94. The product of two diagonal matrices of the same order is a diagonal matrix whose entries are the products of the corresponding diagonal entries of A and B.

96. $3x^2 + 20x - 32 = 0$

$(x + 8)(3x - 4) = 0$

$x = -8, \frac{4}{3}$

98. $4x^2 + 10x - 3 = 0$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(4)(-3)}}{2(4)} = \frac{-10 \pm \sqrt{148}}{8}$$

$$= \frac{-5 \pm \sqrt{37}}{4} \approx -2.771, 0.271$$

100. $3x^3 - 12x^2 + 5x - 20 = 0$

$3x^2(x - 4) + 5(x - 4) = 0$

$(3x^2 + 5)(x - 4) = 0$

$x = 4, \pm\sqrt{\frac{5}{3}}i$

102. $\ln\left(\frac{100}{e^2}\right) = \ln 100 - \ln e^2$

$= \ln 100 - 2$

104. $\ln[x^2(x - 2)^3] = \ln x^2 + \ln(x - 2)^3$

$= 2 \ln x + 3 \ln(x - 2)$

106. $3 \ln 4 - \frac{1}{3}\ln(x^2 + 3) = \ln 4^3 - \ln(x^2 + 3)^{1/3}$

$$= \ln\left[\frac{64}{(x^2 + 3)^{1/3}}\right]$$

108. $\frac{1}{2}[2 \ln(x + 5) + \ln x - \ln(x - 8)] = \ln(x + 5) + \ln x^{1/2} - \ln(x - 8)^{1/2}$

$$= \ln\left[\frac{(x + 5)\sqrt{x}}{\sqrt{x - 8}}\right]$$

110. $\begin{bmatrix} -1 & 4 & -2 & \vdots & 22 \\ 2 & 5 & -3 & \vdots & 28 \\ 6 & -1 & -1 & \vdots & -4 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & 0 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 5 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix}$

Answer: $(0, 5, -1)$

Section 8.3 The Inverse of a Square Matrix

Solutions to Even-Numbered Exercises

2. $AB = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 - 1 & 1 - 1 \\ -2 + 2 & -1 = 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 - 1 & -2 + 2 \\ 1 - 1 & -1 + 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. $AB = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{3}{5} + \frac{2}{5} & \frac{1}{5} - \frac{1}{5} \\ \frac{6}{5} - \frac{6}{5} & \frac{2}{5} + \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} + \frac{2}{5} & -\frac{3}{5} + \frac{3}{5} \\ -\frac{2}{5} + \frac{2}{5} & \frac{2}{5} + \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

6. $AB = \begin{bmatrix} -4 & 1 & 5 \\ -1 & 2 & 4 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} -\frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{4} & -1 & -\frac{11}{4} \\ -\frac{1}{4} & 1 & \frac{7}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} -\frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{4} & -1 & -\frac{11}{4} \\ -\frac{1}{4} & 1 & \frac{7}{4} \end{bmatrix}\begin{bmatrix} -4 & 1 & 5 \\ -1 & 2 & 4 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8. $AB = \frac{1}{3}\begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -3 & 1 & 1 & -3 \\ -3 & -1 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ -3 & -2 & 1 & 0 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3-3+0+3 & -1-1+0+2 & -1+2+0-1 & 3-3+0+0 \\ -3+3+0+0 & 1+1+1+0 & 1-2+1+0 & -3+3+0+0 \\ 3-3+0+0 & -1-1+2+0 & -1+2+2+0 & 3-3+0+0 \\ 0+3+0-3 & 0+1+1-2 & 0-2+1+1 & 0+3+0+0 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = I_4$

$BA = \frac{1}{3}\begin{bmatrix} -3 & 1 & 1 & -3 \\ -3 & -1 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ -3 & -2 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3+3-0+3 & -3-1+1+3 & 0+1+2-3 & 3+0+0-3 \\ 3-1-2+0 & -3+1+2+3 & 0-1+4-3 & 3+0+0-3 \\ 0+1-1+0 & 0-1+1+0 & 0+1+2+0 & 0+0+0+0 \\ 3-2-1+0 & -3+2+1+0 & 0-2+2+0 & 3+0+0+0 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = I_4$

10. $AB = \begin{bmatrix} 11 & -12 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} -1 & 6 \\ -1 & \frac{11}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

12. $AB = \begin{bmatrix} 4 & 0 & -2 \\ 1 & 2 & -4 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 0.28 & -0.12 & 0.08 \\ -0.02 & 0.08 & 0.28 \\ 0.06 & -0.24 & 0.16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

14. $[A \;\vdots\; I] = \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right]$

$$-3R_1 + R_2 \rightarrow \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

$$-2R_2 + R_1 \rightarrow \left[\begin{array}{cc:cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right] = [I \;\vdots\; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

16. $[A \;\vdots\; I] = \left[\begin{array}{cc:cc} -7 & 33 & 1 & 0 \\ 4 & -19 & 0 & 1 \end{array}\right]$

$$2R_2 + R_1 \rightarrow \left[\begin{array}{cc:cc} 1 & -5 & 1 & 2 \\ 4 & -19 & 0 & 1 \end{array}\right]$$

$$-4R_1 + R_2 \rightarrow \left[\begin{array}{cc:cc} 1 & -5 & 1 & 2 \\ 0 & 1 & -4 & -7 \end{array}\right]$$

$$5R_2 + R_1 \rightarrow \left[\begin{array}{cc:cc} 1 & 0 & -19 & -33 \\ 0 & 1 & -4 & -7 \end{array}\right] = [I \;\vdots\; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -19 & -33 \\ -4 & -7 \end{bmatrix}$$

18. $[A \;\vdots\; I] = \left[\begin{array}{cc:cc} 2 & 4 & 1 & 0 \\ 4 & 8 & 0 & 1 \end{array}\right]$

$$-2R_1 + R_2 \rightarrow \left[\begin{array}{cc:cc} 2 & 4 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right]$$

The two zeros in the second row imply that the inverse does not exist.

20. $A = \begin{bmatrix} -2 & 5 \\ 6 & -15 \\ 0 & 1 \end{bmatrix}$ A has no inverse because it is not square.

22. $[A \;\vdots\; I] = \left[\begin{array}{ccc:ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{array}\right]$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & -2 & -5 & 1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} -2R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & -4 & 7 & -2 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right]$$

$$\begin{array}{r} 4R_3 + R_1 \rightarrow \\ -3R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -13 & 6 & 4 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right] = [I \;\vdots\; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$$

24. $[A \ \vdots \ I] = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 5 & 5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -3R_1 + R_2 \longrightarrow \\ -2R_1 + R_3 \longrightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 \\ 0 & 5 & 5 & -2 & 0 & 1 \end{array}\right]$

Since the first three entries of row 2 are all zeros, the inverse of A does not exist.

26.

$$[A \ \vdots \ I] = \left[\begin{array}{cccc:cccc} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 6 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{1}{2}R_2 \longrightarrow \\ \\ \frac{1}{5}R_4 \longrightarrow \end{array} \left[\begin{array}{cccc:cccc} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

$$\begin{array}{r} -3R_2 + R_1 \longrightarrow \\ R_3 + R_2 \longrightarrow \\ -R_4 + R_3 \longrightarrow \\ \\ \end{array} \left[\begin{array}{cccc:cccc} 1 & 0 & -8 & -9 & 1 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 0 & 4 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 1 & -\frac{1}{5} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

$$\begin{array}{r} -4R_3 + R_1 \longrightarrow \\ -4R_4 + R_2 \longrightarrow \\ \frac{1}{2}R_3 \longrightarrow \\ \\ \end{array} \left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 1 & -\frac{3}{2} & -4 & \frac{4}{5} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 & -\frac{4}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{10} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

$$\begin{array}{r} 9R_4 + R_1 \longrightarrow \\ \\ \\ \\ \end{array} \left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 1 & -\frac{3}{2} & -4 & \frac{13}{5} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 & -\frac{4}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{10} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right] = [I \ \vdots \ A^{-1}]$$

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 10 & -15 & -40 & 26 \\ 0 & 5 & 10 & -8 \\ 0 & 0 & -5 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

28. $A = \begin{bmatrix} 10 & 5 & -7 \\ -5 & 1 & 4 \\ 3 & 2 & -2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -10 & -4 & 27 \\ 2 & 1 & -5 \\ -13 & -5 & 35 \end{bmatrix}$

30. $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ -4 & 4 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -2 & 0 \\ 14 & -17 & 2 \\ -16 & 20 & -2 \end{bmatrix}$

32. $A = \begin{bmatrix} -\frac{5}{6} & \frac{1}{3} & \frac{11}{6} \\ 0 & \frac{2}{3} & 2 \\ 1 & -\frac{1}{2} & -\frac{5}{2} \end{bmatrix}$ A^{-1} does not exist.

34. $A = \begin{bmatrix} 0.6 & 0 & -0.3 \\ 0.7 & -1 & 0.2 \\ 1 & 0 & -0.9 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3.75 & 0 & -1.25 \\ 3.4583 & -1 & -1.375 \\ 4.1667 & 0 & -2.5 \end{bmatrix}$

36. $A = \begin{bmatrix} 4 & 8 & -7 & 14 \\ 2 & 5 & -4 & 6 \\ 0 & 2 & 1 & -7 \\ 3 & 6 & -5 & 10 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 27 & -10 & 4 & -29 \\ -16 & 5 & -2 & 18 \\ -17 & 4 & -2 & 20 \\ -7 & 2 & -1 & 8 \end{bmatrix}$

38. $A = \begin{bmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 11 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -24 & 7 & 1 & -2 \\ -10 & 3 & 0 & -1 \\ -29 & 7 & 3 & -2 \\ 12 & -3 & -1 & 1 \end{bmatrix}$

40. $\begin{bmatrix} -12 & 3 \\ 5 & -2 \end{bmatrix}^{-1} = \dfrac{1}{(-12)(-2) - (5)(3)} \begin{bmatrix} -2 & -3 \\ -5 & 12 \end{bmatrix} = \dfrac{1}{9} \begin{bmatrix} -2 & -3 \\ -5 & -12 \end{bmatrix}$

42. $\begin{bmatrix} -\frac{1}{4} & \frac{9}{4} \\ \frac{5}{3} & \frac{8}{9} \end{bmatrix}^{-1} = \dfrac{1}{\left(-\frac{1}{4}\right)\left(\frac{8}{9}\right) - \left(\frac{5}{3}\right)\left(\frac{9}{4}\right)} \begin{bmatrix} \frac{8}{9} & -\frac{9}{4} \\ -\frac{5}{3} & -\frac{1}{4} \end{bmatrix} = \dfrac{-36}{143} \begin{bmatrix} \frac{8}{9} & -\frac{9}{4} \\ -\frac{5}{3} & -\frac{1}{4} \end{bmatrix}$

$= \dfrac{1}{143} \begin{bmatrix} -32 & 81 \\ 60 & 9 \end{bmatrix}$

44. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

Answer: $(6, 3)$

46. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \end{bmatrix}$

Answer: $(-7, -4)$

48. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ -9 \end{bmatrix}$

Answer: $(1, 7, -9)$

50. $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -24 & 7 & 1 & -2 \\ -10 & 3 & 0 & -1 \\ -29 & 7 & 3 & -2 \\ 12 & -3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} -32 \\ -13 \\ -37 \\ 15 \end{bmatrix}$

Answer: $(-32, -13, -37, 15)$

52. $A = \begin{bmatrix} 18 & 12 \\ 30 & 24 \end{bmatrix}$. $A^{-1} = \dfrac{1}{18(24) - 12(30)} \begin{bmatrix} 24 & -12 \\ -30 & 18 \end{bmatrix} = \dfrac{1}{12} \begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}b = \dfrac{1}{12} \begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 13 \\ 23 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$

54. $A = \begin{bmatrix} 0.2 & -0.6 \\ -1 & 1.4 \end{bmatrix}$. $A^{-1} = \dfrac{-1}{8} \begin{bmatrix} 35 & 15 \\ 25 & 5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{-1}{8} \begin{bmatrix} 35 & 15 \\ 25 & 5 \end{bmatrix} \begin{bmatrix} 2.4 \\ -8.8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

56. $A = \begin{bmatrix} \frac{5}{6} & -1 \\ \frac{4}{3} & -\frac{7}{2} \end{bmatrix}$. $A^{-1} = \dfrac{1}{\left(\frac{5}{6}\right)\left(-\frac{7}{2}\right) - \left(\frac{4}{3}\right)(-1)} \begin{bmatrix} -\frac{7}{2} & 1 \\ -\frac{4}{3} & \frac{5}{6} \end{bmatrix} = \dfrac{-12}{19} \begin{bmatrix} -\frac{7}{2} & 1 \\ -\frac{4}{3} & \frac{5}{6} \end{bmatrix}$

$= \dfrac{1}{19} \begin{bmatrix} 42 & -12 \\ 16 & -10 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{19} \begin{bmatrix} 42 & -12 \\ 16 & -10 \end{bmatrix} \begin{bmatrix} -20 \\ -51 \end{bmatrix} = \begin{bmatrix} -12 \\ 10 \end{bmatrix}$

58. $A = \begin{bmatrix} 4 & -2 & 3 \\ 2 & 2 & 5 \\ 8 & -5 & -2 \end{bmatrix}$

$A^{-1} = \dfrac{1}{82}\begin{bmatrix} -21 & 19 & 16 \\ -44 & 32 & 14 \\ 26 & -4 & -12 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{1}{82}\begin{bmatrix} -21 & 19 & 16 \\ -44 & 32 & 14 \\ 26 & -4 & -12 \end{bmatrix}\begin{bmatrix} -2 \\ 16 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ -2 \end{bmatrix}$

Answer: $(5, 8, -2)$

60. $A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 5 & -9 \\ 5 & 9 & 17 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 7 \\ 13 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$

62. $A = \begin{bmatrix} -8 & 7 & -10 \\ 12 & 3 & -5 \\ 15 & -9 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -151 \\ 86 \\ 187 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B = \begin{bmatrix} 10 \\ -3 \\ 5 \end{bmatrix}$

64. $A = \begin{bmatrix} 2 & 5 & 0 & 1 \\ 1 & 4 & 2 & -2 \\ 2 & -2 & 5 & 1 \\ 1 & 0 & 0 & -3 \end{bmatrix}$

$A^{-1} \approx \begin{bmatrix} 0.338 & -0.352 & 0.141 & 0.394 \\ 0.042 & 0.164 & -0.066 & -0.117 \\ -0.141 & 0.230 & 0.108 & -0.164 \\ 0.113 & -0.117 & 0.047 & -0.202 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0.338 & -0.352 & 0.141 & 0.394 \\ 0.042 & 0.164 & -0.066 & -0.117 \\ -0.141 & 0.230 & 0.108 & -0.164 \\ 0.113 & -0.117 & 0.047 & -0.202 \end{bmatrix}\begin{bmatrix} 11 \\ -7 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 6.21 \\ -0.77 \\ -2.67 \\ 2.40 \end{bmatrix}$

Answer: $(6.21, -0.77, -2.67, 2.40)$

For 66 and 68 use $A = \begin{bmatrix} 1 & 1 & 1 \\ 0.065 & 0.07 & 0.09 \\ 0 & 2 & -1 \end{bmatrix}$. Using the methods of this section, we have $A^{-1} = \frac{1}{11}\begin{bmatrix} 50 & -600 & -4 \\ -13 & 200 & 5 \\ -26 & 400 & -1 \end{bmatrix}$.

66. $X = A^{-1}B = \dfrac{1}{11}\begin{bmatrix} 50 & -600 & -4 \\ -13 & 200 & 5 \\ -26 & 400 & -1 \end{bmatrix}\begin{bmatrix} 45{,}000 \\ 3750 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 15{,}000 \\ 30{,}000 \end{bmatrix}$

Answer: \$0 in AAA bonds, \$15,000 in A bonds, and \$30,000 in B bonds.

68. $X = A^{-1}B = \dfrac{1}{11}\begin{bmatrix} 50 & -600 & -4 \\ -13 & 200 & 5 \\ -26 & 400 & -1 \end{bmatrix}\begin{bmatrix} 500{,}000 \\ 38{,}000 \\ 0 \end{bmatrix} = \begin{bmatrix} 200{,}000 \\ 100{,}000 \\ 200{,}000 \end{bmatrix}$

Answer: \$200,000 in AAA bonds, \$100,000 in A bonds, and \$200,000 in B bonds.

70. $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{bmatrix}$

$$A^{-1} = \frac{1}{14}\begin{bmatrix} 5 & -4 & 4 \\ -4 & 6 & 8 \\ 1 & 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 5 & -4 & 4 \\ -4 & 6 & 8 \\ 1 & 2 & -2 \end{bmatrix}\begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix} = \begin{matrix} \frac{5}{7} \\ \frac{10}{7} \\ \frac{15}{7} \end{matrix}$$

Answer: $I_1 = \frac{5}{7}$ amps, $I_2 = \frac{10}{7}$ amps, $I_3 = \frac{15}{7}$ amps

72. False. Consider:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The product is the identity, but the matrices are not inverses of each other (not $n \times n$).

74. The inverse matrix remained the same for each system.

76. (a) Given $A = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$, $A^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 \\ 0 & \frac{1}{a_{22}} \end{bmatrix}$, $a_{11} \neq 0, a_{22} \neq 0$

Given $A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$, $A^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & 0 \\ 0 & \frac{1}{a_{22}} & 0 \\ 0 & 0 & \frac{1}{a_{33}} \end{bmatrix}$, $a_{11}, a_{22}, a_{33} \neq 0$

(b) In general, the inverse of the diagonal matrix A is

$$\begin{bmatrix} \frac{1}{a_{11}} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{a_{22}} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{a_{33}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{a_{nn}} \end{bmatrix} \text{(assuming } a_{ii} \neq 0\text{)}$$