92. False. You need to know how many terms are in the sequence.

94. $a_1 = -y$ $\quad a_6 = 24y$

$a_2 = -y + 5y = 4y$ $\quad a_7 = 29y$

$a_3 = 9y$ $\quad a_8 = 34y$

$a_4 = 14y$ $\quad a_9 = 39y$

$a_5 = 19y$ $\quad a_{10} = 44y$

96. $S_{20} = \frac{20}{2}(a_1 + [a_1 + 19(3)]) = 650$

$650 = 10(2a_1 + 57)$

$65 = 2a_1 + 57$

$a_1 = 4$

98. $\begin{bmatrix} 2 & -1 & 7 & \vdots & -10 \\ 3 & 2 & -4 & \vdots & 17 \\ 6 & -5 & 1 & \vdots & -20 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & 5 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix}$

Answer: $(1, 5, -1)$

100. $\begin{vmatrix} 0 & 0 & 1 \\ 4 & -3 & 1 \\ 2 & 6 & 1 \end{vmatrix} = 30$

Area $= \frac{1}{2}(30) = 15$ square units

102. $\dfrac{6!}{5!\,2!} = \dfrac{6 \cdot 5!}{5!\,2} = 3$

Section 9.3 Geometric Sequences and Series

Solutions to Even-Numbered Exercises

2. $3, 15, 75, 375, \ldots$

Geometric sequence, $r = 5$

4. $1, -2, 4, -8, \ldots$

Geometric sequence, $r = -2$

6. $3, 0.6, 0.12, 0.024$

Geometric sequence, $r = 0.2$

8. $9, -6, 4, -\frac{8}{3}, \ldots$

Geometric sequence, $r = -\frac{2}{3}$

10. $\frac{1}{5}, \frac{2}{7}, \frac{3}{9}, \frac{4}{11}, \ldots$

Not a geometric sequence

12. $a_1 = 10, r = 2$

$a_2 = 10(2) = 20$

$a_3 = 20(2) = 40$

$a_4 = 40(2) = 80$

$a_5 = 80(2) = 160$

14. $a_1 = 2, r = \frac{1}{3}$

$a_2 = 2\left(\frac{1}{3}\right) = \frac{2}{3}$

$a_3 = \frac{2}{3}\left(\frac{1}{3}\right) = \frac{2}{9}$

$a_4 = \frac{2}{9}\left(\frac{1}{3}\right) = \frac{2}{27}$

$a_5 = \frac{2}{27}\left(\frac{1}{3}\right) = \frac{2}{81}$

16. $a_1 = 6, r = -\frac{1}{4}$

$a_1 = 6$

$a_2 = 6\left(-\frac{1}{4}\right)^1 = -\frac{3}{2}$

$a_3 = 6\left(-\frac{1}{4}\right)^2 = \frac{3}{8}$

$a_4 = 6\left(-\frac{1}{4}\right)^3 = -\frac{3}{32}$

$a_5 = 6\left(-\frac{1}{4}\right)^4 = \frac{3}{128}$

18. $a_1 = 0.4 = \frac{2}{5}, r = \frac{5}{2}$

$a_2 = \frac{2}{5}\left(\frac{5}{2}\right) = 1$

$a_3 = 1\left(\frac{5}{2}\right) = \frac{5}{2}$

$a_4 = \frac{5}{2}\left(\frac{5}{2}\right) = \frac{25}{4}$

$a_5 = \frac{25}{4}\left(\frac{5}{2}\right) = \frac{125}{8}$

20. $a_1 = 4, r = \sqrt{3}$

$a_2 = 4\sqrt{3}$

$a_3 = 4\sqrt{3}(\sqrt{3}) = 12$

$a_4 = 12(\sqrt{3}) = 12\sqrt{3}$

$a_5 = 12\sqrt{3}(\sqrt{3}) = 36$

22. $a_1 = 81, a_{k+1} = \frac{1}{3}a_k$

$a_1 = 81$

$a_2 = \frac{1}{3}(81) = 27$

$a_3 = \frac{1}{3}(27) = 9$

$a_4 = \frac{1}{3}(9) = 3$

$a_5 = \frac{1}{3}(3) = 1$

$r = \frac{1}{3}, a_n = 243\left(\frac{1}{3}\right)^n$

24. $a_1 = 5, a_{k+1} = -2a_k$

$a_1 = 5$

$a_2 = -2(5) = -10$

$a_3 = -2(-10) = 20$

$a_4 = -2(20) = -40$

$a_5 = -2(-40) = 80$

$r = -2, a_n = 5(-2)^{n-1}$

26. $a_1 = 36, \quad a_{k+1} = -\frac{2}{3}a_k$

$a_1 = 36$

$a_2 = -\frac{2}{3}(36) = -24$

$a_3 = -\frac{2}{3}(-24) = 16$

$a_4 = -\frac{2}{3}(16) = -\frac{32}{3}$

$a_5 = -\frac{2}{3}\left(-\frac{32}{3}\right) = \frac{64}{9}$

$r = -\frac{2}{3}, a_n = 36\left(-\frac{2}{3}\right)^{n-1}$

28. $a_1 = 5, \quad r = \frac{3}{2}, \quad n = 8$

$a_n = a_1 r^{n-1}$

$a_8 = 5\left(\frac{3}{2}\right)^7 = \frac{10,935}{128}$

30. $a_1 = 8, \quad r = \sqrt{5}, \quad n = 9$

$a_n = a_1 r^{n-1}$

$a_9 = 8\left(\sqrt{5}\right)^8 = 5000$

32. $a_1 = 1000, \quad r = 1.005,$

$n = 11$

$a_n = a_1 r^{n-1}$

$a_{11} = 1000(1.005)^{10}$

≈ 1051.14

34. $a_2 = 3, \quad a_5 = \frac{3}{64}, \quad n = 1$

$a_2 r^3 = a_5$

$3r^3 = \frac{3}{64}$

$r^3 = \frac{1}{64}$

$r = \frac{1}{4}$

$a_2 = a_1 r$

$3 = a_1\left(\frac{1}{4}\right)$

$a_1 = 12$

36. $a_3 = \frac{16}{3}, \quad a_5 = \frac{64}{27}, \quad n = 7$

$a_3 r^2 = a_5$

$\frac{16}{3} r^2 = \frac{64}{27}$

$r^2 = \frac{4}{9}$

$r = \pm\frac{2}{3}$

$a_7 = a_5 r^2 = \frac{64}{27}\left(\pm\frac{2}{3}\right)^2 = \frac{256}{243}$

38. 3, 36, 432

$r = \frac{36}{3} = 12$

$a_7 = 3(12)^6 = 8,957,952$

40. 4, 8, 16, . . .

$r = \frac{8}{4} = 2$

$a_{22} = 4(2)^{21} = 8,388,608$

42. $\frac{1}{2}$, 8, 128

$r = \frac{8}{\left(\frac{1}{2}\right)} = 16$

$a_8 = \frac{1}{2}(16)^7 = 134,217,728$

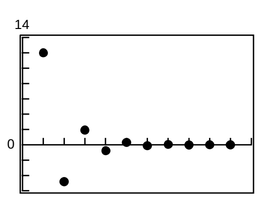
44. $a_n = 18\left(-\frac{2}{3}\right)^{n-1}$

$r = \left(-\frac{2}{3}\right) > -1$, so that the sequence alternates as it approaches 0. Matches (c).

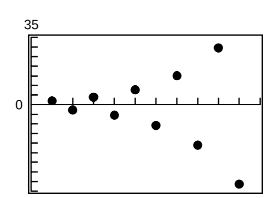
46. $a_n = 18\left(-\frac{3}{2}\right)^{n-1}$

$r = \left(-\frac{3}{2}\right) < -1$, so the sequence alternates as it approaches $\pm\infty$. Matches (d).

48. $a_n = 12(-0.4)^{n-1}$



50. $a_n = 2(-1.4)^{n-1}$



52. $8, 12, 18, 27, \frac{81}{2}, \ldots$

$S_1 = 8$

$S_2 = 8 + 12 = 20$

$S_3 = 8 + 12 + 18 = 38$

$S_4 = 8 + 12 + 18 + 27 = 65$

54. $\sum_{n=1}^{\infty} 4(0.2)^{n-1}$

n	1	2	3	4	5	6	7	8	9	10
S_n	4	4.8	4.96	4.992	4.9984	4.99968	4.999936	4.9999872	≈ 5	≈ 5

56. $\sum_{n=1}^{9} (-2)^{n-1} \Longrightarrow a_1 = 1, r = -2$

$S_9 = \dfrac{1(1-(-2)^9)}{1-(-2)} = 171$

58. $\sum_{i=1}^{6} 32\left(\frac{1}{4}\right)^{i-1} \Longrightarrow a_1 = 32, r = \frac{1}{4}$

$S_6 = 32\dfrac{(1-(1/4)^6)}{1-(1/4)} = \dfrac{1365}{32}$

60. $\sum_{n=0}^{15} 2\left(\frac{4}{3}\right)^n = \sum_{n=1}^{16} 2\left(\frac{4}{3}\right)^{n-1} \Longrightarrow a_1 = 2, r = \frac{4}{3}$

$S_{16} = 2\left(\dfrac{1-(4/3)^{16}}{1-(4/3)}\right) \approx 592.65$

62. $\sum_{i=1}^{10} 5\left(-\frac{1}{3}\right)^{i-1} \Longrightarrow a_1 = 5, r = -\frac{1}{3}$

$S_{10} = 5\left(\dfrac{1-(-1/3)^{10}}{1-(-1/3)}\right) \approx 3.75$

64. $\sum_{n=0}^{6} 500(1.04)^n = \sum_{n=1}^{7} 500(1.04)^{n-1} \Longrightarrow a_1 = 500, r = 1.04$

$S_7 = 500\left(\dfrac{1-(1.04)^7}{1-1.04}\right) \approx 3949.15$

66. $7 + 14 + 28 + \cdots + 896$

$r = 2$ and $896 = 7(2)^{n-1} \Longrightarrow n = 8$

$\sum_{n=1}^{8} 7(2)^{n-1}$

68. $15 - 3 + \frac{3}{5} - \cdots - \frac{3}{625}$

$r = -0.2$ and $-\frac{3}{625} = 15(-0.2)^{n-1} \Longrightarrow n = 6$

$\sum_{n=1}^{6} 15(-0.2)^{n-1}$

70. $a_1 = 2, r = \frac{2}{3}$

$\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n = \dfrac{a_1}{1-r} = \dfrac{2}{1-(2/3)} = 6$

72. $a_1 = 2, r = -\frac{2}{3}$

$\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^n = \dfrac{a_1}{1-r} = \dfrac{2}{1-(-2/3)} = \dfrac{6}{5}$

74. $a_1 = 1, r = \frac{1}{10}$

$\sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n = \dfrac{a_1}{1-r} = \dfrac{1}{1-(1/10)} = \dfrac{10}{9}$

76. $\sum_{n=1}^{\infty} \frac{1}{2}(2)^n$ does not have a finite sum $(2 > 1)$

78. $a_1 = 4, r = 0.2$

$\sum_{n=0}^{\infty} 4(0.2)^n = \dfrac{a_1}{1-r} = \dfrac{4}{1-0.2} = \dfrac{4}{0.8} = 5$

80. $a_1 = -10, r = 0.2$

$\sum_{n=0}^{\infty} -10(0.2)^n = \dfrac{a_1}{1-r} = \dfrac{-10}{1-0.2} = \dfrac{-10}{0.8} = \dfrac{-25}{2}$

82. $9 + 6 + 4 + \frac{8}{3} + \cdots = \sum_{n=0}^{\infty} 9\left(\frac{2}{3}\right)^n = \frac{9}{1 - \frac{2}{3}} = \frac{9}{\frac{1}{3}} = 27$

84. $-6 + 5 - \frac{25}{6} + \frac{125}{36} - \cdots = \sum_{n=0}^{\infty} -6\left(-\frac{5}{6}\right)^n = \frac{-6}{1 - \left(-\frac{5}{6}\right)} = \frac{-6}{\frac{11}{6}} = \frac{-36}{11} \approx -3.2727$

86. $0.\overline{297} = \sum_{n=0}^{\infty} 0.297(0.001)^n = \frac{0.297}{1 - 0.001} = \frac{0.297}{0.999} = \frac{297}{999} = \frac{11}{37}$

88. $1.3\overline{8} = 1.3 + \sum_{n=0}^{\infty} 0.08(0.1)^n = 1.3 + \frac{0.08}{1 - 0.1} = 1.3 + \frac{0.08}{0.9} = 1\frac{3}{10} + \frac{4}{45} = 1\frac{7}{18} = \frac{25}{18}$

90. $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.07}{n}\right)^{n(20)}$

(a) $n = 1,\quad A = 2500(1 + 0.07)^{20} \approx 9674.21$

(b) $n = 2,\quad A = 2500\left(1 + \frac{0.07}{2}\right)^{2(20)} \approx 9898.15$

(c) $n = 4,\quad A = 2500\left(1 + \frac{0.07}{4}\right)^{4(20)} \approx 10{,}015.98$

(d) $n = 12,\quad A = 2500\left(1 + \frac{0.07}{12}\right)^{12(20)} \approx 10{,}096.85$

(e) $n = 365,\quad A = 2500\left(1 + \frac{0.07}{365}\right)^{365(20)} \approx 10{,}136.64$

92. P = population after n years

P_0 = initial population = 350,000

r = rate of increase = 1.3%

n = number of years = 30

$P = P_0(1 + r)^n = 350{,}000(1.013)^{30} \approx 515{,}646$

94. $A = \sum_{n=1}^{60} 50\left(1 + \frac{0.08}{12}\right)^n$

$$= 50\left(\frac{151}{150}\right)\left[\frac{1 - \left(\frac{151}{150}\right)^{60}}{1 - \left(\frac{151}{150}\right)}\right] \approx \$3698.34$$

96. Let $N = 12t$ be the total number of deposits.

$$A = Pe^{r/12} + Pe^{2r/12} + \cdots + Pe^{Nr/12} = \sum_{n=1}^{N} Pe^{r/12 \cdot n}$$

$$= Pe^{r/12}\frac{(1 - (e^{r/12})^N)}{(1 - e^{r/12})}$$

$$= Pe^{r/12}\frac{(1 - (e^{r/12})^{12t})}{1 - e^{r/12}}$$

$$= \frac{Pe^{r/12}(e^{rt} - 1)}{(e^{r/12} - 1)}$$

98. $P = \$75$, $r = 9\%$, $t = 25$ years

(a) Compounded monthly: $A = 75\left[\left(1 + \frac{0.09}{12}\right)^{12(25)} - 1\right]\left(1 + \frac{12}{0.09}\right) \approx \$84{,}714.78$

(b) Compounded continuously: $A = \frac{75e^{0.09/12}(e^{0.09(25)} - 1)}{e^{0.09/12} - 1} \approx \$85{,}196.05$

100. $P = \$20$, $r = 6\%$, $t = 50$ years

(a) Compounded monthly: $A = 20\left[\left(1 + \frac{0.06}{12}\right)^{12(50)} - 1\right]\left(1 + \frac{12}{0.06}\right) \approx \$76{,}122.54$

(b) Compounded continuously: $A = \frac{20e^{0.06/12}(e^{0.06(50)} - 1)}{e^{0.06/12} - 1} \approx \$76{,}533.16$

102. $W = \$2000,\ t = 20,\ r = 9\%$

$$P = W\left(\frac{12}{r}\right)\left[1 - \left(1 + \frac{r}{12}\right)^{-12t}\right]$$

$$P = 2000\left(\frac{12}{0.09}\right)\left[1 - \left(1 + \frac{0.09}{12}\right)^{-12(20)}\right] \approx \$222{,}289.91$$

104. $27^2\left(\frac{1}{9}\right) + 27^2\left(\frac{1}{9}\right)\left(\frac{8}{9}\right) + 27^2\left(\frac{1}{9}\right)\left(\frac{8}{9}\right)^2 + 27^2\left(\frac{1}{9}\right)\left(\frac{8}{9}\right)^2$

$$= \sum_{n=0}^{3} 27^2\left(\frac{1}{9}\right)\left(\frac{8}{9}\right)^n = \frac{2465}{9} \approx 273.89 \text{ square inches}$$

106. (a) Total distance $= \sum_{n=0}^{\infty} 32(0.81)^n - 16 = \dfrac{32}{1 - 0.81} - 16 \approx 152.42$ feet

(b) $t = 1 + 2\sum_{n=1}^{\infty} (0.9)^n = 1 + 2\left[\dfrac{0.9}{1 - 0.9}\right] = 19$ seconds

108. $a_n = 30{,}000(1.05)^{n-1}$

$$T = \sum_{n=1}^{40} 30{,}000(1.05)^{n-1} = 30{,}000\frac{(1 - 1.05^{40})}{(1 - 1.05)} \approx \$3{,}623{,}993.23$$

110. False. You multiply the first term by the common ratio raised to the $(n - 1)$ power.

112. $a_1 = 3,\ r = \dfrac{x}{2}$

$a_2 = 3\left(\dfrac{x}{2}\right) = \dfrac{3x}{2}$

$a_3 = \dfrac{3x}{2}\left(\dfrac{x}{2}\right) = \dfrac{3x^2}{4}$

$a_4 = \dfrac{3x^2}{4}\left(\dfrac{x}{2}\right) = \dfrac{3x^3}{8}$

$a_5 = \dfrac{3x^3}{8}\left(\dfrac{x}{2}\right) = \dfrac{3x^4}{16}$

114. $a_1 = 5,\ r = 2x$

$a_1 = 5$

$a_2 = 5(2x)^1 = 10x$

$a_3 = 5(2x)^2 = 20x^2$

$a_4 = 5(2x)^3 = 40x^3$

$a_5 = 5(2x)^4 = 80x^4$

116. $a_1 = 100,\ r = e^x,\ n = 9$

$a_n = a_1 r^{n-1}$

$a_9 = 100(e^x)^8 = 100e^{8x}$

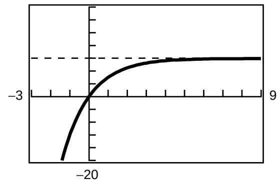
118. $a_1 = 1,\ r = -\dfrac{x}{3},\ n = 7$

$a_n = a_1 r^{n-1}$

$a_7 = 1\left(-\dfrac{x}{3}\right)^6 = \dfrac{x^6}{729}$

120. $f(x) = 6\left[\dfrac{1 - 0.5^x}{1 - 0.5}\right]$

$$\sum_{n=0}^{\infty} 6\left(\frac{1}{2}\right)^n = \frac{6}{1 - \frac{1}{2}} = 12$$



The horizontal asymptote of $f(x)$ is $y = 12$. This corresponds to the sum of the series.