CHAPTER 9
Sequences, Series, and Probability

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Section 9.1 Sequences and Series

Solutions to Even-Numbered Exercises

2. $a_n = 4n - 7$

$a_1 = 4(1) - 7 = -3$

$a_2 = 4(2) - 7 = 1$

$a_3 = 4(3) - 7 = 5$

$a_4 = 4(4) - 7 = 9$

$a_5 = 4(5) - 7 = 13$

4. $a_n = \left(\frac{1}{2}\right)^n$

$a_1 = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$

$a_2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$a_3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$a_4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$a_5 = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

6. $a_n = \left(-\frac{1}{2}\right)^n$

$a_1 = \left(-\frac{1}{2}\right)^1 = -\frac{1}{2}$

$a_2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

$a_3 = \left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$

$a_4 = \left(-\frac{1}{2}\right)^4 = \frac{1}{16}$

$a_5 = \left(-\frac{1}{2}\right)^5 = -\frac{1}{32}$

8. $a_n = \dfrac{n}{n+1}$

$a_1 = \dfrac{1}{1+1} = \dfrac{1}{2}$

$a_2 = \dfrac{2}{2+1} = \dfrac{2}{3}$

$a_3 = \dfrac{3}{3+1} = \dfrac{3}{4}$

$a_4 = \dfrac{4}{4+1} = \dfrac{4}{5}$

$a_5 = \dfrac{5}{5+1} = \dfrac{5}{6}$

10. $a_n = \dfrac{3n^2 - n + 4}{2n^2 + 1}$

$a_1 = \dfrac{3(1)^2 - 1 + 4}{2(1)^2 + 1} = 2$

$a_2 = \dfrac{3(2)^2 - 2 + 4}{2(2)^2 + 1} = \dfrac{14}{9}$

$a_3 = \dfrac{3(3) - 3 + 4}{2(3)^2 + 1} = \dfrac{28}{19}$

$a_4 = \dfrac{3(4) - 4 + 4}{2(4)^2 + 1} = \dfrac{16}{11}$

$a_5 = \dfrac{3(5)^2 - 5 + 4}{2(5)^2 + 1} = \dfrac{74}{51}$

12. $a_n = \dfrac{1 + (-1)^n}{2n}$.

$a_1 = \dfrac{1-1}{2} = 0$

$a_2 = \dfrac{1+1}{2(2)} = \dfrac{1}{2}$

$a_3 = \dfrac{1-1}{2(3)} = 0$

$a_4 = \dfrac{1+1}{2(4)} = \dfrac{1}{4}$

$a_5 = \dfrac{1-1}{2(5)} = 0$

14. $a_n = \dfrac{3^n}{4^n}$

$a_1 = \dfrac{3^1}{4^1} = \dfrac{3}{4}$

$a_2 = \dfrac{3^2}{4^2} = \dfrac{9}{16}$

$a_3 = \dfrac{3^3}{4^3} = \dfrac{27}{64}$

$a_4 = \dfrac{3^4}{4^4} = \dfrac{81}{256}$

$a_5 = \dfrac{3^5}{4^5} = \dfrac{243}{1024}$

16. $a_n = \dfrac{10}{n^{2/3}} = \dfrac{10}{\sqrt[3]{n^2}}$

$a_1 = \dfrac{10}{1} = 10$

$a_2 = \dfrac{10}{\sqrt[3]{2^2}} = \dfrac{10}{\sqrt[3]{4}}$

$a_3 = \dfrac{10}{\sqrt[3]{3^2}} = \dfrac{10}{\sqrt[3]{9}}$

$a_4 = \dfrac{10}{\sqrt[3]{4^2}} = \dfrac{10}{\sqrt[3]{16}}$

$a_5 = \dfrac{10}{\sqrt[3]{5^2}} = \dfrac{10}{\sqrt[3]{25}}$

18. $a_n = \dfrac{n!}{2^n}$

$a_1 = \dfrac{1}{2}$

$a_2 = \dfrac{2}{4} = \dfrac{1}{2}$

$a_3 = \dfrac{6}{8} = \dfrac{3}{4}$

$a_4 = \dfrac{24}{16} = \dfrac{3}{2}$

$a_5 = \dfrac{120}{32} = \dfrac{15}{4}$

20. $a_n = (-1)^n\left(\dfrac{n}{n+1}\right)$

$a_1 = (-1)^1\dfrac{1}{1+1} = -\dfrac{1}{2}$

$a_2 = (-1)^2\dfrac{2}{1+2} = \dfrac{2}{3}$

$a_3 = (-1)^3\dfrac{3}{3+1} = -\dfrac{3}{4}$

$a_4 = (-1)^4\dfrac{4}{4+1} = \dfrac{4}{5}$

$a_5 = (-1)^5\dfrac{5}{5+1} = -\dfrac{5}{6}$

22. $a_n = n(n-1)(n-2)$

$a_1 = 1(1-1)(1-2) = 0$

$a_2 = 2(2-1)(2-2) = 0$

$a_3 = 3(3-1)(3-2) = 6$

$a_4 = 4(4-1)(4-2) = 24$

$a_5 = 5(5-1)(5-2) = 60$

24. $a_{16} = (-1)^{15}[16(15)] = -240$

26. $a_8 = \dfrac{8!}{16} = 2520$

28. $a_{15} = \dfrac{4(15)^2 - 15 + 3}{15(14)(17)} = \dfrac{888}{3570} = \dfrac{148}{595}$

30. $a_1 = 15, \quad a_{k+1} = a_k + 3$

$a_1 = 15$

$a_2 = a_1 + 3 = 15 + 3 = 18$

$a_3 = a_2 + 3 = 18 + 3 = 21$

$a_4 = a_3 + 3 = 21 + 3 = 24$

$a_5 = a_4 + 3 = 24 + 3 = 27$

32. $a_1 = 32, \quad a_{k+1} = \dfrac{1}{2}a_k$

$a_1 = 32$

$a_2 = \dfrac{1}{2}a_1 = \dfrac{1}{2}(32) = 16$

$a_3 = \dfrac{1}{2}a_2 = \dfrac{1}{2}(16) = 8$

$a_4 = \dfrac{1}{2}a_3 = \dfrac{1}{2}(8) = 4$

$a_5 = \dfrac{1}{2}a_4 = \dfrac{1}{2}(4) = 2$

34. $a_1 = 52, \; a_2 = 40,$

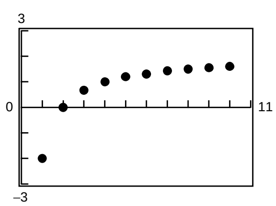
$a_{k+2} = \frac{1}{2}a_{k+1} - a_k$

$a_3 = \frac{1}{2}(40) - 52 = -32$

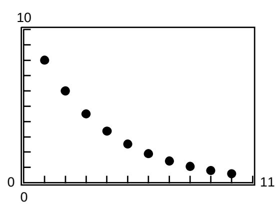
$a_4 = \frac{1}{2}(-32) - 40 = -56$

$a_5 = \frac{1}{2}(-56) + 32 = 4$

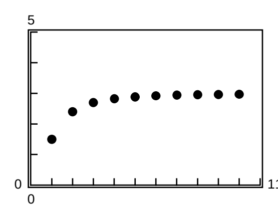
36. $a_n = 2 - \dfrac{4}{n}$



38. $a_n = 8(0.75)^{n-1}$



40. $a_n = \dfrac{3n^2}{n^2 + 1}$



42. $a_n = 2n(n+1)(n+2)$

n	1	2	3	4	5	6	7	8	9	10
a_n	12	48	120	240	420	672	1008	1440	1980	2640

44. $a_n = \dfrac{n!}{(n^2 - 10)}$

n	1	2	3	4	5	6	7	8	9	10
a_n	$-\frac{1}{9}$	$-\frac{1}{3}$	-6	4	8	27.69	129.23	746.67	5110.99	40320

46. $a_n = \dfrac{4n^2}{(n + 2)}$

n	1	2	3	4	5	6	7	8	9	10
a_n	$\frac{4}{3}$	4	7.2	10.67	14.29	18	21.78	25.6	29.45	33.33

48. $a_n = \dfrac{8n}{n + 1}$

$a_n \longrightarrow 8$ as $n \longrightarrow \infty$

$a_1 = 4, \quad a_4 = \dfrac{8(4)}{9} = \dfrac{32}{9}$

Matches graph (b).

50. $a_n = \dfrac{4^n}{n!}$

$a_n \longrightarrow 0$ as $n \longrightarrow \infty$

$a_1 = 4, a_4 = \dfrac{4^4}{4!} = \dfrac{256}{24} = 10\dfrac{2}{3}$

Matches graph (a).

52. $3, 7, 11, 15, 19, \ldots$

$a_n = 4n - 1$

54. $1, \dfrac{1}{4}, \dfrac{1}{9}, \dfrac{1}{16}, \dfrac{1}{25}, \ldots$

$a_n = \dfrac{1}{n^2}$

56. $\dfrac{2}{1}, \dfrac{3}{3}, \dfrac{4}{5}, \dfrac{5}{7}, \dfrac{6}{9}, \ldots$

$a_n = \dfrac{n + 1}{2n - 1}$

58. $\dfrac{1}{3}, \dfrac{-2}{9}, \dfrac{4}{27}, \dfrac{-8}{81}, \ldots$

$a_n = \dfrac{(-1)^{n+1}2^{n-1}}{3^n} = \dfrac{(-2)^{n-1}}{3^n}$

60. $1 + \dfrac{1}{2}, 1 + \dfrac{3}{4}, 1 + \dfrac{7}{8}, 1 + \dfrac{15}{16}, 1 + \dfrac{31}{32}, \ldots$

$a_n = 1 + \dfrac{2^n - 1}{2^n}$

62. $1, 2, \dfrac{2^2}{2}, \dfrac{2^3}{6}, \dfrac{2^4}{24}, \dfrac{2^5}{120}, \ldots$

$a_n = \dfrac{2^{n-1}}{(n - 1)!}$

64. $1, -1, 1, -1, 1, -1, \ldots$

$a_n = (-1)^{n+1}$

66. $a_1 = 25, \quad a_{k+1} = a_k - 5$

$a_1 = 25$

$a_2 = a_1 - 5 = 25 - 5 = 20$

$a_3 = a_2 - 5 = 20 - 5 = 15$

$a_4 = a_3 - 5 = 15 - 5 = 10$

$a_5 = a_4 - 5 = 10 - 5 = 5$

In general, $a_n = 30 - 5n$.

68. $a_1 = 14, \quad a_{k+1} = (-2)a_k$

$a_1 = 14$

$a_2 = (-2)a_1 = (-2)(14) = -28$

$a_3 = (-2)a_2 = (-2)(-28) = 56$

$a_4 = (-2)a_3 = (-2)(56) = -112$

$a_5 = (-2)(a_4) = (-2)(-112) = 224$

In general, $a_n = 14(-2)^{n-1}$.

70. $\dfrac{4!}{7!} = \dfrac{4!}{7 \cdot 6 \cdot 5 \cdot 4!} = \dfrac{1}{210}$

72. $\dfrac{25!}{23!} = \dfrac{25 \cdot 24 \cdot 23!}{23!} = 600$

74. $\dfrac{10!\,3!}{4!\,6!} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6! \cdot 3!}{4 \cdot 3! \cdot 6!} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7}{4} = 1260$

76. $\dfrac{(n + 2)!}{n!} = \dfrac{(n + 2)(n + 1)n!}{n!} = (n + 2)(n + 1)$

78. $\dfrac{(2n + 2)!}{(2n)!} = \dfrac{(2n + 2)(2n + 1)(2n)!}{(2n)!}$

$= (2n + 2)(2n + 1)$

80. $\displaystyle\sum_{i=1}^{6} (3i - 1) = (3 \cdot 1 - 1) + (3 \cdot 2 - 1) + (3 \cdot 3 - 1) + (3 \cdot 4 - 1) + (3 \cdot 5 - 1) + (3 \cdot 6 - 1) = 57$

82. $\sum_{k=1}^{5} 6 = 6 + 6 + 6 + 6 + 6 = 30$

84. $\sum_{k=0}^{5} 3i^2 = 3\sum_{i=0}^{5} i^2 = 3(0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2) = 165$

86. $\sum_{j=3}^{5} \frac{1}{j} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$

88. $\sum_{k=2}^{5} (k + 1)(k - 3) = (2 + 1)(2 - 3) + (3 + 1)(3 - 3) + (4 + 1)(4 - 3) + (5 + 1)(5 - 3) = 14$

90. $\sum_{j=0}^{4} (-2)^j = (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4 = 11$

92. $\sum_{j=1}^{10} \frac{3}{j + 1} \approx 6.06$

94. $\sum_{k=0}^{4} \frac{(-1)^k}{k!} = \frac{3}{8} = 0.375$

96. $\frac{5}{1 + 1} + \frac{5}{1 + 2} + \frac{5}{1 + 3} + \cdots + \frac{5}{1 + 15} = \sum_{i=1}^{15} \frac{5}{1 + i} \approx 11.904$

98. $\left[1 - \left(\frac{1}{6}\right)^2\right] + \left[1 - \left(\frac{2}{6}\right)^2\right] + \cdots + \left[1 - \left(\frac{6}{6}\right)^2\right] = \sum_{k=1}^{6}\left[1 - \left(\frac{k}{6}\right)^2\right] \approx 3.472$

100. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{128} = \frac{1}{2^0} - \frac{1}{2^1} + \frac{1}{2^2} - \frac{1}{2^3} + \cdots - \frac{1}{2^7} = \sum_{n=0}^{7}\left(-\frac{1}{2}\right)^n \approx 0.664$

102. $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{10 \cdot 12} = \sum_{k=1}^{10} \frac{1}{k(k + 2)} \approx 0.663$

104. $\frac{1}{2} + \frac{2}{4} + \frac{6}{8} + \frac{24}{16} + \frac{120}{32} + \frac{720}{64} = \sum_{k=1}^{6} \frac{k!}{2^k} = 18.25$

106. $\sum_{i=1}^{5} 2\left(\frac{1}{3}\right)^i = \frac{242}{243} \approx 0.9959$

108. $\sum_{n=1}^{4} 8\left(-\frac{1}{4}\right)^n = \frac{-51}{32} \approx -1.59375$

110. $\sum_{k=1}^{\infty} \frac{4}{10^k} = 4[0.1 + 0.01 + 0.001 + \cdots] = 4[0.111\cdots] = 4\left(\frac{1}{9}\right) = \frac{4}{9}$

112. $\sum_{i=1}^{\infty} 2\left(\frac{1}{10}\right)^i = 2[0.1 + 0.01 + 0.001 + \cdots] = 2[0.111\cdots] = 2\left(\frac{1}{9}\right) = \frac{2}{9}$

114. (a) $A_1 = 100(101)[(1.01)^1 - 1] = \$101.00$

$A_2 = 100(101)[(1.01)^2 - 1] = \$203.01$

$A_3 = 100(101)[(1.01)^3 - 1] \approx \$306.04$

$A_4 = 100(101)[(1.01)^4 - 1] \approx \$410.10$

$A_5 = 100(101)[(1.01)^5 - 1] \approx \$515.20$

$A_6 = 100(101)[(1.01)^6 - 1] \approx \$621.35$

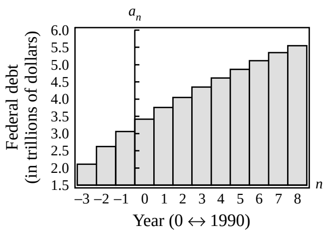
(b) $A_{60} = 100(101)[(1.01)^{60} - 1] \approx \$8248.64$

(c) $A_{240} = 100(101)[(1.01)^{240} - 1] \approx \$99{,}914.79$

116. $a_n\sqrt{11.7 + 2.4n},\ n = -3, -2, \ldots, 8$

n	-3	-2	-1	0	1	2	3	4	5	6	7	8
a_n	2.12	2.63	3.05	3.42	3.75	4.06	4.35	4.62	4.87	5.11	5.34	5.56

The debt is growing as time goes on.



118. $a_n = 4.27 + 0.29n - 2.93\ln(n),\ n = 10, 11, \ldots, 18$

$$\sum_{n=10}^{\infty} a_n \approx \$5.84$$

120. True: $\displaystyle\sum_{j=1}^{4} 2^j = 2^1 + 2^2 + 2^3 + 2^4 = \sum_{j=3}^{6} 2^{j-2}$

122. $$b_n = \frac{a_{n+1}}{a_n} = \frac{a_n + a_{n-1}}{a_n} = 1 + \frac{a_{n-1}}{a_n} = 1 + \frac{1}{\dfrac{a_n}{a_{n-1}}} = 1 + \frac{1}{b_{n-1}}$$

124. $a_n = \dfrac{x^n}{n!}$

$a_1 = \dfrac{x}{1} = x$

$a_2 = \dfrac{x^2}{2!} = \dfrac{x^2}{2}$

$a_3 = \dfrac{x^3}{3!} = \dfrac{x^3}{6}$

$a_4 = \dfrac{x^4}{4!} = \dfrac{x^4}{24}$

$a_5 = \dfrac{x^5}{5!} = \dfrac{x^5}{120}$

126. $a_n = \dfrac{(-1)^n x^{2n}}{(2n)!}$

$a_1 = \dfrac{-x^2}{2}$

$a_2 = \dfrac{x^4}{4!} = \dfrac{x^4}{24}$

$a_3 = \dfrac{-x^6}{6!} = \dfrac{-x^6}{720}$

$a_4 = \dfrac{x^8}{8!} = \dfrac{x^8}{40{,}320}$

$a_5 = \dfrac{-x^{10}}{10!} = \dfrac{-x^{10}}{3{,}628{,}800}$

128. $\left[\begin{array}{rr:r} -4 & 1 & -7 \\ 6 & -9 & 3 \end{array}\right]$

130. (a) $A - B = \begin{bmatrix} 8 & 1 \\ -3 & 7 \end{bmatrix}$

(b) $2B - 3A = \begin{bmatrix} -22 & -7 \\ 3 & -18 \end{bmatrix}$

(c) $AB = \begin{bmatrix} 18 & 9 \\ 18 & 0 \end{bmatrix}$

(d) $BA = \begin{bmatrix} 0 & 6 \\ 27 & 18 \end{bmatrix}$