = 180° - 41.6° - 85.2° \approx 53.1°$

32. $\|\mathbf{u}\| = 100, \|\mathbf{v}\| = 250, \theta = \frac{\pi}{6}$

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta \\ &= (100)(250)\cos\frac{\pi}{6} \\ &= 25{,}000\cdot\frac{\sqrt{3}}{2} \\ &= 12{,}500\sqrt{3}\end{aligned}$$

34. $\mathbf{u} = \langle 15, 45\rangle, \mathbf{v} = \langle -5, 12\rangle$

$\mathbf{u} \neq k\mathbf{v} \Longrightarrow$ Not parallel

$\mathbf{u}\cdot\mathbf{v} \neq 0 \Longrightarrow$ Not orthogonal

Neither

36. $\mathbf{u} = \mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j}$

$\mathbf{u} \neq k\mathbf{v} \Longrightarrow$ Not parallel

$\mathbf{u}\cdot\mathbf{v} \neq 0 \Longrightarrow$ Not orthogonal

Neither

38. $-4\mathbf{v} = -4(-2\mathbf{i} - \mathbf{j}) = 8\mathbf{i} + 4\mathbf{j} = \mathbf{u} \Longrightarrow$ parallel

40. $\mathbf{u} = \langle 4, 2\rangle, \mathbf{v} = \langle 1, -2\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = 0\langle 1, -2\rangle = \langle 0, 0\rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 4, 2\rangle - \langle 0, 0\rangle = \langle 4, 2\rangle$$

42. $\mathbf{u} = \langle -5, -1\rangle, \mathbf{v} = \langle -1, 1\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{4}{2}\langle -1, 1\rangle = 2\langle -1, 1\rangle$$

$$\begin{aligned}\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 &= \langle -5, -1\rangle - 2\langle -1, 1\rangle \\ &= \langle -3, -3\rangle = 3\langle -1, -1\rangle\end{aligned}$$

44. Because **u** and **v** are parallel, the projection of **u** onto **v** is **u**.

46. Because **u** and **v** are orthogonal, the projection of **u** onto **v** is **0**.

48. For **v** to be orthogonal to $\mathbf{u} = \langle -8, 1 \rangle$, their dot product must be zero. Two possibilities: $\langle 1, 8 \rangle$, $\langle -1, -8 \rangle$

50. $\mathbf{u} = -\frac{5}{2}\mathbf{i} - 3\mathbf{j}$

For **v** to be orthogonal to **u**, $\mathbf{u} \cdot \mathbf{v}$ must be equal to 0.

Two possibilities: $\mathbf{v} = 3\mathbf{i} - \frac{5}{2}\mathbf{j}$

$\mathbf{v} = -3\mathbf{i} + \frac{5}{2}\mathbf{j}$

52. $P = (1, 3)$, $Q = (-3, 5)$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$

work $= \|\text{proj}_{\overrightarrow{PQ}}\,\mathbf{v}\|\,\|\overrightarrow{PQ}\|$ where

$\overrightarrow{PQ} = \langle -4, 2 \rangle$ and $\mathbf{v} = \langle -2, 3 \rangle$.

$$\text{proj}_{\overrightarrow{PQ}}\,\mathbf{v} = \left(\frac{\mathbf{v} \cdot \overrightarrow{PQ}}{\|\overrightarrow{PQ}\|^2}\right)\overrightarrow{PQ} = \left(\frac{14}{20}\right)\langle -4, 2 \rangle$$

$$\text{work} = \|\text{proj}_{\overrightarrow{PQ}}\,\mathbf{v}\|\,\|\overrightarrow{PQ}\| = \left(\frac{14\sqrt{20}}{20}\right)\left(\sqrt{20}\right) = 14$$

54. $\mathbf{u} = \langle 1245, 2600 \rangle$

$\mathbf{v} = \langle 12.20, 8.50 \rangle$

Increase prices by 5%: $1.05\mathbf{v}$

$$\begin{aligned} \mathbf{u} \cdot 1.05\mathbf{v} &= 1.05\mathbf{u} \cdot \mathbf{v} \\ &= 1.05[1245(12.20) + 2600(8.50)] \\ &= 1.05(37{,}289) \\ &= \$39{,}153.45 \end{aligned}$$

56. (a) $\mathbf{F} = -36{,}000\mathbf{j}$ Gravitational force

$\mathbf{v} = (\cos 12°)\mathbf{i} + (\sin 12°)\mathbf{j}$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{F} = \left(\frac{\mathbf{F} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = (\mathbf{F} \cdot \mathbf{v}) \approx -7484.8\mathbf{v}$$

The magnitude of this force is 7484.8; therefore, a force of 7484.8 pounds is needed to keep the truck from rolling down the hill.

(b) $\mathbf{w}_2 = \mathbf{F} - \mathbf{w}_1 = -36{,}000\mathbf{j} + 7484.8[(\cos 12°)\mathbf{i} + (\sin 12°)\mathbf{j}]$

$= [(7484.8 \cos 12°)\mathbf{i} + (7484.8 \sin 12° - 36{,}000)\mathbf{j}]$

$\|\mathbf{w}_2\| \approx 35{,}213.3$ pounds

58. work $= (2400)(5) = 12{,}000$ foot-pounds

60. work $= (\cos 35°)(15, 691)(800)$

$\approx 10{,}282{,}651$ newton $-$ meters

62. work $= (\cos 20°)(25)(12) \approx 281.9$ ft-lbs

64. True. $\mathbf{u} \cdot \mathbf{v} = -16 < 0$

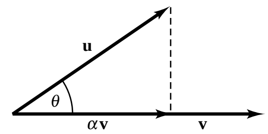
66. (a) $\mathbf{u} \cdot \mathbf{v} = 0 \implies$ **u** and **v** are orthogonal and $\theta = \frac{\pi}{2}$.

(b) $\mathbf{u} \cdot \mathbf{v} > 0 \implies \cos\theta > 0 \implies 0 \le \theta < \frac{\pi}{2}$

(c) $\mathbf{u} \cdot \mathbf{v} < 0 \implies \cos\theta < 0 \implies \frac{\pi}{2} < \theta \le \pi$

68. Since $\text{proj}_{\mathbf{v}}\mathbf{u}$ is a scalar multiple of $\mathbf{v}$, you can write $\text{proj}_{\mathbf{v}}\mathbf{u} = \alpha\mathbf{v}$. If $\alpha > 0$, then $\cos\theta > 0$ and

$$\|\text{proj}_{\mathbf{v}}\mathbf{u}\| = \|\alpha\mathbf{v}\| = \|\mathbf{u}\|\cos\theta = \frac{\|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta}{\|\mathbf{v}\|} = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|}$$

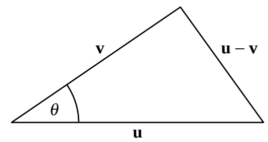


Thus, $\alpha = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}$ and $\text{proj}_{\mathbf{v}}\mathbf{u} = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$.

70. Use the Law of Cosines on the triangle.

$$\|\mathbf{u}-\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\,\|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta$$

$$= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\,\mathbf{u}\cdot\mathbf{v}$$



72. $\mathbf{u}\cdot(c\mathbf{v} + d\mathbf{w}) = \mathbf{u}\cdot(c\mathbf{v}) + \mathbf{u}\cdot(d\mathbf{w})$
$= c(\mathbf{u}\cdot\mathbf{v}) + d(\mathbf{u}\cdot\mathbf{w})$
$= c0 + d0 = 0$

74. $\sin\left(x + \dfrac{3\pi}{2}\right) - \sin\left(x - \dfrac{3\pi}{2}\right) = 0$

$-\cos x - \cos x = 0$

$\cos x = 0$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

76. $\tan(x + \pi) - \cos\left(x + \dfrac{5\pi}{2}\right) = 0$

$\tan x + \sin x = 0$

$\sin x\left(\dfrac{1}{\cos x} + 1\right) = 0$

$\sin x = 0$ or $\cos x = -1$

$x = 0, \pi$ $\quad x = \pi$

Answers: $x = 0, \pi$

78. $s = \dfrac{10.42 + 6.83 + 13.97}{2} = 15.61$

Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(15.61)(5.19)(8.78)(1.64)} \approx 34.2$ sq. units

80. (a) $\mathbf{u} + \mathbf{v} = \langle -2, 10\rangle$

(b) $2\mathbf{v} - \mathbf{u} = \langle 11, 14\rangle$

(c) $3\mathbf{u} - 5\mathbf{v} = \langle -30, -34\rangle$

82. (a) $\mathbf{u} + \mathbf{v} = \langle 18, 12\rangle$

(b) $2\mathbf{v} - \mathbf{u} = \langle 12, -6\rangle$

(c) $3\mathbf{u} - 5\mathbf{v} = \langle -26, 20\rangle$

84. $1.3(739) = \$960.70$

86. Let x be speed of current

$\dfrac{35}{18 - x} + \dfrac{35}{18 + x} = 4$

$\dfrac{1260}{(18 - x)(18 + x)} = 4$

$315 = (18 - x)(18 + x) = 324 - x^2$

$x^2 = 9$

$x = 3$ mph